QUICK ESTIMATION OF DAMPING FROM FREE DAMPED OSCILLOGRAMS

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ABSTRACT

A quick and fairly accurate estimate of the damping ratio, D, of a linear single-degree-of-freedom system can be obtained from an oscillogram of free damped motion by using the relation

$$D \simeq \frac{0.75}{n} ,$$

where n is the number of cycles of motion in the length of record required for the amplitude of the envelope of the motion to decrease to 1% of its initial value.

NOMENCLATURE

C	constant	T	time constant
D	damping ratio	α	phase angle
m	number of time constants	ω_n	damped circular frequency
n	number of periods (or cycles) of motion	ω_n	undamped circular freq.
q	dependent variable	τ_n	undamped period
t	independent variable		

I. INTRODUCTION

The differential equation

$$\ddot{q} + 2D\omega_n \dot{q} + \omega_n^2 q = 0 \qquad \text{Equation (1)}$$

represents the transient motion of a damped linear single-degree-of-freedom second-order system. For the case of subcritical damping, a typical example of transient motion is shown in Figure 1. The well-known solution of Equation (1) for the case of subcritical damping is

$$q = C\, e^{-D\omega_n t} \cos(\omega_d t + \alpha) \qquad \text{Equation (2)}$$

The most common method for evaluating the damping ratio, D, requires determination of the logarithmic decrement. As few people carry in their heads values of natural logarithms, this method usually requires the use of a slide rule or tables of natural logarithms. In addition, the method requires measurement of succeeding maxima of q(t).

Time Constant Method

Use of the envelope of q(t) and the concept of the time constant makes it possible to estimate damping ratios with neither a single overt measurement nor reference to tables. The time constant method does require, however, memorization of the simple formula

$$D \simeq \frac{0.75}{n}, \qquad \text{Equation (3)}$$

where n is the approximate number of cycles of motion in which the amplitude of the envelope decreases to 1/100 of its initial value. In Figure 1, for example, a rapid estimate indicates a value for n of approximately 3, giving an approximate value of 0.25 for the damping ratio.

Derivation

The approximate relation (3) may be derived quite easily. The time constant, T, of the decaying exponential function $e^{-D\omega_n t}$ is defined as the time required for the function to decrease to 1/e times its initial value. From this definition, the relationship

$$D\omega_n T = 1 \qquad \text{Equation (4)}$$

between the time constant and the damping ratio can be found. Equation (4) can be rearranged to give

$$D = \frac{1}{2\pi} \frac{\tau_n}{T} \qquad \text{Equation (5)}$$

By noting that a length of record with a time duration of m time constants will have n periods of motion*, and using the approximation that the damped period is approximately equal to the undamped period, Equation (5) may be written in the form

$$D \simeq \frac{1}{2\pi} \frac{m}{n} \qquad \text{Equation (6)}$$

The position on an oscillogram where the signal amplitude is of the order of 1% of its initial value can usually be estimated. In many cases, an amplitude of 1% will be equal approximately to the thickness of the trace. The length of record between the initial value position and the 1% position represents a time interval of approximately 4.6 time constants. Substituting m = 4.6 in Equation (6) gives

$$D \simeq \frac{1}{2\pi} \frac{4.6}{n} \qquad \text{Equation (7)}$$

to which Equation (3) is a close approximation.

Accuracy

The foregoing analysis shows the time constant method to depend for its accuracy on proper estimation of two quantities: that point on the oscillogram where the amplitude is 1% of its initial value, and the number of cycles of motion between the points of initial and 1% amplitude. Estimates of the damping ratio to within 20% of the correct value can be made within seconds. Greater accuracy can be achieved by measuring, rather than estimating, the two required quantities.

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* m and n need not be integers

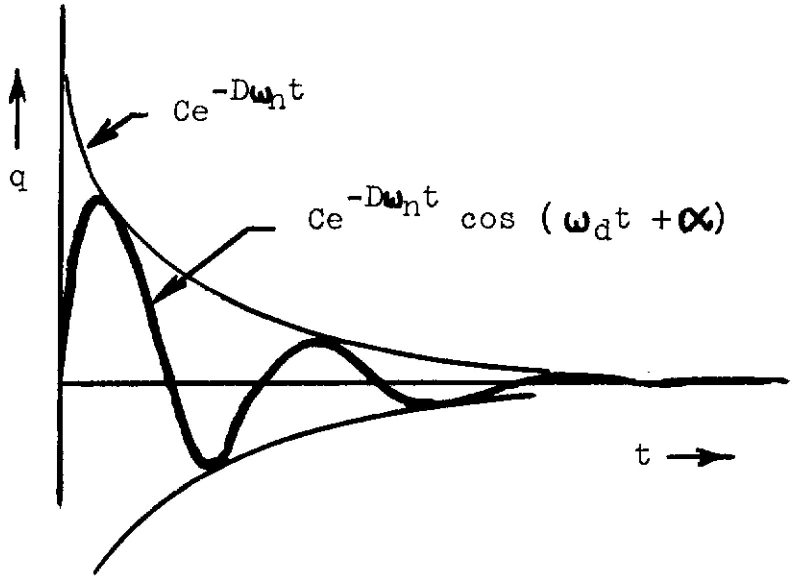

Figure 1

Transient Motion and Envelope of Motion for System with Subcritical Damping Represented by

$$\ddot{q} + 2D\omega_n\dot{q} + \omega_n^2 q = 0$$