A REGULARIZATION METHOD FOR CALCULATION OF STRESSES, DEFORMATIONS, AND ELASTIC MODES

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A realistic approach using orthogonalization of the structure is presented for obtaining the elastic response of highly redundant airframe systems. The regularized method is an orderly procedure for formulating the mathematical structural idealization in a manner that will require a minimum of calendar time from conception to solution by high-speed digital computers. The method has been under development since 1960, and was directed toward improving the digital analysis procedures developed for and employed on the XB-70 wing. This vehicle provided a realistic application of the analysis methods to a large, redundant, airframe structure. The objectives attained were: (a) realistic accuracy, (b) minimum programming and computer time, (c) reduction of potential program data errors, and (d) rapid cycle design data for configuration optimization. Since static and dynamic analyses requirements are interdependent, the necessary detail of a static analysis can delay the attainment of a compatible dynamic matrix. Conversely, the expedient alternative of approximating elastic characteristics by a coarse grid network analogy sacrifices accuracy for time. Regularization materially reduced the time lag between static and dynamic analyses of redundant structures, and still provides highly accurate solutions.

INTRODUCTION

In recent years there has been extensive effort put forth for more accurate description of the elastic characteristics of aerospace vehicles using finite element approaches for structural analyses. For example, Figure 1 shows the XB-70 structural loading grid system. Most of the existing finite element analysis methods may be classified into two general approaches, the matrix force method and the matrix displacement method. A third approach has been introduced by Klein (Reference 3), which may be classified as a "mixed force-displacement" method. The general matrix methods for indeterminate structural analysis have been documented by Argyris (Reference 4), by Pestel and Leckie (Reference 5), by Gallagher, Rattinger, and Archer (Reference 6) and by Warren, Castle, and Gloria (Reference 7). Matrix analysis methods for complex structures are being extended to include both geometric and material nonlinearities, layered configurations, orthotropic shells, and thermal effects. Within the accuracy of the tools available for the use of these analysis methods, computer programs have the capability of defining the structural characteristics of complex structural configurations. The regularization method is not an attempt to replace or discredit these methods. Regularization is a method for making the refined analysis more correct, reducing both the time and cost for an adequate analysis, and for aiding in the proper configuration selection for the required vehicle mission. The accuracy is obtained by (1) reduction of potential program data errors, (2) rapid cycle design data for configuration optimization, and (3) providing a good boundary loading for detailed stress distribution problem areas.

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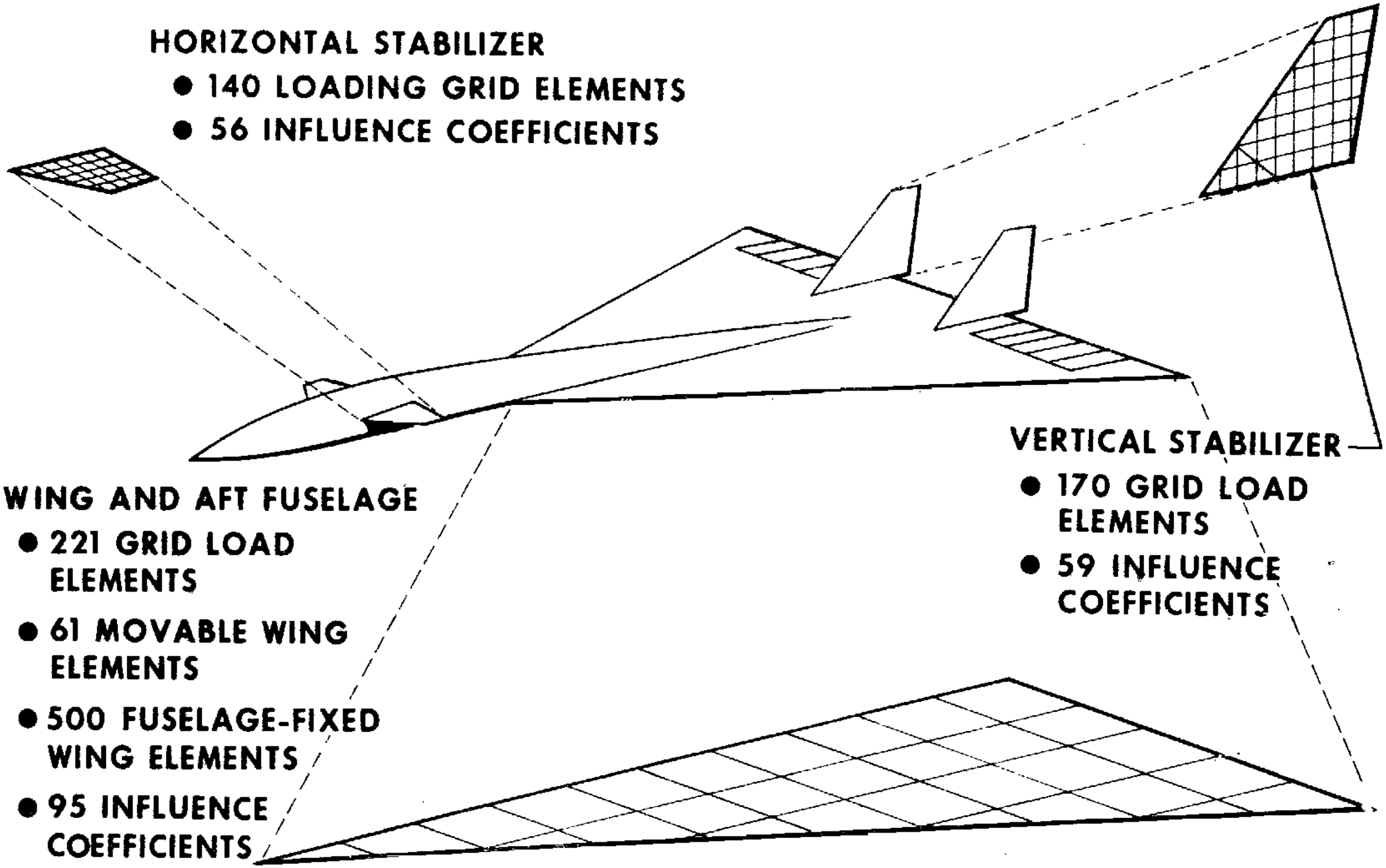


