THE DESIGN OF STABILITY AUGMENTATION SYSTEMS FOR DECOUPLING AIRCRAFT RESPONSES

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FOREWORD

This report was originally prepared by Lt Rhall E. Pope as a thesis in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Science from Purdue University, Lafayette, Indiana. Lt. Pope's thesis advisor was Professor Robert L. Swaim, PhD.

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ABSTRACT

During the landing phase of flight, STOL aircraft exhibit undesirable coupled response in both the longitudinal and lateral-directional modes of flight. There is also a need for stability augmentation for these aircraft due to low longitudinal and directional dynamic stability and low static longitudinal stability.

Application of Gilbert's decoupling procedure is proposed to eliminate the undesirable coupling effects and to develop a simpler method of designing a stability augmentation system. Five decoupled configurations were investigated. In the longitudinal mode, pitch response controlled by longitudinal stick was decoupled, first, from angle of attack, second, from flight path angle, and third from airspeed all of which were controlled by throttle. In the lateral directional mode, roll response controlled by lateral stick was decoupled first from yaw, and second from sideslip which were both controlled by rudder pedals.

The linearized aircraft equations are given by

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where x is an n order state vector, u is an m control, y is an m output vector, and A, B, and C are matrices of appropriate size. A class of control laws of the form

$$u = Fx + Gv$$

when v is an m order input vector will decouple the system provided necessary and sufficient conditions are met. Particular F and G matrices are determined by specifying the transient response of each single input, single output subsystem. This specification was determined digitally with emphasis on not exceeding aircraft control authority limits.

An analog simulation was carried out to obtain pilot comments.

The general conclusion reached was that decoupling theory applied to aircraft systems did simplify the design of stability augmentation systems. However, it does not allow for the potential for control authority saturation. It was also found that it was possible to provide exponential attitude response. A sensitivity analysis showed that decoupled response was very sensitive to errors in measurements of feedback variables but was relatively insensitive to aircraft model errors.

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LIST OF SYMBOLS

A	....	Open-loop stability matrix
B	....	Control input matrix
C	....	Output matrix
C	....	Stability coefficient
D	....	Conditional decoupling matrix
d	....	Normal deviation from glide slope (ft.)
d_s	....	Localizer deviation (ft.)
F	....	Feedback matrix
G	....	Feedforward matrix
H	....	Transfer function matrix
h	....	Altitude (ft.)
I	....	Moment of inertia
I	....	Identity matrix
J	....	Controllable decoupling matrix
K	....	Uncontrollable decoupling matrix
L	....	Rolling moment
L_i	....	Turbulence scale (i = u, v, or w)
M	....	Pitching moment
m	....	Mass
m	....	Number of controls
N	....	Yawing moment
n	....	Number of states
Q	....	Canonical transformation matrix

R	....	Slant range
S	....	Wing area
S	....	System
s	....	Laplace transform variable
T	....	Factored power spectral density
t	....	Time
U_o	....	Reference velocity (ft/sec, and knots)
u	....	Longitudinal perturbation to reference velocity (ft/sec)
u	....	Control vector
v	....	Control input vector
v	....	Lateral perturbation to reference velocity (ft/sec)
w	....	Normal perturbation to reference velocity
X	....	Longitudinal force
x	....	State vector
Y	....	Lateral force
y	....	Output vector
Z	....	Normal force
z	....	Normal displacement

Greek Symbols

α	....	Angle of attack
α	....	Closed loop numerator
β	....	Sideslip angle
Γ	....	Initial flightpath angle
γ	....	Flightpath angle

δ	Glide-slope deviation (deg.)
δ	Perturbation
ζ	Damping ratio
η	Localizer deviation (deg.)
θ	Pitch
λ	Gain parameter
π	Closed-loop specified poles
ρ	Uncontrollable gain parameter
ρ	Air density
Σ	Summation
σ	Transient response parameter
ϕ	Roll
ψ	Yaw
ψ	Closed-loop denominator
Ω	Spatial frequency
ω	Frequency

Superscripts

*	Integrator decoupled transformation
—	Integrator decoupled system
—	Scaled simulation variable
c	Controllable
u	Uncontrollable
^	Canonical decoupled system
~	Controllable decoupled system

Subscripts

D	....	Drag
D_u	....	Change in drag due to u
d	....	Dutch roll
e	....	Elevator
e_{stick}	....	Longitudinal stick
g	....	Gust
o	....	Initial value
ph	....	Phugoid
r	....	Rudder
r_{pedal}	....	Rudder pedal
s	....	Lateral deviation from beam
s	....	Spoiler
s_{stick}	....	Lateral stick
sp	....	Short period
T	....	Thrust
$T_{throttle}$	....	Throttle
T_c	....	Commanded thrust
u	....	Longitudinal velocity component
v	....	Lateral velocity component
w	....	Normal velocity component

CHAPTER 1

INTRODUCTION

The increasing congestion, both surface and air, around today's airports can be directly attributed to the large runway areas needed for conventional aircraft, coupled with the limited space available for these runways. To solve the real estate problem, airports are usually located on the outskirts of a community. However, since in most cases, communities are served by only one airport, air congestion is an immediate consequence. The surface congestion usually results from short-sightedness on the part of community planners in failing to provide sufficient and varied modes of transportation from points in the community to the airport. A popular and probably the most demonstrable example of this situation occurs in the Northeast Corridor.[1]

A prospective solution to this problem has been the use of V/STOL (vertical and/or short take-off and landing) aircraft.[1-12] This solution has been evident for nearly two decades, but has never reached practical operation due in part to two reasons. First, until very recently,[47] there have been no specific handling qualities officially defined as yet for V/STOL aircraft.[13] Second, and partly as a consequence of the first reason, very few theoretical and experimental V/STOL aircraft have reached production stage due to control problems unique to V/STOL aircraft. These problems vary in severity with the complexity of the respective aircraft: i.e., a STOL (short take-off and landing) aircraft with deflected slipstream configuration will have less severe control problems than a tilt wing or vectored thrust VTOL (vertical take-off and landing) aircraft. A STOL vehicle is without the mechanical complexity and added pilot control workload of a VTOL vehicle.[13-15] It does not undergo large variations in vehicle characteristics as demonstrated by VTOL vehicles in the transition region of flight.[13] A STOL vehicle can utilize much of the existing technology developed for conventional aircraft. Practically speaking, at the present moment, the airlines would prefer a STOL aircraft over a VTOL aircraft. This decision is the result of economic considerations as well as a poor service image generated by existing helicopter (VTOL) carriers, the prospect of a limited acceptance by the public, and of course, the all important noise problems.[11] For these reasons, it seems certain that the first large scale application* of the V/STOL solution to the surface and air congestion problem will be the use of STOL aircraft. Consequently, I will concern myself with the analyses

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* Smaller scale use of STOL aircraft is currently in operation: e.g., use of DeHavilland Twin Otter STOL Aircraft by Air Wisconsin out of Purdue Airport.

of the control problems associated with STOL vehicles only.

Aircraft are usually mathematically represented by linearized, small perturbation equations of motion about a trim flight condition with time fixed coefficients (see Appendix A). Since the equations are time fixed, any analysis must begin with the selection of a particular aircraft configuration for a particular phase of flight. Since the landing phase is the most critical and since for comparison purposes there has been extensive research previously conducted on control problems occurring during landing, I will restrict my analysis to the control problems of STOL vehicles in the landing configuration. The landing configuration for STOL aircraft usually refers to an aircraft with fully deflected flaps enabling it to land at around 7½° flight path angle, 800 feet/minute sink rate, and a speed of around 60 knots.[16] With respect to this configuration, the following control problems exist:

1. Large sideslip excursions during turn entries
2. Adverse yaw effects
3. Low directional stability
4. Low longitudinal stability
5. Large pitching moments with power changes

References 17-20 agree that the "ability to maintain the desired bank angle in turbulent air has been the most critical requirement for lateral control of STOL aircraft at... landing speeds. Precise control is required because small bank angles generate large yaw rates at low speeds which quickly produce heading changes."[20] Reference 17 states, "When the aircraft is banked into a turn, a turn rate in the desired direction does not develop for several seconds, and large excursions in sideslip angles result." Likewise, reference 19 states, "The flight study...showed that low directional stability, low directional damping, and adverse yaw due to lateral control were responsible for the large sideslip excursions during maneuvers in the landing approach or during flight in gusty air at low speeds." And finally reference 18 states, "The problem of controlling sideslip at the low airspeeds required for STOL operation is due for the most part to low directional stability and damping." An important control problem then is to eliminate any sideslip build-up which might occur during a turn entry.

Adverse yaw effects and low directional stability contribute to the sideslip buildup problem,[19] but they are also problems in themselves. Reference 18 states, "The main source of control cross coupling comes from adverse yaw due to aileron deflection." Various methods have been used to reduce adverse yaw, e.g., the use of differential propeller pitch.[16] However, additional controls add complexity and expense. Since it is a common problem, adverse yaw or turn coordination is represented numerically by the ratio of the peak sideslip excursion to the bank angle ($\Delta\beta/\Delta\phi$) developed during rapid turn entries.[20] Low directional stability is indicated by the long period (12 seconds per cycle for the NC-130B) of large sideslip angles and an appreciable time period to establish the desired turn rate.[17,18] References 16 and 17 note the low directional stability and the low directional damping of the Brequet 941. Also concerning the directional mode, reference 17 states, "At high values of L_r (roll due to yaw rate) large spiral instability resulted." To summarize the control problems in the lateral directional mode: Low directional stability and adverse yaw contribute to the problem of eliminating sideslip during turn entries.

There are fewer and less severe control problems in the longitudinal mode than in the lateral directional mode. References 16 and 17 state that low longitudinal stability and large pitching moment changes with power changes have been observed with STOL aircraft and these characteristics require moderate pilot effort to correct. Reference 20 notes that "little longitudinal control was needed during the approach because flight was controlled primarily by engine power and moderate angle of attack excursions...could be corrected."

Decoupling Approach

An analysis of the control problems mentioned above presents three immediate conclusions:

1. Significant cross-coupling between sideslip and roll and between yaw and roll exists in the lateral-directional mode.
2. Significant cross-coupling between forward speed and pitch exists in the longitudinal mode.
3. A stability augmentation system (SAS) is needed for STOL vehicles.

