MODELING MATERIAL DAMPING USING AUGMENTING THERMODYNAMIC FIELDS (ATF)

George A. Lesieutre
SPARTA, Inc.
Laguna Hills, CA

ABSTRACT

A method of modeling frequency-dependent material damping (and moduli) in engineering structural dynamics analysis is reported. Motivated by results from materials science, augmenting thermodynamic fields (ATF) are introduced to interact with the usual mechanical displacement field. The methods of irreversible thermodynamics are used to develop consistent coupled material constitutive relations and partial differential equations of evolution (PDE). The resulting PDE are implemented for numerical solution within the computational framework of the finite element method and the results are compatible with conventional structural analysis techniques. This ATF damping modeling method is illustrated for the case of longitudinal vibration of a rod.

INTRODUCTION

SUMMARY

Vibration damping is an important aspect of many engineering systems. Knowledge of damping characteristics is especially critical when the dynamic response of a flexible structure is to be controlled by a high performance control system. Current analytical treatments of damping in engineering structural dynamics are inaccurate and inadequate, often founded on mathematical convenience rather than on an understanding of the underlying fundamental physical processes.

A new, physically–motivated method of modeling material damping in structural dynamics analysis is presented. The method combines elements of the microscopic approach of materials science and the macroscopic approach of continuum structural dynamics using irreversible thermodynamics.

BACKGROUND

Material damping is likely to be an important, perhaps dominant, contributor to damping in "monolithic" structures and to on-orbit damping in precision spacecraft. In common built-up structures which operate in the atmosphere, air damping and joint damping typically dominate system damping. However, air damping is clearly eliminated in space, and the effects of joint damping will be reduced because of requirements for precision ("tight" joints) and typically low vibration levels (friction "lockup").

Material damping is generally a complex function of frequency, temperature, type of deformation, amplitude, and structural geometry. Figure 1, adapted from Elasticity and Anelasticity of Metals [Zener, 1948] illustrates the typical frequency-dependence of material damping. Current popular treatments of damping in structural dynamics are not physically-motivated and are unable to reproduce this fundamental behavior.

Several methods for incorporating material damping into structural models have been used, and continue to be used within the engineering community. These methods include viscous damping, frequency-dependent viscous damping, hysteretic damping, complex modulus, structural damping, viscoelasticity, hereditary integrals (memory functions), and modal damping [Bert, 1973]. Each has some utility, but each suffers from one flaw or another. Although some potentially accurate models exist (*e.g.*, viscoelasticity), they are not widely used in the engineering community—perhaps because of the lack of physical motivation for, or the difficulty of use of, such models.

RELATED RESEARCH

Several results in the recent engineering literature are related to this work and are briefly discussed below.

Golla, Hughes, and McTavish (GHM) of the University of Toronto have recently developed a time-domain finite element formulation of viscoelastic material damping [Golla and Hughes, 1985; McTavish and Hughes, 1987]. Their work was motivated by the same general perceived need as this work, but was guided by the observation that experimental results, often recorded in the frequency domain, are of little direct use in time-domain models. The results reported here resemble theirs in some ways, for example, in the introduction of additional "dissipation coordinates." However, the results differ in important ways: no attempt is made to provide a physical interpretation of the GHM dissipation coordinates as thermodynamic field variables; the GHM model is restricted to consideration of what is termed here "microstructural