Figure 1. XB-70 Structural Loading - Grid Systems

The flying qualities of large aircraft are extremely dependent upon the deflected shape and the stiffness of the structure under loading. The ability to predict and verify the elastic characteristics of a structure, in order to ensure aerodynamic stability and to achieve desired flying qualities, is required for proper vehicle selection as well as for the detailed analysis that follows.

All redundant structural analysis is basically a measuring device or "yard stick" to solve for the relative ability of the individual bays or elements of the mathematical analogy to react as a structural system. Regularization is a process of keeping the measuring device simple. For an example, a coil spring can be visualized as an element of a structure. Few, if any, internal load analysts would attempt to describe the actual geometric shape of the spring when only the stiffness is needed for the overall problem. A simple axial and/or torsional member would be used to represent the spring. In similar fashion, regularization circumvents the local details of real structure to first obtain the overall solution. The solution may be checked immediately by using three-dimensional Cathode Ray Tube (CRT) plots, such as shown in Figure 2, to detect the presence or absence of discontinuities.

REGULARIZATION RATIONALE

The technique of regularization is the substitution of a simplified geometric analogy of equivalent elemental stiffnesses for the complexities involved in the geometry of the actual structure. The geometric effects are simulated in the regularized analogy while keeping the analogy a measuring device of the actual structural system.

There are two basic principles which lead to checked-out results in a minimum time:

A. Orthogonalization of the structure.

The analogy members are either parallel or perpendicular to all other analogy members. The elements describing the covers are rectangular in shape and it is desirable that they be nearly square (aspect ratio approximately of one).

B. Modification of webs.

Cutouts are simulated by weakened members. If the webs in ribs and spars of the analogy do not exist, the properties assigned these elements are made negligibly small relative to the structural sizing.

When a diagonal member must be used (front spar of a delta-shaped wing) the spacing of the spanwise and chordwise members are set so that the diagonal member intersects these members only at their node points.

With the analogy structure regularized, it follows that the section numbers describing the analogy can be regularized. This is done in the following manner:

a. The section numbers of adjacent parallel members differ by a constant.

b. All other section numbers can be determined by adding a constant to the values in Item a.

For example, in Figure 3, the identification numbers of a shear panel/axial member analogy are determined by nine values. These are: the number of members required in each of two directions, the two initial identification values at point A, and five constants. By specifying the analogy size and the location of point A in Figure 4, along with the data in Figure 3, the identification can be completed by a digital computer program. Major components can be described completely using the shapes shown in Figure 4.