In the lateral-directional mode, adverse yaw is a coupling between roll and yaw; i.e., there is an unwanted yaw response together with the desired roll response to an aileron deflection. There is also unwanted coupling between side slip and roll in turn maneuvers. In turn maneuvers, most pilots strive for coordinated turns. A coordinated turn is one in which there is no side acceleration or side force and therefore no side slip.[21] In order to achieve a coordinated turn, the pilot must compensate for side slip buildup with rudder deflections. The less the pilot has to compensate, the more he will like to fly the aircraft.

As mentioned before, prior to December 1970, there had been no handling qualities specifically defined for STOL vehicles. Two of the most definitive studies, however, realized the difficulties that could be encountered with the coupling effects. Reference 22 states, "Throughout the speed and height range covered by these recommendations, the application of any roll control input necessary to satisfy roll control requirements, the other controls being held fixed, should not result in yaw motion, side slip or pitch attitude change which causes any objecticnable or dangerous flight condition." And reference 23 states, "In rudder and elevator cockpit control, fixed rolls to the maximum designated bank attitude at all altitudes and permissable speeds, the resulting yaw motion, side slip angle, and normal acceleration shall neither exceed structural limits nor cause dangerous flight conditions such as uncontrollable oscillations." Neither recommendation is very specific, but both allow for the fact that dangerous flight conditions could result from cross coupling of control output responses.

Besides the coupling problems, it has been noted that STOL vehicles have both low directional and longitudinal stability. If the vehicle has passed the configuration design stage of development, this condition can be rectified with the introduction of a stability augmentation system. Two examples of the effectiveness of SAS's applied to STOL vehicles can be found in references 17 and 19.

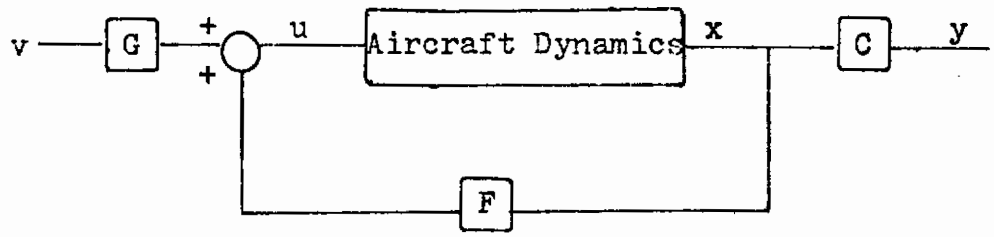
Recognizing, therefore, the problems caused by coupling effects and the low stability inherent in STOL vehicles, it would appear that a stability augmentation system which also decoupled would be ideal. A decoupled system with stability augmentation is defined as a system in which each input affects one and only one output and in which the desired transient response can be obtained. The combination of augmentation and decoupling is not a new idea. Several reports have been published on the subject, differing mainly in the decoupling method used. These can be divided into

1. High Gain Decoupling
2. Optimal Blending
3. Optimal Decoupling
4. Analytic Decoupling

Before proceeding further, a more exact description of a system is necessary. Consider the system shown in Figure 1-1, where the aircraft dynamics are represented by:

$$\dot{x} = Ax + Bu \tag{1-1}$$

where x is an n order state vector; u is an m order control vector with m≤n and A and B are n x n and n x m constant matrices, respectively. Consider also the output y given by



Block Diagram Representation of Closed Loop Aircraft System

Figure 1-1

$$y = Cx \tag{1-2}$$

where C is an m x n constant matrix and y is an m output vector. The decoupling problem is: given a control law of the form

$$u = Fx + Gv \tag{1-3}$$

where v is an m order open loop control vector and F and G are m x n and m x m constant matrices respectively, choose F and G such that the closed loop transfer function matrix

$$H = C\,(Is - A - BF)^{-1}\,BG \tag{1-4}$$

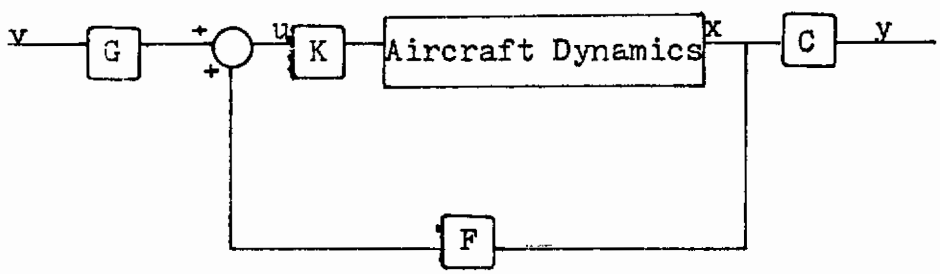
in the relation

$$y = Hv \tag{1-5}$$

is diagonal and non-singular.[24]

The High Gain decoupling method[14] inserts a feedforward gain matrix K as shown in Figure 1-2. By varying the elements of K, it is possible to obtain a closed-loop transfer function matrix whose off diagonal elements are small compared to the diagonal elements. The disadvantages of this method are that K must be chosen on a trial and error basis and that, obviously, the results are only an approximation to a de-coupled system.

Optimal Blending is defined as the transition process of phasing between the moment producing devices used in hover and those used in conventional flight. The control blender is that element of the flight control system which accepts commands from the pilot as inputs and commands motions of the various control surfaces of the vehicle as outputs, proportioned according to a predetermined function of thrust incidence angle. The design is partially based on generating



High Gain Decoupling

Figure 1-2

single axis moments in response to single axis stick or pedal inputs. The cost of control is measured on the basis of the blender output (control surface deflections) and this cost is then minimized using a quadratic performance index. The major disadvantage as stated in Reference 24 is that one cannot obtain pure moments in the roll and yaw axis since the blending is a function of a single flight parameter (thrust incidence angle) and, therefore, uncoupled moments can at best be generated only along the transition trajectory for which the blender was designed.

The basic technique of the optimal decoupling control method[25] is to construct an ideal decoupled model and apply quadratic optimal control theory to minimize the integral quadratic error between the system and model states. The advantage is simultaneous decoupling and optimization; but the error minimization, while theoretically feasible, may be difficult to implement.

The analytic decoupling methods seem to promise the best results when applied to V/STOL aircraft dynamics. In general, decoupling a system has always been desirable from a control engineer's point of view. Reference 26 states: "The response and dynamic characteristics may be easily designed one loop at a time without having to alter several loops when an interacting one is present" after a system has been decoupled. The first generally recognized attempt to decouple a system was performed on a jet engine system by Boksenbom and Hood.[27]

This early work evolved into treatment of multivariable systems, in general, with major contributions being made by Freeman[28,29], Kavanaugh[30], and Bohn[31]. The work most pertinent to this proposal, however, are the contributions of Morgan[32], Rekasius[33], Falb and Wolovich[34,35,36] and Gilbert[24,37,38].

In reference 32, Morgan initiated the current approach to the decoupling problem. His results were limited in the fact that the system given by Equations 1 through 5 could be decoupled only if CB was non-singular. Rekasius[33] extended Morgan's results and outlined an essentially trial and error procedure for specifying some of the systems poles along with the decoupling. In references 34 and 35, Falb and Wolovich presented all the essentials needed for decoupling a system. These essentials included: 1) necessary and sufficient conditions for decoupling; 2) description of a restricted class of control laws which decouple; and 3) necessary and sufficient conditions on F and G for decoupling. E. G. Gilbert, however, has brought the design technique to a high level of completeness. The following chapter presents a brief summary of Gilbert's decoupling method together with an explanatory example.

CHAPTER 2

GILBERT'S METHOD

Gilbert has presented an efficient means for determining both the class of all F and G which decouple a plant and the structure of the transfer function of each single-input single-output subsystem of the decoupled closed loop system.[24] His method can be broken down into three basic steps:

1. Check for ability to decouple
2. Determine the structure of the decoupled system
3. Calculate any compensation necessary

Since the calculations become unwieldy for systems of $n \geq 3$ (Reference 38 describes a computer program which handles problems of order $n \leq 25$ and $m \leq 10$), let us go through the steps outlined above for an arbitrary system, $n = m = 2$. Referring again to the systems represented by equations 1 and 2, let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \tag{2-1}$$

The necessary and sufficient conditions for decoupling are dependent on the non-singularity of the matrix D (B^* in reference 35) which is defined as

$$D = \begin{bmatrix} C_1 A^{d_1} B \\ C_2 A^{d_2} B \\ \vdots \\ C_m A^{d_m} B \end{bmatrix} \tag{2-2}$$

where C_i denotes the i - th row of C, i = 1,... m and d_i is defined as[35]

$$d_i = \min \left\{ j: \quad C_i A^j B \neq 0 \qquad j = 0,1 \ldots n-1 \right\}$$

$$d_i = n - 1 \text{ if } \quad C_i A^j B = 0 \qquad \text{for all } j \tag{2-3}$$

For our example $d_1 = d_2 = 0$ and therefore

$$D = \begin{bmatrix} 6 & 4 \\ 10 & 7 \end{bmatrix} \tag{2-4}$$

which is non-singular and thus the example system can be decoupled. This corresponds to Step 1 of Gilbert's method.

In Step 2, the structure of the decoupled system is determined. Calculate F^* and G^* using the equations

$$F^* = -D^{-1} A^* \tag{2-5}$$

where

$$A^* = \begin{bmatrix} C_1 A^{d_1+1} \\ \vdots \\ C_m A^{d_m+1} \end{bmatrix} \tag{2-6}$$

and

$$G^* = D^{-1} \tag{2-7}$$

Now, if we let

$$\bar{A} = A + BF^* \qquad \bar{B} = BG^* \qquad \bar{C} = C \tag{2-8}$$

our system is transformed to an integrator decoupled (ID) form

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}u \\ y &= \bar{C}x \end{aligned} \tag{2-9}$$

The integrator decoupled terminology is justified by the fact that

$$\bar{H}\ (s,\ 0,\ I) = \text{diagonal}\ (s^{-d_i-1}) \quad i\ 1,\ldots m \tag{2-10}$$

or that the plant transfer matrix relating y(s) and u(s) is diagonal with its i - th element being $1/s^{d_i+1}$.