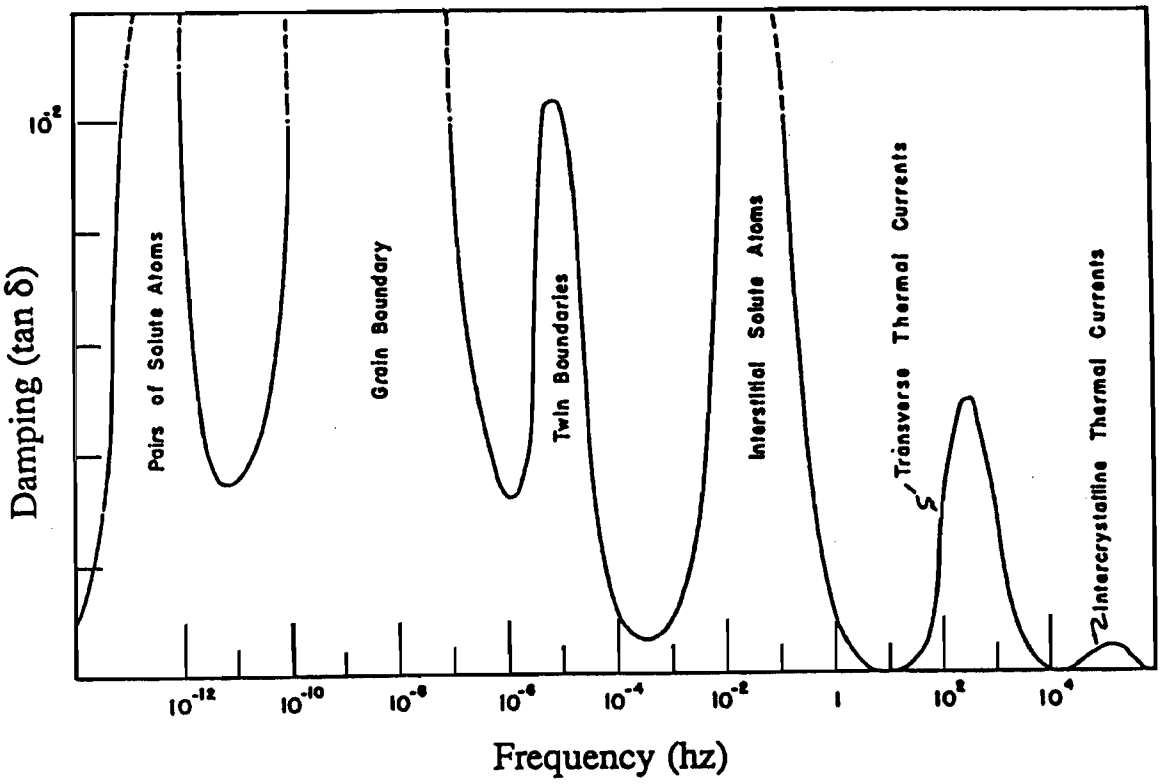


Figure 1: Typical Frequency-Dependence of Material Damping (from [Zener, 1948])

damping," in which the governing thermodynamic relaxation is essentially a scalar process; the "dissipation coordinates" of GHM are internal to individual elements, while the augmenting thermodynamic fields of ATF are continuous from element to element; and the mathematical form of the GHM augmenting equations is restricted to be second order, the same as that of the fundamental equations of mechanical vibration. As shown in their examples, however, the GHM technique can be successfully used to fit a portion of an experimentally-determined curve of damping versus frequency, and standard structural analysis tools can be used to solve the resulting equations.

Both ATF and GHM have advantages over the more conventional modal strain energy (MSE) modeling method in that they are time-domain models, modal damping is calculated concurrently with modal frequency (no look-up tables or iterative procedures are required to converge on both), and the resulting complex modes more accurately reflect the relative phase of vibration at various points on a structure. The ATF method is primarily distinguished from the GHM approach in that it is a *direct* time-domain formulation, amenable to numerical treatment using conventional finite element methods.

D.J. Segalman of Sandia National Laboratories has recently addressed the calculation of stiffness and damping matrices for structures made from linearly viscoelastic materials [Segalman, 1987]. His is essentially a perturbation technique: the perturbation solution for a "slightly viscoelastic" structure is required to match the corresponding solution for a "slightly damped" structure. He works exclusively in the time domain and avoids introducing additional coordinates, although the resulting stiffness and damping matrices are generally unsymmetric. How the assumption of "small viscoelasticity" limits the utility of the approach is currently under investigation.

Torvik and Bagley of the Air Force Institute of Technology have also developed a relevant model of material damping [Torvik and Bagley, 1987]. The core of their approach is the use of fractional time derivatives in material constitutive equations. Their development was motivated by the observation that the frequency dependence observed in real materials is often weaker than the dependence predicted by first-order viscoelastic models. With four and five parameter models, they have been able to accurately represent the elastic and dissipative behavior of over 100 materials over frequency ranges as broad as 8 decades. For most viscoelastic polymeric materials they have examined, the parameter representing the order of differentiation is in the range 1/2 to 2/3. The application of the general fractional derivative approach to time-domain analysis, however, is cumbersome and is an area of continuing research.