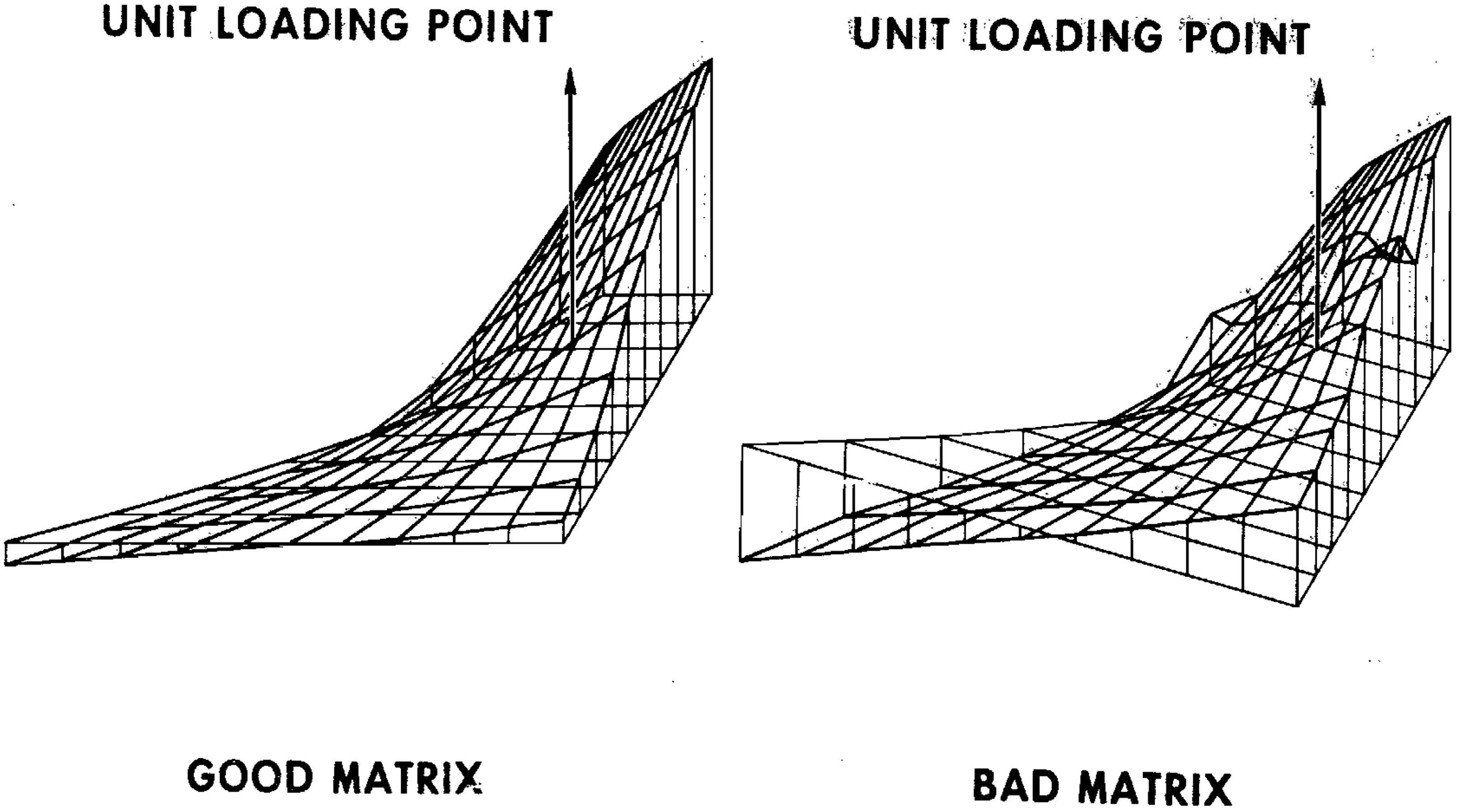


Figure 2. Structural Deflections from Point Load and Influence Coefficient

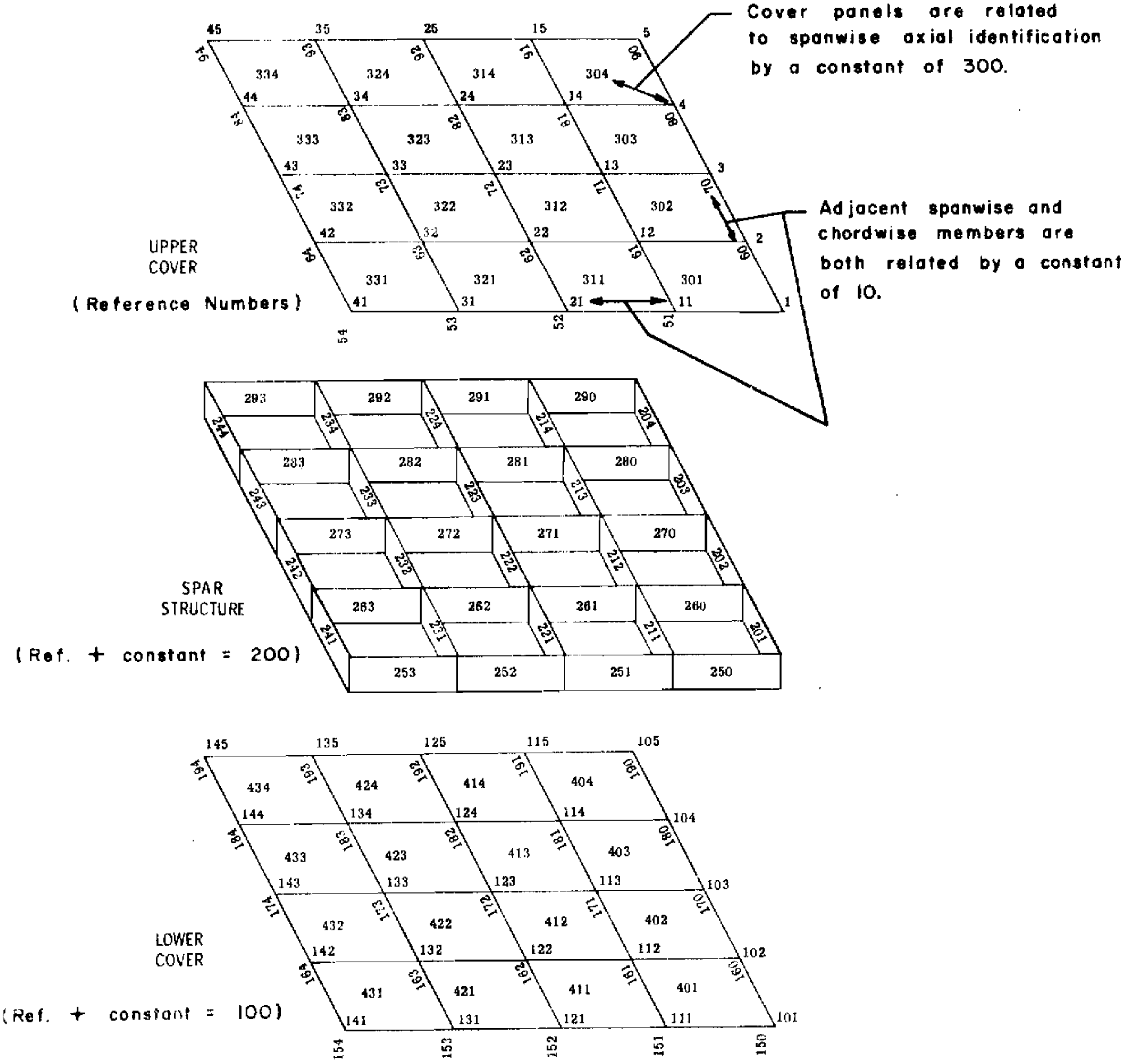
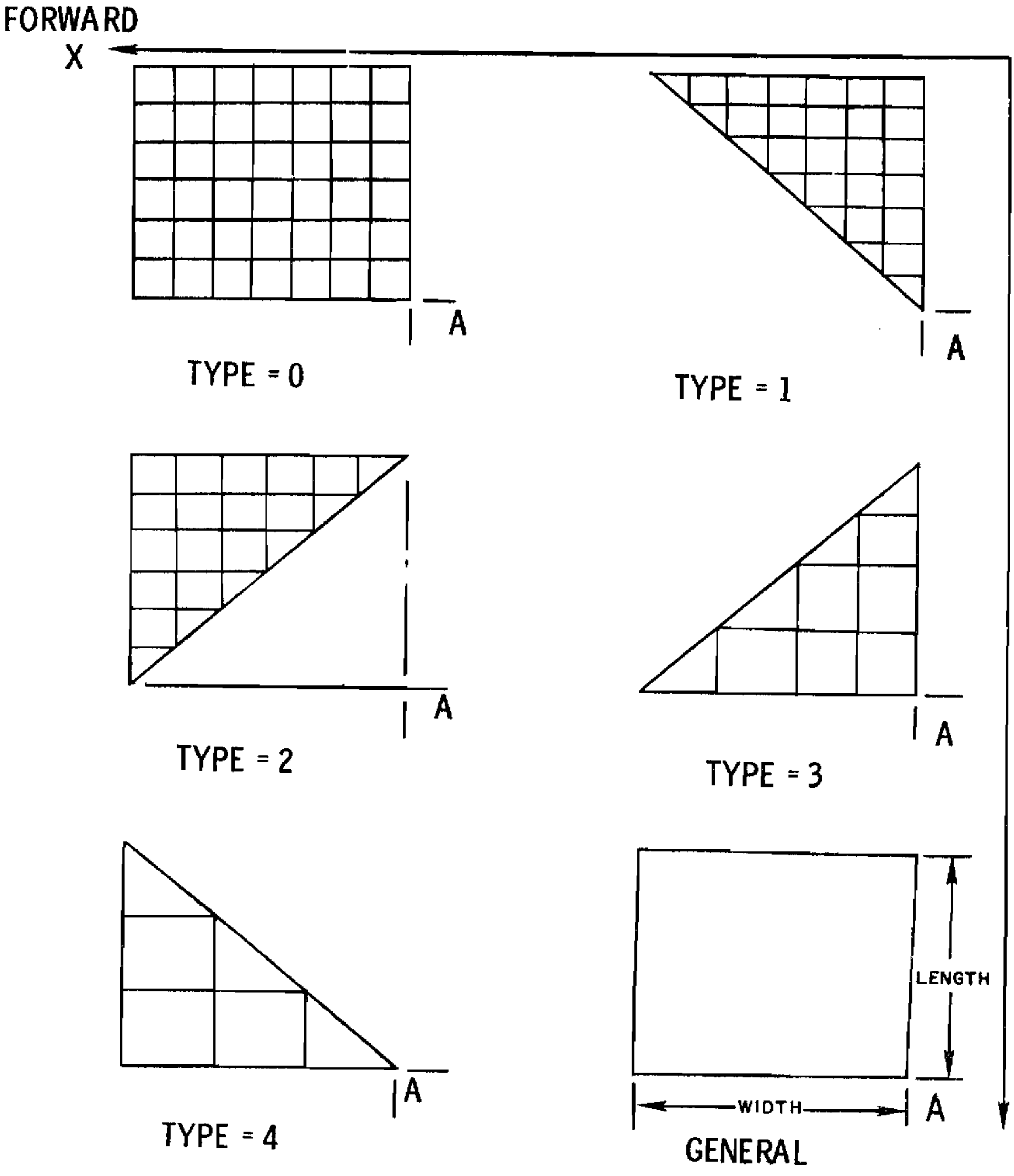