It is a direct calculation to show that if

$$\bar{F} = DF + A^* \qquad \bar{G} = DG \tag{2-11}$$

then $H\ (s,\ F,\ G) = \bar{H}\ (s,\ \bar{F},\ \bar{G})$ or that S and $\bar{S}$ are control law equivalent (CLE). The systems S and $\bar{S}$ are CLE in the sense that there is a one-to-one relationship between $\{F,\ G\}$ and $\{\bar{F},\ \bar{G}\}$ such that $H\ (s,\ F,\ G) = H\ (s,\ \bar{F},\ \bar{G})$. For the ID system $\bar{S}$ it is also a direct calculation to show that

$$\bar{D} = I \qquad \text{and} \qquad \bar{A}^* = 0 \tag{2-12}$$

Applying this to our example

$$F^* = \begin{bmatrix} 4 & -5 \\ 5 & 6 \end{bmatrix}$$

$$G^* = \begin{bmatrix} 7/2 & -2 \\ -5 & 3 \end{bmatrix}$$

$$H(s,F^*,G^*) = \begin{bmatrix} 1/s & 0 \\ 0 & 1/s \end{bmatrix}$$

$$\bar{D} = I \qquad \bar{A}^* = 0 \tag{2-13}$$

At this point the system has been decoupled. For the sake of brevity, therefore, reference to the example system will be terminated here.

Continuing in Step 2, the integrator decoupled system $\bar{S}$ is checked for controllability. If it is not controllable, another transformation is introduced which separates the controllable part $\bar{S}^c$ from the uncontrollable part $\bar{S}^u$. The system $\bar{S}^c$ is again CLE to the ID system $\bar{S}$.

The final stage of Step 2 is to transform $\bar{S}^c$ to canonically decoupled form. Let new state variables be designated $\hat{x}$, defined in terms of x and the non-singular matrix Q as

$$\hat{x} = Qx \tag{2-14}$$

The state equations in terms of $\hat{x}$ are

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u = Q(A + BF^*)Q^{-1}\hat{x} + QBG^*u$$

$$y = \hat{C}\hat{x} = CQ^{-1}\hat{x} \tag{2-15}$$

where Q is defined by

$$Q = \begin{bmatrix} Q_1 \\ \vdots \\ Q_{m}1 \end{bmatrix} \qquad Q_i = \begin{bmatrix} c_i^{\,c} \\ c_i^{\,c} A^c \\ \vdots \\ c_i^{\,c} (A^c)^{d_i} \\ q_i^{\,1} \\ \vdots \\ q_i^{\,p_i - d_i - 1} \end{bmatrix} \qquad i = 1,\dots m \tag{2-16}$$

$$Q_{m+1} = \begin{bmatrix} q_{m+1}^{\,1} \\ \vdots \\ q_{m+1}^{\,p_{m+1}} \end{bmatrix}$$

where the c superscript refers to the controllable part matrices and the last p_i-d_i rows are any row vectors, which together with the first d_i+1 rows, form a basis for the p_i dimensional (row) vector space Q_i and where $p_i \geq 0$, i 1,...m, $p_{m+1} = r_{m+1} \geq 0$, $p_{m+2} = r_{m+2}$, $r_i \geq 0$, i = 1,...m. With the system in canonically decoupled form, we may proceed to Step 3.

It is shown that a necessary and sufficient condition for the decoupling of the CD system $\hat{S}(s,\hat{F},\hat{G})$ is that

$$F = \begin{bmatrix} \theta_1 & 0 & \cdots & 0 & 0 & \theta_1^u \\ 0 & \theta_2 & \cdots & 0 & 0 & \theta_2^u \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & \theta_m & 0 & \theta_m^u \end{bmatrix} \quad G = \text{diagonal } (\lambda_1 \ldots \lambda_m) \tag{2-17}$$

where θ_i, i = 1,...m is the 1 x p_i row matrix and is found by compensating the i-th single-input single-output subsystem i and λ_i is the desired gain of subsystem i. The choice of θ_i^u involves the "uncontrollable part" of $\hat{S}$ and hense has no effect on the diagonal elements $\hat{h}_i(s,\hat{F},\hat{G})$ which again are CLE to $h_i(s,F,G)$. The compensation matrices for equations (2-1) and (2-2) are then

$$F = -D^{-1} A^* + \sum_{i=1}^{m} \sum_{k=1}^{p_i} (\sigma_{ik} - \pi_{ik}) J_k^i + \sum_{i=1}^{m} \sum_{k=1}^{r_{m+2}} \rho_{ik} K_{ik} \tag{2-18}$$

$$G = \sum_{i=1}^{m} \lambda_i G_i \tag{2-19}$$

where the following parameters are obtained from the original system S(A,B,C): integers $p_i>0$, $i=1,...m$; integers $r_i\geq 0$, $i=1...m+2$ ($r_i<p_i$, $i=1,...m$ and $r_{m+2}\neq 0$ if and only if S is not controllable); polynomials $\alpha_i(s)=s^{r_i}-\alpha_1 s^{r_i-1}...\alpha_{r_i}$, $i=1,...m+2$ (if $r_i=0, \alpha_i(s)=1$); mxm matrices G_i, $i=1,...m$; mxn matrices J_k^i, $i=1,...m$, $k=1,...p_i$; mxn matrices K_{ik}, $i=1,...m$, $k=1,...r_{m+2}$ (the K_{ik} are not defined if $r_{m+2}=0$); mxn matrix A^*; and where $\pi_{ik}=\alpha_{ik}$, $k=1,...r_i$, $\pi_{ik}=0$, $k=r_{i+}1...p_i$; and λ_i, σ_{ik}, ρ_{ik} are arbitrary real numbers ($\lambda_i\neq 0$, $i=1,...m$). The last term in (23) is missing if $r_{m+2}=0$.

The elements of the diagonal matrix H(s,F,G) may then be represented by

$$h_i(s,F,G)=\frac{\lambda_i\alpha_i(s)}{\psi_i\ (s,\sigma_i)} \tag{2-20}$$

$$\psi_i(s,\sigma_i)=s^{p_i}-\sigma_{i1}s^{p_i-1}...-\sigma_{ip_i}$$

$$\sigma_i=(\sigma_{i1}...\sigma_{ip_i}) \tag{2-21}$$

where λ_i and σ_i, $i=1,...m$ may be chosen arbitrarily.

The characteristic equation of the closed loop system is

$$q(s,F)=\det(Is-A-BF)=\alpha_{m+1}(s)+\alpha_{m+2}(s)\prod_{i=1}^{m}\psi_i(s,\sigma_i) \tag{2-22}$$

In summary, if matrix D of equation (2-2) is non-singular, the system can be decoupled. The structure of each single-input single-output subsystem is given by equations (2-20), (2-21), and (2-22) where the λ's and σ's are determined by the designer. All other parameters are obtained from the computer program mentioned in reference 38. Appendix C presents the flow chart for this program. Once a particular set of $h_i(s,F,G)$ is specified, equations (2-18) and (2-19) give the corresponding control law $\{F,G\}$.

CHAPTER 3

DIGITAL ANALYSIS

As mentioned in Chapter 1, the landing phase of flight is most critical. Most simulations use the equations of motion associated with this phase and there is, therefore, considerable data available. Appendix A shows that the equations of motion for a STOL aircraft in the landing phase can be separated into longitudinal and lateral-directional equations which are uncoupled from each other. To begin the analysis of the application of Gilbert's decoupling method to a STOL aircraft system, let us first look at the longitudinal dynamics for the landing phase.

Longitudinal Dynamics

The longitudinal equations are represented by four state variables: u - forward speed, θ - pitch, $\dot{\theta}$ - pitch rate, and $\dot{z}$ - vertical velocity, and by two control variables δ_e - elevator deflection, and δ_t - thrust changes. The two control variables are driven by two cockpit control inputs: $\delta_{e_{stick}}$ - longitudinal stick deflection, and $\delta_{t_{throttle}}$ - throttle setting, respectively. Clearly, there are fewer control inputs than state variables (m<n). Consequently, decoupling in the sense that each input affect one and only

one output is not possible. Since there are but two control inputs to three state variables (if θ and $\dot{\theta}$ are combined into one variable), the most advantageous arrangement is to assign the two control inputs, one each, to two state variables such that the uncontrolled variable is not severely affected by inputs to the other state variables. "Uncontrolled variable" here is taken to mean the variable to which no control input has been assigned.

This brings us to the question of what control inputs should be assigned to what state variables.

In the longitudinal mode for STOL aircraft, large pitching moment changes occur with power changes. Despite this, pitch attitude is usually controlled by elevator deflections through longitudinal stick positioning while the throttle is used to control vertical speed. The remaining variable, airspeed, is controlled through pitch attitude in many cases and by throttle in cases where it is desirable that pitch attitude be maintained. Obviously, power changes affect response in pitch attitude, and airspeed and the same is true with elevator deflections. As hinted at above, different controls can be applied to obtain the same response. The question then arises, "What control does the pilot consider primary in controlling a particular variable?" The choice is a function of the task. As stated previously, this analysis is concerned with the landing approach phase of flight and the pilot's ability to track a localizer and glide slope. Traditionally, designers have always considered attitude loops, i.e., those in which roll is controlled by the ailerons and pitch by the elevator, as the most important single loop aircraft control systems[39] With longitudinal stick deflection controlling pitch, the throttle is left to control either forward velocity, vertical velocity, or flight path angle γ. (γ is equal to pitch angle θ minus angle of attack α. α may be approximated by $\dot{z}/U_o$ or vertical velocity over unperturbed forward velocity[40].) This analysis will deal with configurations that are combinations of longitudinal stick as primary control of pitch attitude and of throttle as primary control of either I) vertical velocity, II) flight path angle, or III) forward velocity. The analysis will begin with case I.