Other relevant, current work in the engineering aspects of material damping focuses primarily on the development of experimental techniques and measurement of damping in various materials [*c.f.*, Lesieutre, 1988; Crawley, 1983]. In addition,the use of the MSE method for estimating the damping of built-up structures and composite materials from the measured damping of constituents continues to grow.

BACKGROUND AND APPROACH

Material damping is a subject familiar to many scientists, who have used "internal friction" as an investigative tool with which to explore the basic structure of materials; their work is largely unknown in the engineering community. By measuring damping as a function of frequency, temperature, load type, and amplitude, they are able to determine the mobility and activation energies of various microstructural features of materials. They have shown material damping to be the result of the mutual coupling of stress and strain to other material state variables, variables which can change to new values only through kinetic processes such as diffusion. A multitude of internal variables and relaxation mechanisms which range in geometrical scale from crystal lattice dimensions on up to structural dimensions have been identified [Nowick and Berry, 1972].

While it is an oversimplification to state that dynamics is the study of the evolution of momentum, it provides useful contrast to the statement that thermodynamics is the study of the evolution of energy. Material damping, involving the transfer of energy from structural vibration to other forms of energy, is fundamentally a thermodynamic phenomenon. The field of non-equilibrium thermodynamics provides a general framework for the macroscopic description of irreversible processes [deGroot and Mazur, 1962]. Thermodynamic considerations constrain the form of coupled material constitutive equations and guide the development of coupled field equations of evolution.

Given field equations of evolution, the finite element method is arguably the most powerful and popular method for solving them. Piecewise-continuous trial displacement functions are assumed over a local region of the system being analyzed, individual element matrices are computed using the method of weighted residuals (MWR) or variational principles, then assembled into global system matrices. The output of this procedure is a set of discretized equations of motion that are readily solved using standard computational techniques on digital computers.

APPROACH

The physically-significant "internal state variables" of materials science play a central role in this work, motivating the introduction of augmenting thermodynamic fields (ATF) to interact with the usual mechanical displacement field of continuum structural dynamics. The techniques of nonequilibrium, irreversible thermodynamics are used to develop coupled material constitutive equations and coupled partial differential equations of evolution. Constitutive equations of damped materials describe the coupling of all dependent fields (*e.g.*, as the coefficient of thermal expansion couples the displacement and temperature fields in thermoelasticity).

To illustrate the ATF modeling method, general field equations are specialized to the simplest continuum case, *viz.*, that of one-dimensional vibration of an isotropic rod; an alternate form of the governing equations is also investigated. For simplicity, a single augmenting field is employed. In practice, however, additional fields could be used as needed to better approximate experimental data over a frequency range of interest (*e.g.*, by curve-fitting as in the GHM method).