Figure 3. Regularized Section Numbers



X REF, Y REF, ISS, ISC IN DATA REFER TO POINT A. IFSO IS AT OUTBOARD END OF THE DIAGONAL RIB. BEAMS ARE NUMBERED FORWARD, RIBS INBOARD FOR BOUNDARY TIE CONDITIONS.

Figure 4. Analogy Type Control

While the application of a regularized structural analogy benefits the internal load analyst, the true benefit is to the entire design sequence (Figure 5) because the same grid network may be used in each operation. Because of this, the dynamic matrix and the elastic mode shapes/frequencies may be determined in the same computer program that is used to obtain stresses and deflections.

REGULARIZED PROCEDURES

The regularization procedures can be described as follows:

1. Adopt a mathematical model that can be solved exactly.
    - A. All assumptions must be made in the transformation from the structure to the analogy.
    - B. The structural stiffness is described in the regularized analogy.
2. Select an analogy that can be used for both the static and dynamic analysis.
    - A. Since discrete elements are being used, the mass associated with any given node point is best defined as that which gives compatibility between the two analyses.
    - B. If the analogies are inconsistent, but the structural influence coefficient matrix is a product of the static analogy, an unknown and varying amount of error is introduced into the entire design.
3. Keep the degree of coverage consistent for the structure.
    - A. Since the matrix methods force a given internal load distribution between node points, the coverage of the surface should be consistent. In areas requiring a finer grid, the transition should be made gradually; otherwise, large internal couple loads can be developed that are not consistent with the structure.
4. Obtain local detail with local complexity.
    - A. The required detailed stress distribution for static design can be obtained in several ways.
        1. The true structural member loads can be determined by interpretation of the loads from the analogy.
        2. Where interpretation is not possible for the required local stresses, a finer analogy is constructed using the coarse analogy final loads as input.
            - a. The overall solution is not degraded because of local detail.
            - b. Only conditions that design the area must be analyzed; this can be determined by the overall solution.
5. Tailor the analogy to the computer as well as to the structure.
    - A. With any large, highly redundant structure, there is an extensive amount of data required for any solution. Regularization lends itself to computing input, thus eliminating extensive hand tabulation.