For convenience' sake, let us rewrite the equations that represent our system.

$$\dot{x} = Ax + Bu \tag{3-1}$$

$$y = Cx \tag{3-2}$$

Let the state variables be:

$x_1 = u$ = forward speed

$x_2 = \theta$ = pitch

$x_3 = \dot{\theta}$ = pitch rate

$x_4 = \dot{z}$ = vertical velocity

and let the control variables be:

$u_1 = \delta_e$ = elevator deflection

$u_2 = \delta_t$ = thrust change

Since, in this case we are concerned with vertical velocity and pitch rate (pilots generally prefer to control attitude changes by sensing pitch rate), let the output variables be:

$$y_1 = x_3$$
$$y_2 = x_4$$

Substituting the data obtained in Appendix A, equations (1) and (2) become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -.032 & -32.2 & 0 & .133 \\ 0 & 0 & 1.0 & 0 \\ .00137 & 0 & -.743 & -.0014 \\ -.02 & 4.2 & 96.5 & -.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & .00065 \\ 0 & 0 \\ -.989 & -.000007 \\ 3.0 & -.00087 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (3\text{-}3)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (3\text{-}4)$$

Equations (3-3) and (3-4) represent the open loop system or the unaugmented aircraft dynamics. The loop is closed by applying the feedback control law:

$$u = Fx + Gv \tag{3-5}$$

where the control variables are:

$v_1 = \delta_{e\ stick}$ = longitudinal stick displacement

$v_2 = \delta_{t\ throttle}$ = throttle setting

Implementation of equation (3-5) with the proper values for the F and G matrices results in both the decoupling of the system and the evolution of a simpler procedure for designing a stability augmentation system.

Traditional methods for designing SAS's have been the use of root locus and Bode plot analyses. Though some engineers would not agree, these methods are limited because they are graph oriented and do not lend themselves to easy analysis of multiple loop systems. Personal contact with both methods in design analysis problems has involved at best tedious work, though digital computers can be employed to lessen some of the drudgery.

An alternative is the use of modern control techniques. Modern control techniques are the product of computer technology and its use has increased correspondingly with the sophistication of computer applications. Computers have been an invaluable aid in the ability to handle swiftly the iterative processes needed to find maxima and minima which are often encountered in the use of modern control techniques. The approach used here will be a combination of both in that state space representation will eventually lead to transfer function analysis.

Since the majority of previous work in the design of SAS's has employed classical techniques, specification of criteria for the desirable responses of a system has also involved classical control parameters, e.g., crossover frequency, phase angles, etc. These parameters are not applicable to the design analysis of this study. Also, past analyses of aircraft systems have always considered coupled responses and the transfer functions associated with these responses. The decoupling procedure eliminates the traditional transfer functions which determined restrictions on certain parameters, e.g., $\frac{\Delta\beta}{\Delta\phi}$, which had to be satisfied for a good handling aircraft. Until recently, handling qualities specifications have been extremely vague. Reference 41, however, is the first document in which criteria for desirable transient response of V/STOL aircraft is precisely defined. Until very recently,* there have been no official criteria set forth by any government agency, but reference 41 presents much needed initial guidelines.

* Criteria given in Reference 47 is comparable to that taken from Reference 41.

Table 3-1

Pitch Transient Response Criteria

PARAMETER TO BE MEASURED	MINIMUM LEVELS FOR SATISFACTORY OPERATION
1) pitch angular acceleration per unit control deflection (rad / sec^2 / in)	.08 - .12
2) pitch angle after 1 sec.°	2 - 4
3) damping ratio	≤15% overshoot

Table 3-1 presents the minimum values of aircraft response for low speed flight following an abrupt step input of longitudinal stick. "Aircraft whose missions require extensive ... low speed maneuvering should as a minimum meet the upper levels of values shown, while those for which maneuvering is only incidental to the mission and for which thrust vectoring can also be used, should as a minimum meet only the lower values."[41]

In satisfying these requirements together with the decoupling, selection of F and G must guarantee that deflections of control surfaces are within realizable limits. For our model, a full 6-inch deflection aft produces a minus 35° (-.611 rad.) elevator deflection up, and a full 5-inch deflection forward produces a 25° (.436 rad.) elevator deflection down. (See Table A-1).

With these restrictions in mind, we may now apply the decoupling program to Equation (3-3) and (3-4). Pertinent data may be found in Table 3-2.

Substituting the data presented in Table 3-2, Equation (2-20) becomes:

$$h_1 = \frac{\dot{\theta}}{\delta_e \text{ stick}} = \frac{\lambda_1 s}{s^2 - \sigma_{11} s - \sigma_{12}} \tag{3-6}$$

$$h_2 = \frac{\dot{z}}{\delta_T \text{ throttle}} = \frac{\lambda_2}{s - \sigma_{21}} \tag{3-7}$$

and the characteristic equation is:

$$q(s, F, G) = (s + .04356)\ (s^2 - \sigma_{11} s - \sigma_{12})\ (s - \sigma_{21}) \tag{3-8}$$

Since $m<n$, all closed loop poles can not be arbitrarily positioned by the designer, thus the appearance of the first factor on the right-hand side of the equation (3-8). Since this pole is in the left-hand plane, our system remains stable, and no problem is created here.

Besides the actual physical decoupling of aircraft response, use of Gilbert's method in design analysis has another unique advantage. Classical techniques, when applied to multiple loop systems are awkward because they deal with nested loops in which inner loops must be closed and their specifications met before outer loops can be analyzed.

Table 3-2

Computer Output For $\dot{\theta} \rightarrow \delta_{e\ stick}$, $\dot{z} \rightarrow \delta_{T\ throttle}$ Case

$$|D| = .0008814$$

$d_1 = 0$ $p_1 = 2$ $r_1 = 1$ $\pi_{11} = 0$

$d_2 = 0$ $p_2 = 1$ $r_2 = 0$ $\pi_{12} = 0$

$p_3 = 1$ $r_3 = 1$ $\pi_{21} = 0$

$\pi_{31} = -.04356$

$$D^{-1}A^* = \begin{bmatrix} -.00151 & .03335 & 1.4497 & -.001 \\ 17.77779 & -4712.569 & -105748.046 & 341.377 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -.987 & 0 \\ -3403.560 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & .0079 \\ 0 & -1122.04 \end{bmatrix}$$

$$J_1^1 = \begin{bmatrix} 0 & 0 & -.987 & 0 \\ 0 & 0 & -3403.56 & 0 \end{bmatrix}$$

$$J_2^1 = \begin{bmatrix} 0 & -.987 & 0 & 0 \\ 0 & -3403.56 & 0 & 0 \end{bmatrix}$$

$$J_1^2 = \begin{bmatrix} 0 & 0 & 0 & .0079 \\ 0 & 0 & 0 & -1122.04 \end{bmatrix}$$

Use of Gilbert's method permits the designer to analyze each single input-single output system separately. Thus we can look at equations (3-6) and (3-7) separately.

In equation (3-6), λ_1, σ_{11}, and σ_{12} must be chosen so that pitch response satisfies the criteria of Table 3-1 and that elevator deflection limitations are not exceeded. A computer program (Appendix D) was written to obtain the transient response and control surface deflections produced by a step change on a cockpit control. The pitch response can be thought of as the response of a second order system where $\sigma_{11} = -2\zeta\omega$ and $\sigma_{12} = -\omega^2$. It is important to note at this point that any similarity between the transfer functions obtained from Equation (2-20) and standard approximations to aircraft transfer functions is coincidental (Equation (3-6) is similar to the short period approximation). With this in mind, the designer can only guess initially at values for the parameters λ_1, σ_{11}, σ_{12} and then analyze the transient response. The guesses are educated in that the designer can apply the Final Value theorem to determine steady state response and that he also knows a range of values for ζ and ω which produce good response for a second order system. Final Value theorem applied to equation (3-6) with a step input reveals that the steady state pitch angle is $\lambda_1/(-\sigma_{12})$. It is also well known that second order systems with a damping ratio $\zeta \geq .5$ usually have good responses. After making our initial guesses, we substitute the values into equations (2-18) and (2-19) which in this case become:

$$F = -D^{-1}A^{*} + \sum_{i=1}^{2} \sum_{k=1}^{2} \sigma_{ik} J_k^i \tag{3-9}$$

$$G = \sum_{i=1}^{2} \lambda_i G_i \tag{3-10}$$

Note that at this point we have not specified values for σ_{21}, and λ_2. Since $\dot{\theta}$ is decoupled from $\dot{z}$, any values may be substituted and not affect the response of $\dot{\theta}$. For example, let $\sigma_{21} = -1$, $\lambda_2 = 1$. Table 3-3 shows the response of the system to different values of λ_1, σ_{11}, and the corresponding control surface deflections. Note that Table 3-3 presents data which was recorded for Case III. However, pitch response analysis and the data obtained from this analysis would be the same in Cases I, II and III. As will later be seen, the pitch response parameters that were obtained in the Case I analysis, were also used in Cases II and III. The final values settled upon were $\sigma_{11} = -1.6$, $\sigma_{12} = -1$, and $\lambda_1 = .087$. This corresponds to a $\zeta = .8 \text{ sec}^{-1}$ and $\omega = 1(\text{rad})/\text{sec}$. Table 3-4 shows a comparison between pitch response and pitch criteria. We see that both pitch angular acceleration and pitch angle after one second are low, but not by much. Both of these parameters could be increased by increasing λ_1. This would, however, incur the risk of exceeding elevator deflection limitations. Pitch angle after one second could be increased if the damping ratio were increased. However, we are approaching critical damping, and

Run	ω (rad/sec)	ζ (sec^{-1})	σ_{21} (sec)	Stick def (in)	Throttle def (in)	Decoupled Variables (steady state values) θ(rad)	u(ft/sec)	δ_e(max) (rad)	δ_T(max) (lbs)
1	2	.5	-1	1	0	.249	0	-.911	9269
2	2	.5	-1	0	1	0	.99	.001	1095
3	2	.5	-1	1	0	.498	0	-1.82	18,539
4	2	.5	-1	0	1	0	1.99	.003	2191
5	2	.7	-1	1	0	.249	0	-.844	7586
6	2	.5	-1	1	0	.086	0	-.316	3712
7	2	.5	-1	0	1	0	2.099	.003	2301
8	2	.5	-1	0	-1	0	-2.099	-.003	-2301
9	2	.5	-1	1	0	.608	0	-2.23	22,313
9A	2	.5	-1	7.0	0	.605	0	-2.2	22,494
10	2	.5	-1	-5.0	0	-.432	0	1.58	-16,067
11	2	.7	-1	1	0	.249	0	-.844	7865
12	3	.5	-1	1	0	.110	0	-.807	6452
13	1	.5	-1	1	0	.084	0	-.088	1416
14	1	.7	-1	1	0	.085	0	-.085	1720
15	1	.7	-1	7.0	0	.599	0	-.595	8478

Case III Time History Data

Table 3-3

pilots often complain of aircraft that are overdamped as being sluggish. This will be investigated in the simulation in Chapter 4.