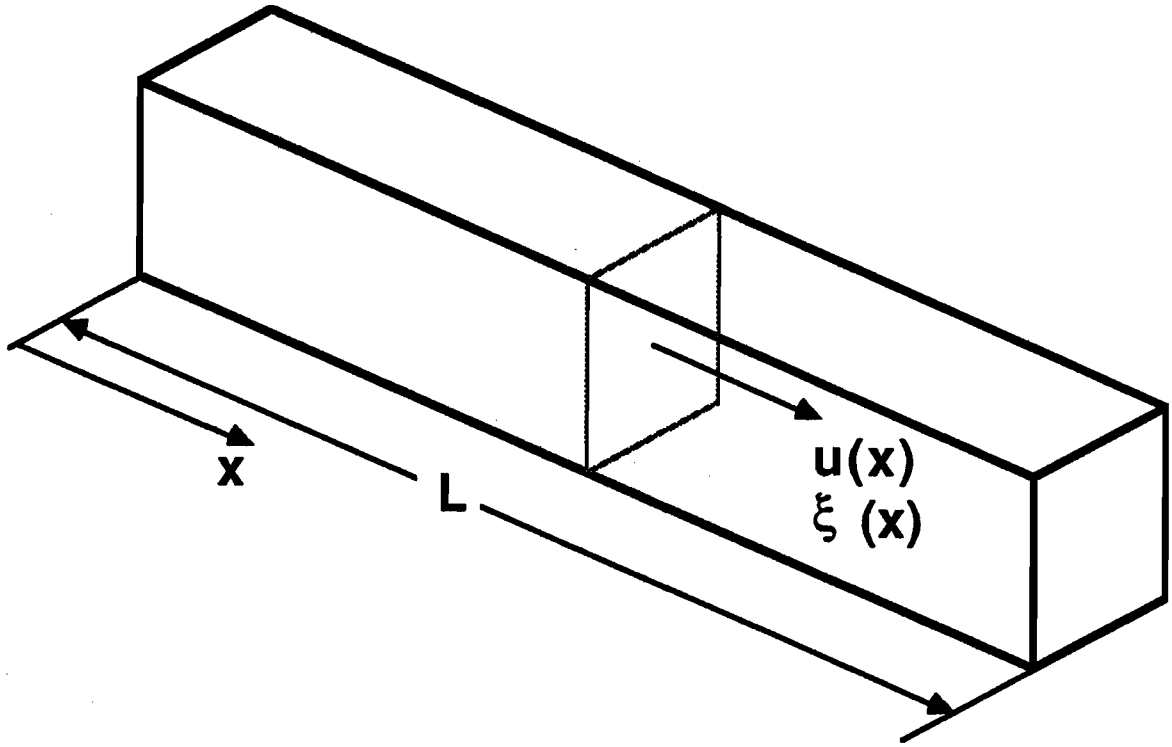
The solution of the coupled partial differential equations are addressed in several ways. Analytical Fourier analysis yields an approximate relationship between damping and frequency, while numerical finite element analysis results in a set of coupled discrete differential equations of evolution. A free vibration eigenvalue problem for these discrete equations can then be solved to yield complex modes.

The method of weighted residuals (MWR) is used to develop damped finite element matrices. Because it can reduce the order and continuity required of assumed approximate displacement fields, integration by parts is an important part of the process of developing element matrices in MWR. However, its use can also present an analyst with a choice between alternate matrices, with no *a priori* rules for choosing between them. A series of numerical experiments was performed for the damped rod elements to identify those matrices superior in terms of convergence to the solution obtained via Fourier analysis. The ATF method readily accommodates the use of multiple materials and ATF-damped elements can be used concurrently with undamped elastic elements.

LONGITUDINAL VIBRATION OF A ROD

GOVERNING EQUATIONS

Consider the case of one-dimensional motion, corresponding to longitudinal vibration of a thin rod, as shown in Figure 2. The mechanical displacement along the rod is denoted by u(x)



Coupled Fields: Mechanical displacement, u(x)
Augmenting thermodynamic, ξ (x)

Figure 2: Example Application—Longitudinal Vibration of a Thin Rod

(strain $\varepsilon(x)=u'(x)$), and the rod has uniform mass density ρ and (unrelaxed) modulus of elasticity E. A single augmenting thermodynamic field, $\xi(x)$, is introduced. The fields which are thermodynamically conjugate to ε and ξ are the stress, σ, and the affinity, A. The affinity can be interpreted as a thermodynamic "force" driving ξ towards equilibrium. The material property δ describes the strength of the coupling of the two dependent fields, u and ξ. In the absence of coupling of the two fields, increments of stress and strain are proportional, with E the relating factor. Analogously, α is the material property that relates changes in A to those in ξ. The material constitutive equations may be found as:

$$\sigma = \frac{\partial f}{\partial \varepsilon} = E\varepsilon - \delta\xi$$

$$A = -\frac{\partial f}{\partial \xi} = \delta\varepsilon - \alpha\xi$$

The usual stress-strain constitutive relations are seen to be augmented by an additional term in ξ, similar to the way in which temperature changes couple to stress and strain. The equation of evolution for the mechanical displacement field is developed from consideration of momentum balance (zero body forces are assumed). The equation of evolution for the augmenting thermodynamic field, ξ, is found through the use of a basic assumption of irreversible thermodynamics, namely that the rate of change of ξ is proportional to A or, in other words, that the rate of change of ξ is proportional to its deviation from an equilibrium value. The result is a set of two coupled partial differential equations in u and ξ:

$$\rho\ddot{u} - Eu'' = -\delta\xi'$$

$$\dot{\xi} + B\xi = \left(\frac{B\delta}{\alpha}\right)u'$$

With appropriate boundary conditions, this set of equations can be shown to be dissipative and well-posed, guaranteeing a solution which depends continuously on the initial conditions. Fourier analysis reveals that the damping and effective modulus are frequency dependent; an approximate equation for the damping ratio (for small damping) is:

$$\zeta = \frac{1}{4}\left(\frac{\delta^2}{E\alpha}\right)\frac{2\left(\frac{\omega}{B}\right)}{\left(1+\left(\frac{\omega}{B}\right)^2\right)}$$