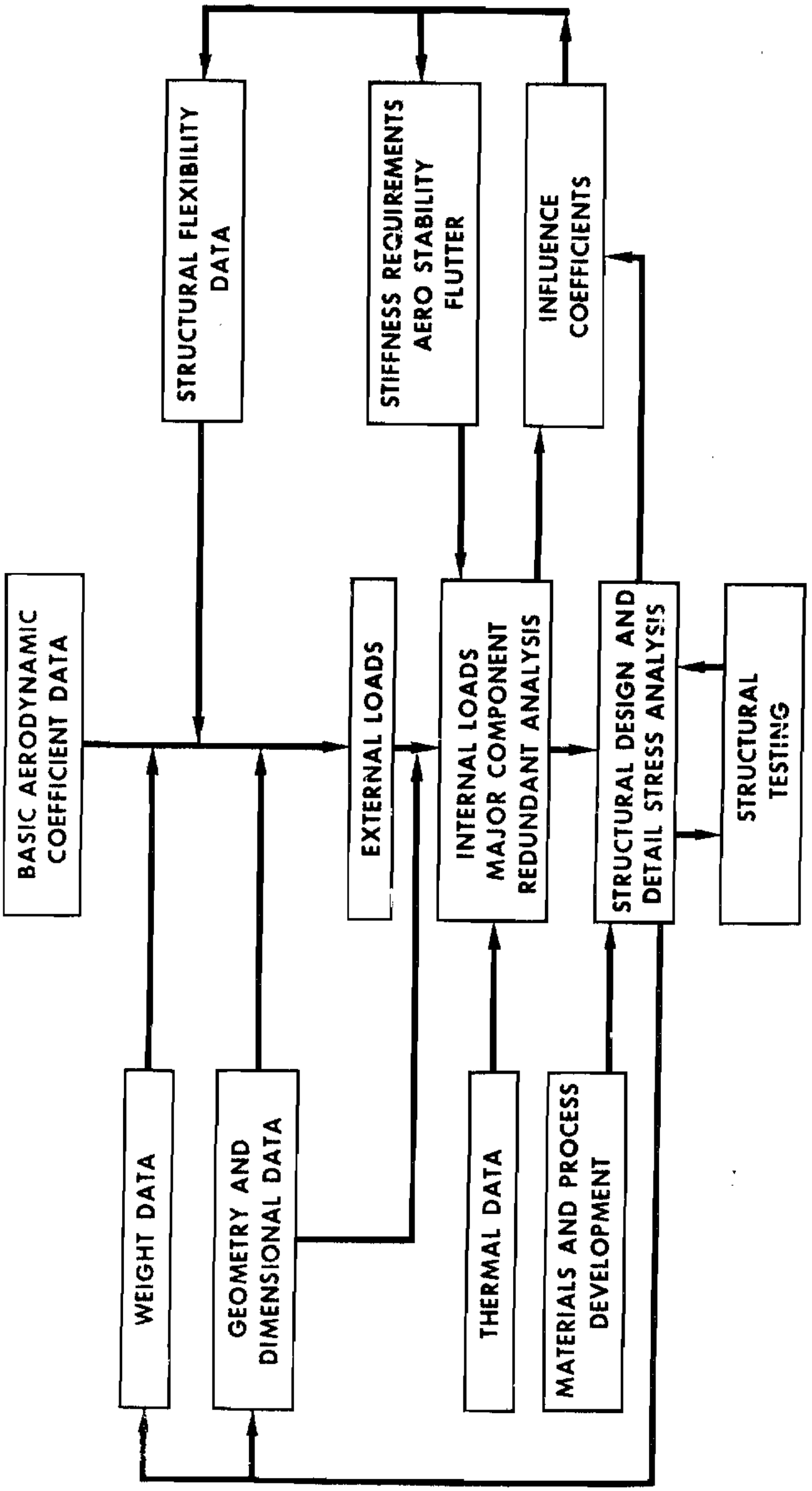


Figure 5. Structural Analysis - Design Sequence

6. Have the system for elastic response available to aid in configuration selection.

   A. For complex structures, the design is often delayed while computer programs are being checked out. By developing computer programs using regularization, large areas of the air vehicles can be optimized while detailed local problem areas are being investigated.

THE DEVELOPMENT OF REGULARIZATION

The history of regularization dates back to March 1955 when Argyris (Reference 1) proposed "a new approach to the problem of cutouts". This method is referred to as "regularization and cutout procedure" in Reference 2. This method was used as early as 1958 on the XB-70.