Table 3-4

Pitch Criteria and Response Comparison

	$\ddot{\theta}$ (rad/sec^2/in)	θ after 1 sec(deg)	Over-shoot	δ_e max def (rad)
Criteria	.08	2	.15	.61
Response	.073	1.71	.01	.504

Equation 3-7 is somewhat simpler to handle. The response is exponential so in picking σ_{21}, we are actually picking a time delay. A time delay of one second should be adequate, making $\sigma_{21} = -1$. In choosing λ_2 we may make use of the fact that with $\sigma_{21} = -1$ steady state response of $\dot{z}$ to a step input is λ_2, and also of the fact that the landing technique used for our model was to hold a three degree angle of attack until flare.[16] Since $\alpha = \dot{z}$ /100 ft./sec. and at steady state $\dot{z} = \lambda_2$, $\lambda_2 = 100$ ft/sec $\cdot$ $3^\circ = 5.3$. However, since $\dot{z}$ has been oriented so that positive is down, the sign of λ_2 must be changed so that a positive throttle, i.e., add power, produces an increase in lift and consequently a negative $\dot{z}$. Thus $\lambda_2 = -5.3$. Applying a one inch positive throttle change, the three degree angle of attack is achieved in approximately seven seconds with, of course, no pitch axis response.

With all five parameters specified the feedback control law $\{ F, G \}$ is completely specified

$$\sigma_{11} = -1.6 \qquad \lambda_1 = .087$$
$$\sigma_{12} = -1 \qquad \lambda_2 = -5.3$$
$$\sigma_{21} = -1$$

$$F = \begin{bmatrix} .00151 & .9536 & .0792 & -.0069 \\ -17.778 & 8116.13 & 111{,}193.75 & 780.663 \end{bmatrix}$$

$$G = \begin{bmatrix} -.086 & -.042 \\ -296.11 & 5946.81 \end{bmatrix}$$

Once F and G have been completely specified, we may look at the response of the "uncontrollable variable" which in this case is perturbation to forward speed, u. The response of u follows what would be expected in a typical aircraft in that for a new positive constant pitch attitude, u is negative and aircraft airspeed decreases. As stated earlier, aircraft speed is often controlled through pitch attitude, so that in this case, u is uncontrollable only in the sense that no specific control has been assigned to it. Through experience with the aircraft's speed response to pitch variations, the pilot does then have effective control of airspeed. As an indication of this speed response, u decreases by 10 feet/second in 8 seconds for a one-inch stick deflection, and increases 6 feet/second in 10 seconds for a one-inch throttle deflection.

On paper, the control configuration appears to be satisfactory. Simulation will further test it in Chapter 4.

A second possible configuration is to continue to control pitch with longitudinal stick, but to use the throttle to control flight path angle γ. Matrices A and B remain the same, but matrix C becomes

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -.01 \end{bmatrix}$$

since flight path angle $\gamma = \theta - \alpha$ and $\alpha = \dot{z}/100$ ft/sec.

Pertinent data from the decoupling program can be found in Table 3-5. Substituting this data in equation 2-20, we obtain

$$h_1 = \frac{\dot{\theta}}{\delta_{e\ \text{stick}}} = \frac{\lambda_1 s}{s^2 - \sigma_{11}s - \sigma_{12}} \qquad (3\text{-}11)$$

$$h_2 = \frac{\gamma}{\delta_{T\text{throttle}}} = \frac{\lambda_2}{s - \sigma_{21}} \qquad (3\text{-}12)$$

and the characteristic equation remains the same:

$$q(s,\ F,\ G) = (s + .04356)\ (s^2 - \sigma_{11}s - \sigma_{12})\ (s - \sigma_{21}) \qquad (3\text{-}13)$$

Pitch parameters σ_{11}, σ_{12} and λ_1 have been obtained in the previous case.

Table 3-5

Computer Output for $\dot{\theta} \rightarrow \delta_{e_{stick}}$, $\gamma \rightarrow \delta_{T_{throttle}}$ Case

$$|D| = .0008814$$

$d_1 = 0$	$p_1 = 2$	$r_1 = 1$	$\pi_{11} = 0$
$d_2 = 0$	$p_2 = 1$	$r_2 = 0$	$\pi_{12} = 0$
	$p_3 = 1$	$r_3 = 1$	$\pi_{21} = 0$
			$\pi_{31} = -.04356$

$$D^{-1}A^* = \begin{bmatrix} -.00151 & .03335 & .70557 & -.001 \\ 17.7779 & -4712.569 & 6455.986 & 341.377 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -.987 & 0 \\ -3403.56 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & -.794 \\ 0 & 112204.032 \end{bmatrix}$$

$$J_1^1 = \begin{bmatrix} 0 & 0 & -.987 & 0 \\ 0 & 0 & -3403.56 & 0 \end{bmatrix}$$

$$J_2^1 = \begin{bmatrix} 0 & -.987 & 0 & 0 \\ 0 & -3403.56 & 0 & 0 \end{bmatrix}$$

$$J_1^2 = \begin{bmatrix} 0 & -.794 & 0 & 0 \\ 0 & 112204.032 & 0 & 0 \end{bmatrix}$$

Flight path angle response like angle of attack response in the previous case is exponential, so again we may let $\sigma_{21} = -1$. The gain λ_2 is also chosen in a similar manner. Reference 42 suggests a flight path angle of $7\frac{1}{2}°$ for landing approaches. Again using the Final Value theorem, steady state γ is equal to λ_2 if $\sigma_{21} = -1$. Therefore, let $\lambda_2 = 7\frac{1}{2} = .13$. Applying a one-inch negative throttle change, γ reaches steady state angle of $-7\frac{1}{2}°$ in approximately six seconds.

With all five parameters again specified, the feedback control law F, G becomes

$$F = \begin{bmatrix} .00151 & 1.7478 & .8736 & -.0069 \\ -17.778 & -104,087.901 & -1010.29 & 780.663 \end{bmatrix}$$

$$G = \begin{bmatrix} -.086 & -.103 \\ -296.11 & 14586.525 \end{bmatrix}$$

$$\sigma_{11} = -1.6 \qquad \lambda_1 = .087$$

$$\sigma_{12} = -1. \qquad \lambda_2 = .13$$

$$\sigma_{21} = -1$$

Again, the "uncontrolled variable" is forward speed u, and the same comments that applied to the previous configuration apply here. Speed response in this case shows that for a one-inch forward stick deflection, u increases by approximately 23.4 feet/second in ten seconds. For a one-inch positive throttle, u increases by approximately fifteen feet/second in ten seconds.

This case is somewhat interesting. Flight path angle γ has been decoupled from pitch θ. However, $\gamma = \theta - \alpha$.

Therefore, a throttle deflection produces no change in pitch and consequently γ is a function of α only. The steady state $-7\frac{1}{2}^\circ$ flight path angle is thus equivalent to a positive $7\frac{1}{2}^\circ$ angle of attack. Likewise, a longitudinal stick deflection will result in a compensating change in angle of attack so that there is no change in flight path angle. For a stick deflection, then, $\alpha = \theta$.

In the previous case, the pilot also had control of flight path angle since he had separate control in θ and α. The previous case also required less thrust and the resulting changes in u were less.

A third possible configuration is to again control pitch response with longitudinal stick deflection, and use throttle to control forward speed. This configuration is more common sense-oriented since it extends the automobile practice of controlling speed with accelerator. Matrices A and B of course remain the same, but

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Pertinent data from the decoupling program is presented in Table 3-6. Substituting this data into equation (2-20) we obtain:

$$h_1 = \frac{\dot{\theta}}{\delta_{e_{stick}}} = \frac{\lambda_1 s}{s^2 - \sigma_{11}s - \sigma_{12}} \qquad (3\text{-}14)$$

$$h_2 = \frac{u}{\delta_{T_{throttle}}} = \frac{\lambda_2}{s - \sigma_{21}} \qquad)3\text{-}16)$$

Table 3-6

Computer Data for $\dot{\theta} \rightarrow \delta_{e_{stick}}$, $u \rightarrow \delta_{T_{throttle}}$ Case

$$|D| = -.0006429$$

$d_1 = 0 \quad p_1 = 2 \quad r_1 = 1 \quad \pi_{11} = 0$

$d_2 = 0 \quad p_2 = 1 \quad r_2 = 0 \quad \pi_{12} = 0$

$p_3 = 1 \quad r_3 = 1 \quad \pi_{21} = 0$

$\pi_{31} = -.1219$

$$D^{-1}A^{*} = \begin{bmatrix} -.00104 & .3506 & .75126 & -.00003 \\ -49.23 & -49{,}538.47 & 0 & 204.61 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -1.0111 & 0 \\ 0 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & -.010889 \\ 0 & 1538.46 \end{bmatrix}$$

$$J_1^1 = \begin{bmatrix} 0 & 0 & -1.0111 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_2^1 = \begin{bmatrix} 0 & -1.0111 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_1^2 = \begin{bmatrix} -010889 & 0 & 0 & 0 \\ 1538.46 & 0 & 0 & 0 \end{bmatrix}$$