This result is in accord with experimental results obtained by materials scientists for many microstructural damping mechanisms although, as noted by Torvik and Bagley, many materials exhibit weaker frequency-dependence. Note that peak damping is observed at ω=B, and that the magnitude of the peak depends on strength of the coupling of the two equations. Also note that, in accord with the principles of irreversible thermodynamics, the entropy generation rate may be expressed as:

$$\sigma' = \frac{1}{T}\left(\frac{\alpha}{\beta}\right)\dot{\xi}^2$$

An alternative formulation of this one-dimensional case may also be considered. For example, the preceding equations can be expressed in terms of γ, the gradient of the ξ-field, as

follows:

$$\rho\ddot{u} - Eu'' = -\delta\gamma$$
$$\dot{\gamma} + B\gamma = \left(\frac{B\delta}{\alpha}\right)u''$$

This formulation contains only even spatial derivatives, and leads to some benefits in numerical solution, such as symmetric element submatrices.

FINITE ELEMENT TREATMENT

The method of weighted residuals (MWR) is used to develop element matrices. The u-γ formulation of the equations has been shown to be better than the u-ξ formulation, and is employed here. Integration by parts is employed, changing the continuity required of the approximating and weighting functions. This approach can yield alternate element matrices and matrix equations, with no *a priori* rules for selection among them. The same functions used to approximate the behavior of the dependent fields in the spatial region bounded by the element are used here as weighting functions—when there is only one dependent field, this is known as Galerkin's method. Let u(x) and its weighting function be approximated by:

$$u(x) = \Phi^T(x)\mathbf{C}\mathbf{q}$$

where q is the vector of nodal mechanical displacements. Employ a similar approximation for γ(x) and its weighting function:

$$\gamma(x) = \Theta^T(x)\mathbf{D}\mathbf{p}$$

where p is the vector of nodal ATF displacements. Substituting the preceding into the appropriate PDE and integrating over the length of the element, one finds:

$$\int_0^L \left\{\rho\mathbf{C}^T\Phi\Phi^T\mathbf{C}\ddot{\mathbf{q}} + E\mathbf{C}^T\Phi'\Phi'^T\mathbf{C}\mathbf{q}\right\}dx = \int_0^L \left\{-\delta\,\mathbf{C}^T\Phi\Theta^T\mathbf{D}\mathbf{p}\right\}dx$$

$$\int_0^L \left\{\mathbf{D}^T\Theta\Theta^T\mathbf{D}\dot{\mathbf{p}} + B\mathbf{D}^T\Theta\Theta^T\mathbf{D}\mathbf{p}\right\}dx = \int_0^L \left\{-\left(\frac{B\delta}{\alpha}\right)\mathbf{D}^T\Theta'\Phi'^T\mathbf{C}\mathbf{q}\right\}dx$$

These sets of equations may be written in first-order form as:

$$\begin{bmatrix}\mathbf{M} & 0 & 0\\ 0 & \mathbf{I} & 0\\ 0 & 0 & \mathbf{C}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{q}}\\ \dot{\mathbf{q}}\\ \dot{\mathbf{p}}\end{Bmatrix} + \begin{bmatrix}0 & \mathbf{K} & \mathbf{B}\\ -\mathbf{I} & 0 & 0\\ 0 & \mathbf{F} & \mathbf{H}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{q}}\\ \mathbf{q}\\ \mathbf{p}\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ 0\end{Bmatrix}$$