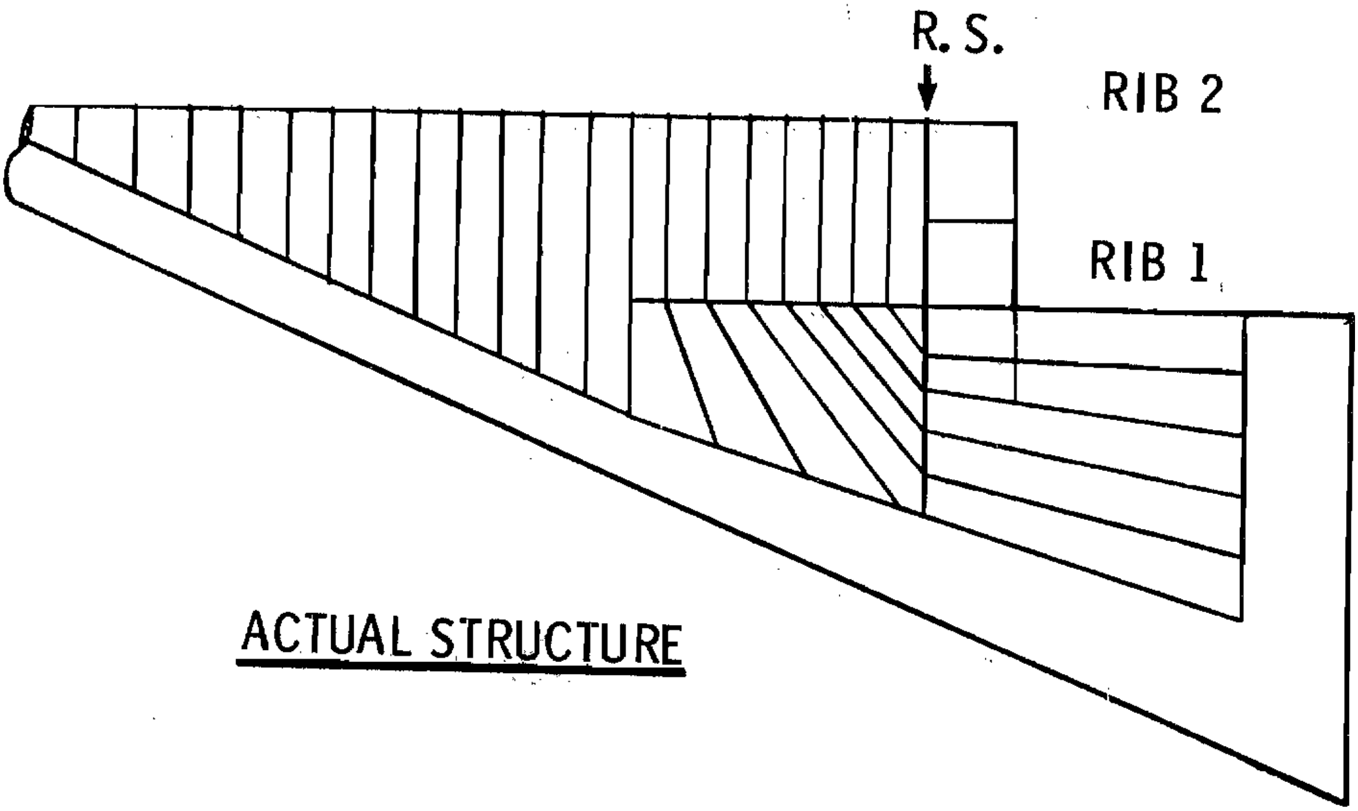
The method as presented here has been under development since 1960, and was directed toward improving the digital procedures used on the XB-70 wing.

For the mathematical model of the XB-70, the matrix force method with a lumped parameter type analogy was used. The structure was broken into several major subsections which were solved independently, then tied together at the common boundaries. Because of the size of the problem, as many simplifying techniques as possible were employed. The regularization and cutout procedure was used on the fuselage as recommended in Reference 2. To verify the results, a typical section was programmed using the regularization procedure and also writing the solution to include the cutouts. It was found that the answers were identical within engineering accuracy. In the wing structure there were partial ribs. For ease of setup, these members were put in for the entire chord and then weakened where no ribs appeared. It was again found that the solution was very good and did not require iterations. The examples used here fall into the description of regularized structure as used in Reference 2. The results in these cases were all considered very satisfactory.

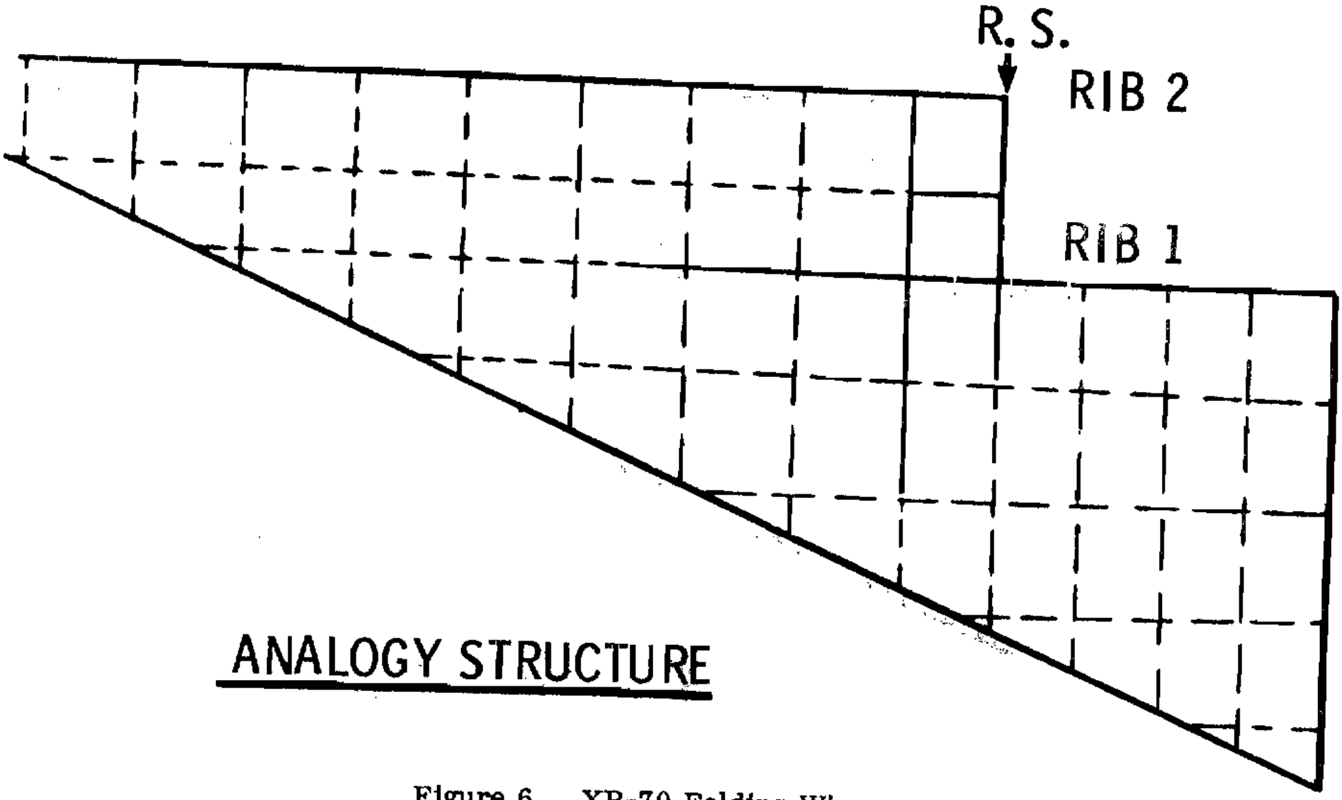
The covers of the XB-70 wing are made of stainless steel honeycomb and the substructure is designed to follow the main bending paths. Figure 6 shows the folding wing spar structure which is oriented first in the chordwise direction, then diagonally, and finally in the spanwise direction.