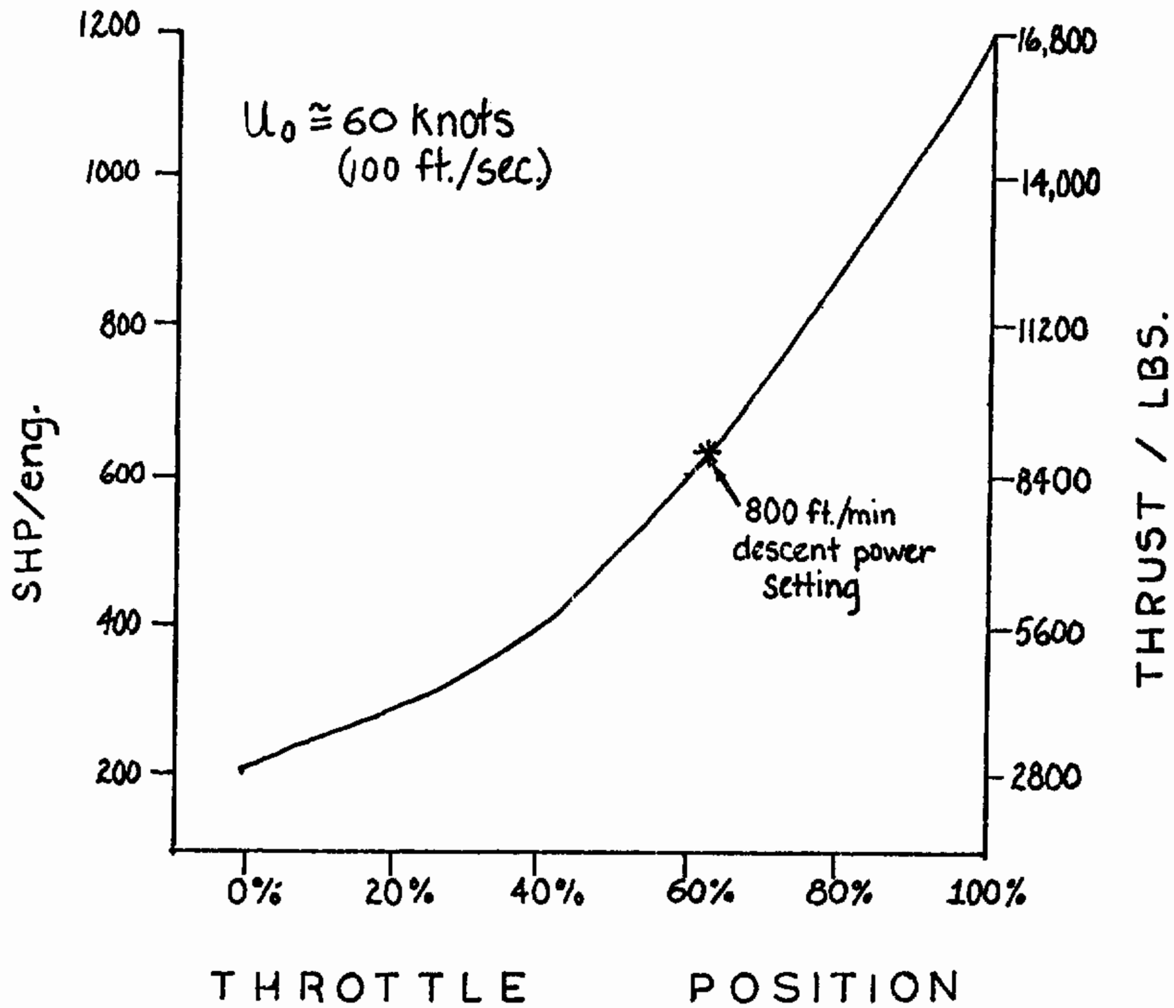


Figure 3-1

THRUST-THROTTLE CURVE

Pitch response parameters have previously been obtained.

Again, because of the exponential nature of the response, let $\sigma_{21} = -1$. Picking the gain λ_2 is slightly more difficult. The Final Value theorem is not very helpful since there are no criteria for specifying response. From Figure 10 in reference 16 and from Figure 3-1 below, it is possible to obtain a relationship between throttle deflection and change in forward speed. From these graphs, it was crudely determined that in the landing configuration, speed increases 2.1 feet/second for a one-inch positive throttle deflection. Therefore, let $\lambda_2 = 2.1$. The value selected for this gain is not critical since the overall desired response is that forward speed remain constant for changes in pitch and vertical velocity.

With all five parameters specified, the feedback control law $\{F, G\}$ for this case becomes:

$$\sigma_{11} = -1.6 \qquad \lambda_1 = .087$$
$$\sigma_{12} = -1 \qquad \lambda_2 = +2.1$$
$$\sigma_{21} = -1$$

$$F = \begin{bmatrix} .01193 & .6605 & .6643 & .00003 \\ 1489.23 & 49{,}538.47 & 0 & -204.615 \end{bmatrix}$$

$$G = \begin{bmatrix} -.088 & -.0229 \\ 0 & 3{,}230.766 \end{bmatrix}$$

The "uncontrolled variable" in this case is angle of attack. Since we are concerned with keeping forward speed constant, the only important response is that due to stick deflection. For a one-inch forward stick deflection, transient angle of attack never exceeds 1° which is within allowable limits.

Reference 16 states, "The approach is made by maintaining a constant angle of attack of 3° while controlling flightpath angle with power." This is equivalent to controlling angle of attack with longitudinal stick deflection and flight path angle with throttle. Matrix C is:

$$C = \begin{bmatrix} 0 & 0 & 0 & .01 \\ 0 & 1 & 0 & -.01 \end{bmatrix}$$

Computer output gives $d_1 = d_2 = 0$, and therefore

$$D = \begin{bmatrix} C_1 B \\ C_2 B \end{bmatrix} = \begin{bmatrix} .03 & .000009 \\ -.03 & .000009 \end{bmatrix}$$

Clearly, the determinant of D is zero and the system consequently cannot be decoupled. A partial explanation is that the second row of the B matrix negates the C_{22} element in the matrix multiplication to get CB. Theoretically, this means we are trying to control α with longitudinal stick deflection and $-\alpha$ with the throttle, which is physically impossible.

This concludes the transient analysis of the longitudinal responses.

Lateral Dynamics

The lateral equations are represented by five state variables: v - slideslip velocity; ϕ - roll rate; ψ - yaw; $\dot{\psi}$ - yaw rate; and by two control variables, δ_s - spoiler deflection; and δ_r - rudder deflection. The two control variables are driven by two cockpit control inputs: $\delta_{s_{stick}}$ - lateral stick deflection, and $\delta_{r_{pedal}}$ - pedal deflection, respectively. As in the longitudinal case, there are fewer cockpit controls than variables so there can be no "pure" decoupling. The control problems stated in Chapter 1 suggest what variables need to be decoupled. Roll attitude should primarily be controlled by lateral stick deflection. As previously mentioned, there is an undesirable yaw response to lateral stick deflection. This suggests decoupling yaw and roll response. Of course, the pilot also wishes to eliminate all sideslip when he is trimmed up on approach. This suggests decoupling roll and sideslip. Both of these configurations will be examined.

First, let us consider the case where lateral stick deflection controls roll response and rudder pedal deflections control yaw response. Let the state variables be defined:

$$x_1 = v = \text{sideslip velocity}$$
$$x_2 = \phi = \text{roll}$$
$$x_3 = \dot{\phi} = \text{roll rate}$$
$$x_4 = \psi = \text{yaw}$$
$$x_5 = \dot{\psi} = \text{yaw rate}$$

and let the control variables be defined:

$$u_1 = \delta_s = \text{spoiler deflection}$$
$$u_2 = \delta_r = \text{rudder deflection}$$

Substituting the data obtained in Appendix A, equations (3-1) and (3-2) become:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -.13 & 32.2 & 0 & -4.2 & -100.0 \\ 0 & 0 & 1.0 & 0 & 0 \\ -.00322 & 0 & -.82 & 0 & .139 \\ 0 & 0 & 0 & 0 & 1.0 \\ .0054 & 0 & -.05 & 0 & -.33 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 5.0 \\ 0 & 0 \\ 1.337 & .0716 \\ 0 & 0 \\ -.125 & -.246 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (3\text{-}17)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad (3\text{-}18)$$

Again, the loop is closed by applying the feedback control law:

$$u = Fx + Gv \qquad (3\text{-}19)$$

where:

$$v_1 = \delta_{s_{stick}} = \text{lateral stick deflection}$$

$$v_2 = \delta_{r_{pedal}} = \text{rudder pedal deflection}$$

For guidelines as to what type of response is desirable let us again refer to reference 41. Table 3-7 presents the minimum values of roll response to a step input.

Spoiler deflection must also fall within acceptable limits which, as Table A-1 in Appendix A shows must be within .79 radians.

Table 3-7

Roll Transient Response Criteria

Parameter to be measured	Minimum levels for satisfactory operation
1) roll angular acceleration/unit control deflection (rad/sec^2/in)	.05 — .25
2) roll angle after one second (deg)	2 — 4
3) damping ratio	$\leq$15% overshoot

Table 3-8 presents the minimum values of yaw response to a step input which must be satisfied for satisfactory operation.

Rudder deflection must be less than .7 rad for both right and left rudder deflections.

These guidelines must be satisfied as we analyze the computer results presented in Table 3-9.