In this treatment, both fields are approximated with linear interpolation functions using:

$$\Phi^T = [1\ x]$$

Figure 3 illustrates the element and the nodal values for the two dependent fields, u and γ. If the elemental degrees of freedom are ordered to facilitate assembly as:

$$\mathbf{x} = [\dot{q}_1\ q_1\ p_1\ \dot{q}_2\ q_2\ p_2]^T$$

the elemental equations may be expressed as:

$$\mathbf{A}\dot{\mathbf{x}} + \mathbf{B}\mathbf{x} = 0$$

and the ATF-damped rod elements are:

$$\mathbf{A} = \begin{bmatrix} \left(\frac{\rho AL}{3}\right) & 0 & 0 & \left(\frac{\rho AL}{6}\right) & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{AL}{3}\right) & 0 & 0 & \left(\frac{AL}{6}\right) \\ \left(\frac{\rho AL}{6}\right) & 0 & 0 & \left(\frac{\rho AL}{3}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \left(\frac{AL}{6}\right) & 0 & 0 & \left(\frac{AL}{3}\right) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & \left(\frac{EA}{L}\right) & \left(\frac{\delta AL}{3}\right) & 0 & \left(-\frac{EA}{L}\right) & \left(\frac{\delta AL}{6}\right) \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \left(\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{3}\right) & 0 & \left(-\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{6}\right) \\ 0 & \left(-\frac{EA}{L}\right) & \left(\frac{\delta AL}{6}\right) & 0 & \left(\frac{EA}{L}\right) & \left(\frac{\delta AL}{3}\right) \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & \left(-\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{6}\right) & 0 & \left(\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{3}\right) \end{bmatrix}$$

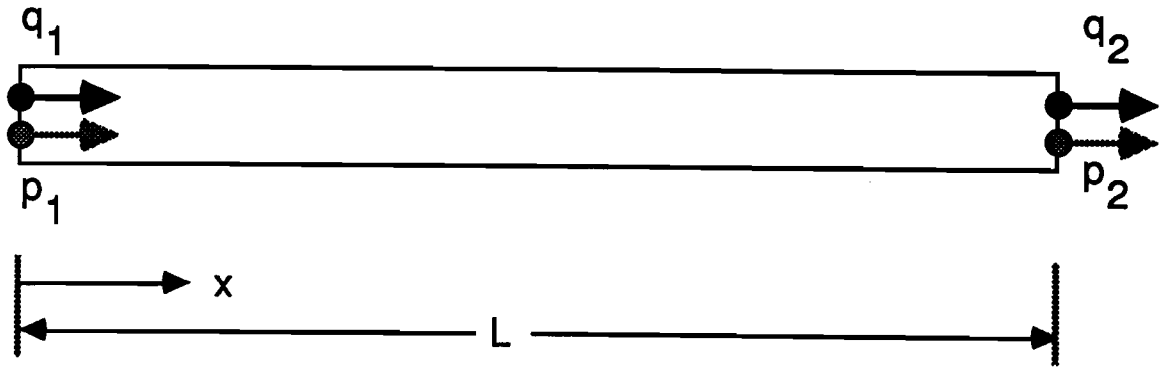
NUMERICAL RESULTS FOR FREE VIBRATION EIGENVALUE PROBLEM

In order to evaluate the performance of this formulation of an ATF-damped rod element, a specific problem is addressed, *viz.*, the determination of the natural modes of vibration of a free-free rod. The results are compared to those obtained using approximate Fourier analysis.

Assuming a solution for x(t)in the form $e^{\lambda t}$, the following eigenvalue problem is defined:

$$[\lambda\mathbf{A} + \mathbf{B}]\mathbf{x} = 0$$

The matrix equations of motion are formulated and this problem is solved to yield complex eigenvalues, λ, and mode shapes, x. Element matrices may be assembled into global system matrices in the usual manner of structural finite element analysis. The damping ratio for each mode is calculated as the ratio of the negative of the real part of the eigenvalue to the total magnitude. The damping ratio, ζ, is then plotted against the magnitude of the eigenvalue. Note that the



Mechanical displacement field

$$u(x) = \left[\left(1 - \frac{x}{L}\right)\left(\frac{x}{L}\right)\right]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

Augmenting thermodynamic field

$$\gamma(x) = \left[\left(1 - \frac{x}{L}\right)\left(\frac{x}{L}\right)\right]\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}$$

Figure 3: The ATF-Damped Linear-u, Linear-γ Rod Element

spectrum of eigenvalues will contain "vibration modes," "relaxation modes," and "rigid-body modes." In the complex plane, the damped vibration modes lay near the imaginary axis, slightly in the LHP with negative real parts; the relaxation modes lie on the negative real axis. These relaxation modes are characteristic of the response of the γ field.