In the early attempts to describe this region, the analogy structure was made as close to the true geometry as possible. In the analogy, the changes in direction of the substructure were represented by point loads at the intersections of members. The deflections from the solution of this structural analogy showed the anticipated trend (the three-dimensional plots showed the effects of orientation changes of substructure), however, the internal loads were not realistic and seemed to be a product of the mathematical analogy. Several attempts were made to improve the quality of the loads, but these efforts failed. Therefore, another approach was tried. Realizing the analogy was needlessly complex, it was decided to simplify the approach. The following reasoning was used.

Plate-like structures are readily analyzed using finite difference methods by dividing the plate into squares. The size of the squares is governed by the accuracy required to obtain an adequate approximation of the load distribution. In typical aircraft structures with nonbuckling covers, elastic bending energy makes up a major part of the total elastic strain energy. The cover stresses are not primarily dependent on the orientation of the substructure, especially if the spars deviate in direction. This will be true as long as there is sufficient substructure to make the covers fully effective in bending. Because of this, the correct shear paths of the true structure are easily approximated by a "squared pattern" of vertical load-carrying members. By keeping the size of the squares small in relation to the total size of the structure, the bending patterns will be virtually unaffected. Correct vertical panel stiffness of the true structure can be approximated by "effective thicknesses" to compensate



ACTUAL STRUCTURE



ANALOGY STRUCTURE

Figure 6. XB-70 Folding Wing

for length variation in the structural analogy model. The results of this procedure established internal load distributions which were closely checked by strain gage during the XB-70 proof tests. The deflection patterns were very close to the patterns obtained from the first model, but were much smoother. Figure 7 shows a spanwise load condition (load per inch) just outboard of the wing fold joint. The analogy used to obtain this distribution is shown in Figure 6.

The primary development of regularization has been for the redundant force method using lumped parameter idealized elements. The static proof test of the XB-70 verified the predicted stress from the regularized solution at the folding wing joint with very good accuracy (approximately 3% average).

The method has been applied to other structures (skin-stringer and truss core sandwich). The results were compared to other analogies of the same structure and both the deflection and load distributions were also well within engineering accuracy.

There are many advantages of the regularization philosophy from a data handling and solution checkout viewpoint. At the time that the original ''unregularized'' analogy was made, all of the redundant equations for special problem areas were coded by hand, and checkout of the solution was a major problem for two reasons. Firstly, errors are inevitable in coding in nonregularized structure, and, secondly, approximations had to be made as to how the loads were carried on irregular members. It was found that it was possible to get identical answers for unit loads carried in vastly different static paths through the structure when the analogy was regularized. The solution was good even when weakened members were used as primary static load paths. With this background, it became a simple matter to write digital computer programs to do the major portion of the work that was previously done by hand coding, thus sharply reducing the probability of coding error.

The key to regularization falls in the properties given to the analogy structure. Each analogy member may range from having the properties weakened (in the case of cutouts or nonexistent spar structure) to representing several spars or stringers. It follows that this approach to complex built-up structures will be valid as far into the design optimization cycle as the vehicle stiffness can be described in this manner. In aircraft design, major sections often are sized in this way throughout the entire design period.

With the foundation laid, it is then possible to automate the entire approach for section properties, redundant equations, static loads, and solution. The input data are given in the most basic form, and the mechanics of expanding the data are performed completely by the computer.

REGULARIZED STRUCTURAL ANALOGY

The most important phase in obtaining an accurate description for the elastic response of a structure is the initial selection of an analogy. There are two extremes possible when selecting the analogy. One of the extremes is to oversimplify, which leads to quick solutions; however, such an approach often does not describe the true structure and can lead to inconclusive results. The other extreme, towards which the stress analyst tends to lean, is to be so refined in the element description and demanding of the exact details of the structure, that the number of unknowns is excessive, the redundant flexibility matrix is ill-conditioned (approaches singularity), and the results arrive too late for proper usage because of checkout difficulties. Both extremes lead to the same result, namely, that a proper description of the air vehicle is not available when required.