Table 3-8

Yaw Transient Response Criteria

Parameter to be measured	Minimum levels for satisfactory operation
1. yaw angular acceleration per unit control deflection (rad/sec^2/in)	.05 — .10
2. time for 15° heading change (seconds)	2

Substituting the data presented in Table 3-9, equation (2-20) becomes

$$h_1 = \frac{\dot{\phi}}{\delta_{s_{stick}}} = \frac{\lambda_1 s}{s^2 - \sigma_{11} s - \sigma_{12}} \qquad (3\text{-}20)$$

$$h_2 = \frac{\dot{\psi}}{\delta_{r_{pedal}}} = \frac{\lambda_2 s}{s^2 - \sigma_{21} s - \sigma_{22}} \qquad (3\text{-}21)$$

and the characteristic equation is:

$$q(s, F, G) = (s + .02356)\,(s^2 - \sigma_{11} s - \sigma_{12})\,(s^2 - \sigma_{21} s - \sigma_{22}) \qquad (3\text{-}22)$$

Again we have the advantages of looking at equations (3-20) and (3-21) separately. We may again use the Final Value theorem and the knowledge we have gained in dealing with second order systems. Table 3-10 presents the transient response of equation (3-20) to step inputs (→indicates

Table 3-9

Computer Data for the $\dot{\phi} \rightarrow \delta_{s_{stick}}$, $\psi \rightarrow \delta_{r_{pedal}}$ Case

$$|D| = -.3202$$

$d_1 = 0$ $\quad$ $p_1 = 2$ $\quad$ $r_1 = 1$ $\quad$ $\pi_{11} = 0$

$d_2 = 0$ $\quad$ $p_2 = 2$ $\quad$ $r_2 = 1$ $\quad$ $\pi_{12} = 0$

$p_3 = 1$ $\quad$ $r_3 = 1$ $\quad$ $\pi_{21} = 0$

$\pi_{22} = 0$

$\pi_{31} = -.02356$

$$D^{-1}A^* = \begin{bmatrix} -.00127 & 0 & -.64165 & 0 & .033 \\ -.02129 & 0 & .52912 & 0 & 1.32356 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} .76885 & 0 \\ -.3907 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & .223 \\ 0 & -4.175 \end{bmatrix}$$

$$J_1^1 = \begin{bmatrix} 0 & 0 & .76885 & 0 & 0 \\ 0 & 0 & -.390678 & 0 & 0 \end{bmatrix}$$

$$J_2^1 = \begin{bmatrix} 0 & .76885 & 0 & 0 & 0 \\ 0 & -.390678 & 0 & 0 & 0 \end{bmatrix}$$

$$J_1^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & .2236 \\ 0 & 0 & 0 & 0 & -4.175356 \end{bmatrix}$$

$$J_2^2 = \begin{bmatrix} 0 & 0 & 0 & .2236 & 0 \\ 0 & 0 & 0 & -4.175356 & 0 \end{bmatrix}$$

Run	σ_{11}	σ_{12}	σ_{21}	σ_{22}	$\delta_{s_{stick}}$ (ins)	$\delta_{r_{pedal}}$ (ins)	STEADY STATE v ft/sec	ϕ rad	ψ rad	$\ddot{\phi}$ (max) (rad/sec^2)	$\ddot{\psi}$ (max) (rad/sec^2)	δ_{s} (max) (rad)	δ_{r} (max) (rad)
1A	-3	-9	-2	-1	0	1	-116 →	0	.999	0	.814	.782	-3.4
2A	-3	-9	-2	-1	1	0	27 →	.111	0	.704	0	.915	.5953
3A	-3	-9	-2	-1	1	0	3.303→	.021	0	.133	0	.173	.071
4A	-3	-9	-2	-1	0	1	-25.826→	0	.232	0	.189	.182	.798
5A	0	0	0	0	1	0	361→	4.9	0 (unstable)	.19	0	1.14	7.53
6A	0	0	0	0	0	1	-169→	-.002	1.427	0	.233	.535	-5.66
7A	-3	0	-2	-1	1	0	24.106→	.315	0	.140	0	.182	.508
8A	-2	-4	-2	-1	1	0	14.38→	.047	0	.155	0	.201	.310
9A	-2.8	-4	-2	-1	1	0	12.94→	.043	0	.140	0	.181	.279
10A	-1	-1	-2	-1	1	0	76.448→	.190	0	.173	0	.222	1.65
11A	-1.4	-1	-2	-1	1	0	66.73→	.190	0	.172	0	.221	1.44
12A	-1.4	-1	-2	-1	4	0	242 →	.76	0	.689	0	.885	5.236
13A	-1.4	-1	-2	-1	4	0	83 →	.61	0	.543	0	.696	1.8
14A	-1.6	-1	-2	-1	1	0	45→	.15	0	.-35	0	.173	.979

Case IV Time History Data

Table 3-10

response has not reached steady state value). The final values settled upon were $\sigma_{11} = -1.6$, $\sigma_{12} = -1$, and $\lambda_1 = .15$. This corresponds to a damping ratio $\zeta = .8\ sec^{-1}$ and a frequency $\omega = 1$ rad/sec. It is not too surprising that the values for σ_{11} and σ_{12} are the same as those obtained in the pitch response analysis. Both are second order systems and these parameters determine a good response to a step input for second order systems.

Table 3-11
Roll Criteria and Response Comparison

	(rad/sec^2/in) $\ddot{\phi}$	after 1 sec (deg) ϕ	overshoot	δ_s(max) def (rad)
Criteria	.05	2	.15	.79
Response	.128	3.1	.01	.71

Table (3-11) shows a comparison between roll response and roll criteria. Roll response for these parameters satisfies all criteria handily.

Equation (3-21) requires similar analysis although the criteria listed are not quite as specific. Table 3-12 shows the transient analysis of yaw response to a step input. The final values selected were $\sigma_{21} = -1.4$, $\sigma_{22} = -1$, and $\lambda_2 = .06$. Again, the values selected are similar to previous ones although the damping ratio has been reduced to .7. Table 3-13 is a comparison between yaw response and yaw criteria. Table 3-13 shows that even though rudder deflection approached the maximum value, the time for a 15° heading change criteria

Run	σ_{11}	σ_{12}	σ_{21}	σ_{22}	$\delta_{s_{stick}}$ (ins)	$\delta_{r_{pedal}}$ (ins)	STEADY STATE ν (ft/sec)	ϕ (rad)	ψ (rad)	$\ddot{\phi}$(max) (rad/sec^2)	$\ddot{\psi}$(max) (rad/sec^2)	δ_s (max) (rad)	δ_r (max) (rad)
15A	-1.6	-1	-1	0	0	1	-180 →	0	.232	0	.210	.267	-4.19
16A	-1.6	-1	-1	-1	0	1	-26.8 →	0	.233	0	.209	.201	-.88
17A	-1.6	-1	-1.4	-1	0	1	-26.1 →	0	.232	0	.201	.193	-.846
18A	-1.6	-1	-1.4	-1	0	1	-6.7 →	0	.06	0	.051	.049	-.218
19A	-1.6	-1	-.7	-.25	0	1	-109 →	0	.929	0	.216	.207	-.909
20A	[illegible]	-1	-.7	-.25	0	1	-27 →	0	.24	0	.055	.053	-.6202
21A	-1.6	-1	-.7	-.25	0	3		0	.72	0	.167	.160	-1.7
22A	-1.6	-1	-.7	-.25	0	-1	27 →	0	.72	0	-.055	-.053	.620
23A	-1.6	-1	-1.4	-1	0	3	-20 →	0	.179	0	.155	.149	-.654

Case IV Time History Data

Table 3-12

could not be satisfied. One solution to this is to apply a wash out filter which would override the SAS when a large heading change had to be made. This will be investigated in Chapter 5.

Table 3-13

Yaw Criteria and Response Comparison

	$\ddot{\psi}$ (rad/sec^2/in)	time for 15° change (sec)	δ_r (max) def (rad)
Criteria	.05	2	.7
Response	.06	unmeasurable	.7

With all six parameters specified, the feedback control law $\{F, G\}$ is

$$\sigma_{11} = -1.6 \qquad \sigma_{21} = -1.4$$
$$\sigma_{12} = -1 \qquad \sigma_{22} = -1$$
$$\lambda_1 = .15 \qquad \lambda_2 = .06$$

$$F = \begin{bmatrix} .0012 & -.76886 & -.58853 & -.2236 & -.346 \\ .02129 & .39068 & .09596 & 4.1754 & 4.52 \end{bmatrix}$$

$$G = \begin{bmatrix} .1163 & .0134 \\ -.0586 & -.2905 \end{bmatrix}$$

The "uncontrolled variable" in this case is v - sideslip velocity. Sideslip angle β can also be measured since it is equal to v/U_o.[40] For a one-inch lateral stick deflection, β builds up to .26 radians after eight seconds. Unquestionably, this should be considered excessive. For a one-inch rudder pedal deflection, β builds to .08 radians in eight seconds. This large buildup in sideslip angle suggests that we should immediately look at the case when sideslip is eliminated by assigning the rudder pedal deflection as its control.

Again, we will control roll response with lateral stick deflection and thus the C matrix becomes:

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Table 3-14 below presents the computer data for this case. Substituting this data into equation (2-20), we obtain:

$$h_1 = \frac{\dot{\phi}}{\delta_{s_{stick}}} = \frac{\lambda_1 s}{s^2 - \sigma_{11}s - \sigma_{12}} \qquad (3\text{-}23)$$

$$h_2 = \frac{v}{\delta_{r_{pedal}}} = \frac{\lambda_2}{s - \sigma_{21}} \qquad (3\text{-}24)$$

and the characteristic equation is

$$q(s, F, G) = (s + 5.107)(s + .1542)(s^2 - \sigma_{11}s - \sigma_{12})(s - \sigma_{21}) \qquad (3\text{-}25)$$

Note that in this case two closed loop poles are specified by the decoupling procedure. Equation (3-23) has previously been analyzed. Selection of values for λ_2 and σ_{21} is not vitally important at this time. There are no specific criteria for sideslip response. These can better be determined through simulation. In this case, we wish only to eliminate sideslip. For convenience, then, let us say

Table 3-14

Computer Data for $\phi \rightarrow \delta_{s_{stick}}$, $v \rightarrow \delta_{r_{pedal}}$ Case

$d_1 = 0$ $\quad p_1 = 2$ $\quad r_1 = 1$ $\quad \pi_{11} = 0$

$d_2 = 0$ $\quad p_2 = 1$ $\quad r_2 = 0$ $\quad \pi_{12} = 0$

$p_3 = 2$ $\quad r_3 = 2$ $\quad \pi_{21} = 0$

$\pi_{31} = -5.107$

$\pi_{32} = -.1542$

$$D^{-1}A^* = \begin{bmatrix} -.00102 & -.34486 & -.613 & .04498 & 1.175 \\ -.026 & 6.44 & 0 & -.84 & -20.0 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} .7479 & 0 \\ 0 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & -.0107 \\ 0 & .2 \end{bmatrix}$$

$$J_1^1 = \begin{bmatrix} 0 & 0 & .7479 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_2^1 = \begin{bmatrix} 0 & .7479 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_1^2 = \begin{bmatrix} -.0107 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\sigma_{21} = -1$ and $\lambda_2 = .23$. The feedback control law then becomes:

$$\sigma_{11} = -1.6 \qquad \sigma_{12} = -1 \qquad \lambda_1 = .15 \qquad\qquad \sigma_{21} = -1 \qquad \lambda_2 = .23$$

$$F = \begin{bmatrix} .01173 & -.403 & -.5854 & -.0449 & -1.175 \\ -.074 & -6.44 & 0 & .84 & 20.0 \end{bmatrix}$$

$$G = \begin{bmatrix} .1122 & -.0025 \\ 0 & .466 \end{bmatrix}$$

The "uncontrolled variable" is yaw angle. For a one-inch lateral stick deflection, yaw angle builds up to .33 radians in 10 seconds with a steady state roll angle of .15 radians. This is, of course, a large heading excursion, but without the sideslip that usually results. Obviously, both of these cases present difficulties.