The numerical parameter values used are:

$E = 7.13e10$
$\rho = 2750$
$L = 10$
$B = 8000$
$\alpha = B$
$\delta = 4.7766e6$

The elastic properties correspond roughly to those of aluminum in SI units. Using the results of the Fourier analysis, values for the "dissipative" properties were chosen to yield a peak damping ratio of 1% at the frequency of the 5th mode; in practice, these would be obtained from experimental data.

Figure 4 shows typical numerical results yielded by this approach, using 20 damped rod elements. The characteristic variation of material damping with frequency is apparent and, as previously noted, conventional damping modeling techniques are incapable of producing such results.

Summary and Conclusions

A physically-motivated material damping model compatible with current computational structural analysis methods has been developed. Referred to as the ATF method, its key feature is the introduction of augmenting thermodynamic fields to interact with the usual displacement field of continuum structural dynamics. Note that an increase in the accuracy of a structural dynamic model comes with a cost of dimensionality—additional coordinates are required to represent additional aspects of material behavior, *viz.*, damping.

Coupled material constitutive equations and partial differential equations of evolution have been developed for microstructurally-damped rods, and ATF-damped finite elements developed and used to solve a free vibration eigenvalue problem. Numerical finite element results compared favorably with results obtained using Fourier analysis.

Although a single augmenting field was generally discussed, the results are readily extended to multiple fields. In addition, although all the elements of the example structures were assumed to be damped, the method is compatible with conventional structural analysis techniques and readily accommodates both damped and undamped elements. With the continued development of better analytical tools such as this ATF method, damping will be modelled more accurately in the design of engineering systems and may ultimately become more accessible to design specification.

Acknowledgments

This work is based on the author's dissertation research [Lesieutre, 1989]. He wishes to thank Professor D.L. Mingori of UCLA for his participatory guidance, and R. Quartararo and H. Rediess for their encouragement. The financial support of SPARTA, Inc. through a tuition reimbursement program and free computer time is noted. A more detailed presentation of this work will be made at the 1989 Structures, Structural Dynamics, and Materials Conference in Mobile, Alabama.