Regularization is a philosophy which leads to a technique for describing any complex highly-redundant airframe system using engineering judgment and experience. In applying the regularization philosophy, the first step is in reverse to the normal approach for indeterminate

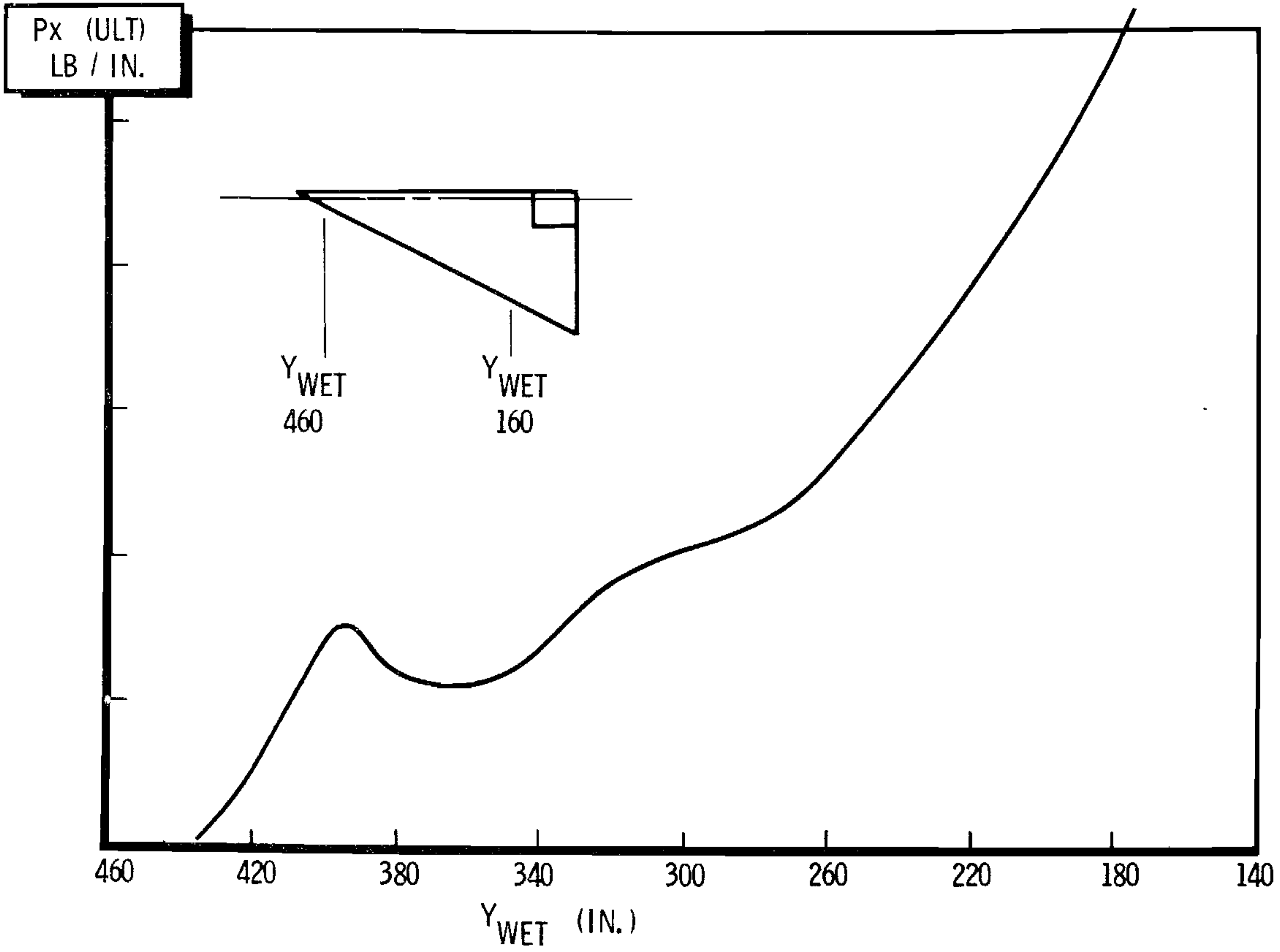


Figure 7. Wing Folding Tip - Spanwise Bending Px (lb/in.)

load analysis. Usually, the approach is to review a structure and make the analogy follow the exact lines of the structure and to have a mathematical element for every member. From this point, members are lumped and modified to reduce the degree of redundancy. It is then determined if the analogy will yield the required results for stresses and deformations. With regularization, the structure is reviewed and it is determined how simple an analogy will describe it for elastic characteristics. The minimum description will always be the number of nodes required to describe the deflected shape for a complete aeroelastic analysis.

In regularization, the complexity of the static analogy is determined by the required deflection results, not by the loads required for the static stress analysis. The local stress distribution problems are solved with as complex solution as is required in the local area. (It should be pointed out that most highly redundant structures contain very few areas that require additional solutions.) The starting point for a static stress analysis is the pressure distributions determined from aeroelastic effects. If the influence coefficients do not describe the lifting surfaces in the same manner as used for the aeroelastic analysis, then any refinements in the static analogy lead to invalid conclusions in regard to the accuracy of internal stresses.

Many of the techniques applied in regularization are not new and have been applied in the past. For example, Reference 1 refers to regularization for describing cutouts in fuselages and partial rib and spars in wing structures.

CONCLUSIONS

In conclusion, the regularized approach will yield accurate, usable results for both static and dynamic analogy of highly redundant structures. The verification of this conclusion is the excellent accuracy obtained on the XB-70. The unique feature of the regularization philosophy is the utilization of engineering judgment in the transition from the real structure to the mathematical model, instead of complication of the equations that describe the model. Detail solutions, for internal load and stress distributions can be obtained subsequently for localized areas, to the degree of sophistication desired, without sacrifice of accuracy or time in solutions of complex and complete airframe systems.

By developing regularization techniques into a system of computer programs for structural analysis, the following advantages may be obtained.

1. The geometry of the analogy will be completely consistent within itself.

2. The same system can be used in both preliminary and production design. The analogy fineness can be controlled by a few input variables; therefore, the degree of coverage is consistent with the required output.

3. Failsafe analysis can be obtained readily by automatically weakening elements of the analogy structure or eliminating the members from the analogy entirely.

4. With all members of the analogy perpendicular, cross-coupling effects, such as Poisson's ratio, can be accounted for easily.

5. Any type of general mechanical loads and thermal gradients can be applied to the analog structure.

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