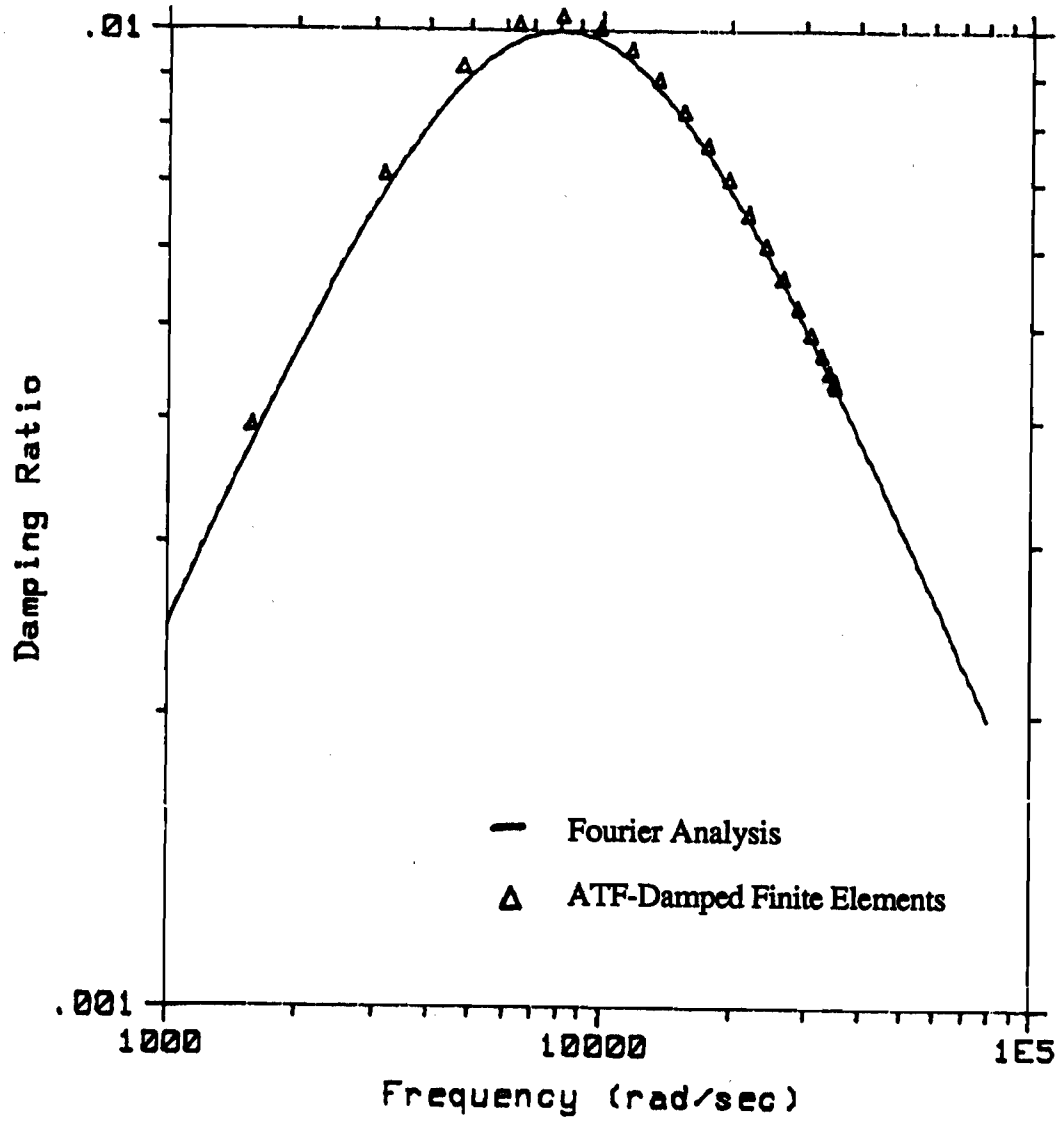


Figure 4: Damping versus Frequency for 20 ATF-Damped Rod Finite Elements and Fourier Analysis

REFERENCES

Bert, C.W., "Material Damping: An Introductory Review of Mathematical Models, Measures, and Experimental Techniques," *J Sound and Vibration*, Vol. 29, No. 2, 1973, pp. 129-153.

Crawley, E.F., and Mohr, D.G., "Experimental Measurements of Material Damping in Free-Fall With Tuneable Excitation," AIAA Paper 83-0858, May 1983.

deGroot, S.R., and Mazur, P., *Non-Equiiibrium Thermodynamics*, North-Holland (Interscience), 1962.

Golla, D.F., and Hughes, P.C., "Dynamics of Viscoelastic Structures - A Time-Domain, Finite Element Formulation," *J Appl Mech*, Vol. 52, December, 1985, pp. 897-906.

Lesieutre, George A., Eckel, Andrew J., and DiCarlo, James A., "Temperature-Dependent Damping of Some Commercial Graphite Fibers," *Proceedings of the NASA/DoD Composites Materials and Structures Conference*, January, 1988, Cocoa Beach, FL.

Lesieutre, G.A., "Finite Element Modeling of Frequency-Dependent Material Damping using Augmenting Thermodynamic Fields," Ph.D. Dissertation, Aerospace Engineering, University of California, Los Angeles, 1989 (expected).

McTavish, D.J., and Hughes, P.C., "Finite Element Modeling of Linear Viscoelastic Structures," *Proceedings of the 11th ASME Biennial Conference on Mechanical Vibration and Noise*, September, 1987, Boston, MA.

Nowick, A.S., and Berry, B.S., *Anelastic Relaxation in Crystalline Solids*, Academic Press, 1972.

Segalman, D.J., "Calculation of Damping Matrices for Linearly Viscoelastic Structures," *Proceedings of the 11th ASME Biennial Conference on Mechanical Vibration and Noise*, September, 1987, Boston, MA.

Torvik, P.J., and Bagley, D.L., "Fractional Derivatives in the Description of Damping Materials and Phenomena," *Proceedings of the 11th ASME Biennial Conference on Mechanical Vibration and Noise*, Boston, Massachusetts, September, 1987.

Zener, C.M., *Elasticity and Anelasticity of Metals*, University of Chicago Press, 1948.