CHAPTER 4

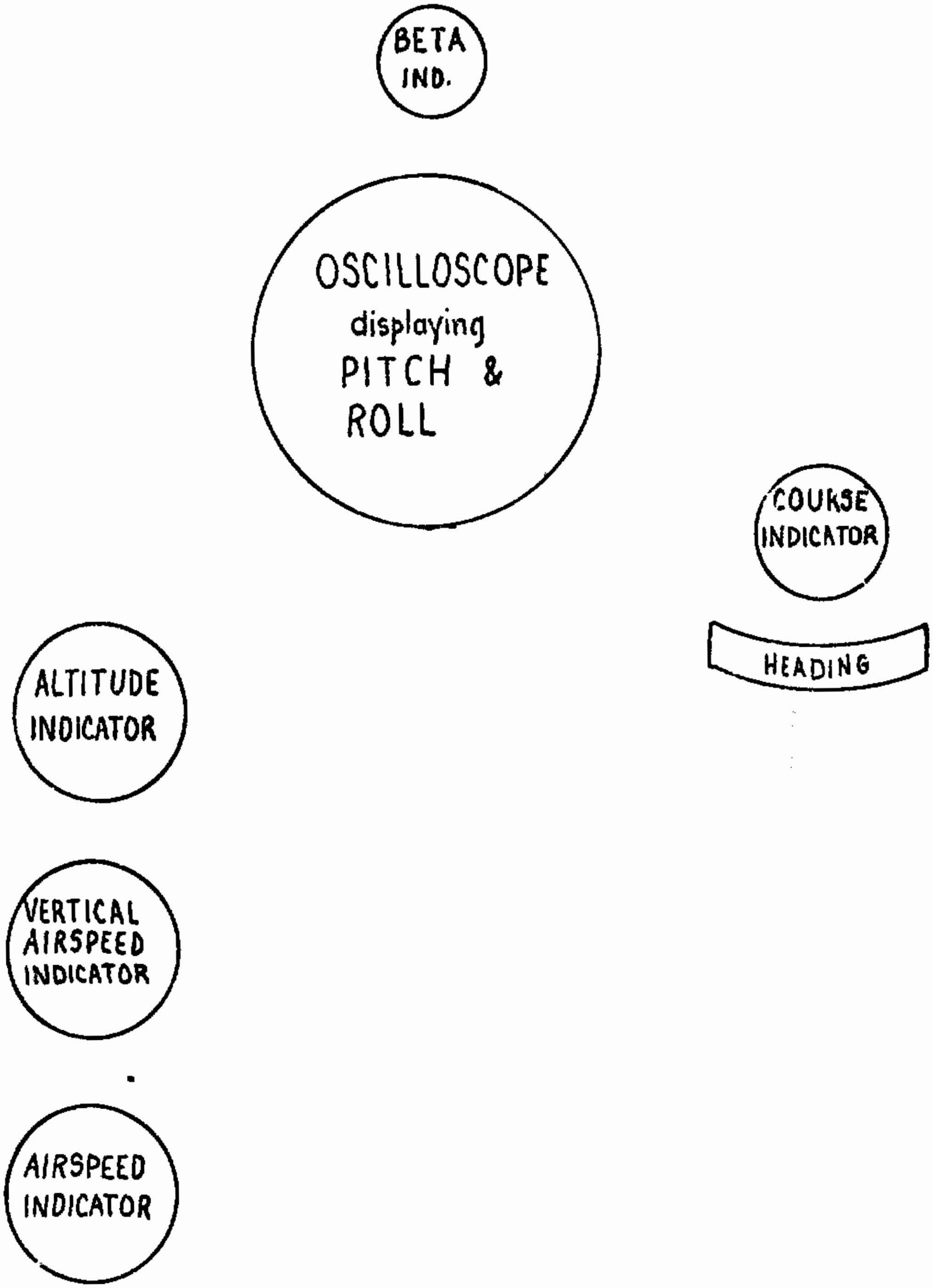
ANALOG SIMULATION

All analog simulation was conducted at the Air Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, Ohio.

Facilities consisted of a fixed-base simulator cockpit driven by two Applied Dynamics, Inc. AD-64 general purpose analog computers. The controls provided were longitudinal and lateral stick deflection, throttle or power setting, and rudder pedals. Instrument display was novel (Shown in Figure 4-1).

The task the pilot was given was to track the ILS beam from an altitude of 1000 feet to a flare altitude of 50 feet. Glide slope and localizer deviations are represented by δ and η respectively. Initially, $\delta = -.25^\circ$ and $\eta = 0^\circ$, indicating that the pilot was lined up with the localizer but was beneath the glide slope. His initial task, then, was to capture the glide slope.

Wind gusts were added in the z and v axis to determine their effect on control response given completely accurate state measurement. A secondary purpose for their implementation was to produce a more realistic environment for the pilots. Wind gust model calculations are given in Appendix B.



Instrument Display

Figure 4-1

Analog equations of motion with scaling factors are given in Appendix E. Gains on throttle, longitudinal and lateral stick, and rudder pedal were adjusted to present maximum control surface deflection per maximum cockpit control deflection. Preliminary results indicated that feedback to rudder might produce rudder deflections that exceed aircraft specifications and so a limiter was placed on rudder deflection. No other control surface deflection limiters were employed and none were needed.

A one second delay was placed on throttle-power dynamics, i.e.:

$$\delta_{T_c} = \frac{1}{s + 1} \delta_{T_{throttle}} \qquad (4\text{-}1)$$

It is common practice to place a .1 second delay on elevator, spoiler, and rudder response, but due to a scarcity of integrators, this practice was not followed.

Preliminary Observations

Chapter Three's digital analysis has presented all closed-loop attitude (i.e., pitch, roll, and yaw) response in terms of command angle inputs (angular response is proportional to cockpit deflections). Early simulations with decoupling feedback applied demonstrated that command angle response did not appear natural to the pilots. Command angle response had been analyzed in the past and found to be unsatisfactory, at least in the lateral directional case.[48]

It was necessary, therefore, to determine whether command rate (angular rate proportional to cockpit control deflection) response could be achieved with the resulting single-input, single-output transfer functions.

Referring to Equation (3-14)

$$\frac{\dot{\theta}}{\delta_{e_{stick}}} = \frac{\lambda_1 s}{(s^2 - \sigma_{11} s - \sigma_{12})} \tag{4-2}$$

application of the Final Value theorem shows that for a step input, $\dot{\theta}$ reaches a steady state value of zero. This is the command angle option. Command rate capability can be obtained by letting $\sigma_{12} = 0$, thus cancelling a numerator and denominator s. The $\frac{\dot{\theta}}{\delta_{e_{stick}}}$ transfer function thus becomes

$$\frac{\dot{\theta}}{\delta_{e_{stick}}} = \frac{\lambda_1}{s - \sigma_{11}} \tag{4-3}$$

which is the command rate response desired but with an exponential rather than an oscillatory response. Roll and yaw response are similarly treated. The term σ_{11} represents $1/T_d$, where T_d is the time delay of the system. For all three axes, T_d was taken to be equal to 1 second.

Simulation

Five pilots were tested. Although non-pilots could have been used, pilots were chosen since I felt that pilots could better achieve the coordination required for combination throttle-stick inputs. This coordinative experience, it turned out, was a slight liability, as will be explained in the pilot comment section. Pilots were also chosen since they could more accurately predict the natural response of the aircraft and determine whether the feedback decoupling altered this response. Pilots with experience in low speed flight would have been preferred, but were hard to find. Three of the five pilots had flown low speed aircraft, and of the remaining two, one had been involved with a STOL simulation previously. Save one light plane pilot, those tested would describe themselves as high performance vehicle pilots. Since the pilot responses desired were general, this low speed flying experience deficiency did not appear to be critical.

A sample test run schedule is given in Table 4-1, although various changes were made in it throughout the simulations. Pilot # 1 flew combinations of the longitudinal and lateral decoupled configurations and, for each configuration, made an additional run in calm air. Runs 6 and 7 were eliminated from the simulation runs by pilots 3, 4, and 5 since these runs had been proved conclusively by pilots 1 and 2 to be uncontrollable. Thirteen parameters, which included the 6 degrees of freedom of the aircraft, the 4 control producing devices, glide slope deviation δ, localizer deviation η, and altitude h, were monitored on strip chart recorders. At the end of each run, pilots made both written and oral comments. Appendix F contains a background briefing guide

Table 4-1

Test Schedule

Runs	Longitudinal Configuration	Lateral Configuration
1	open loop	open loop
2	command angle $\dot{\theta} \rightarrow \delta_{e_{stick}}$ $u \quad \delta_{T_{throttle}}$	open loop
3	command rate $\dot{\theta} \rightarrow \delta_{e_{stick}}$ $u \rightarrow \delta_{T_{throttle}}$	open loop
4	command rate $\dot{\theta} \rightarrow \delta_{e_{stick}}$ $\dot{z} \rightarrow \delta_{T_{throttle}}$	open loop
5	command rate $\dot{\theta} \rightarrow \delta_{e_{stick}}$ $\gamma \rightarrow \delta_{T_{throttle}}$	open loop
6	open loop	command angle $\dot{\phi} \rightarrow \delta_{s_{stick}}$ command angle $\psi \rightarrow \delta_{r_{pedal}}$
7	open loop	command rate $\dot{\phi} \rightarrow \delta_{s_{stick}}$ command rate $\dot{\psi} \rightarrow \delta_{r_{pedal}}$
8	open loop	command angle $\dot{\phi} \rightarrow \delta_{s_{stick}}$ $v \rightarrow \delta_{r_{pedal}}$
9	open loop	command rate $\dot{\phi} \rightarrow \delta_{s_{stick}}$ $v \rightarrow \delta_{r_{pedal}}$

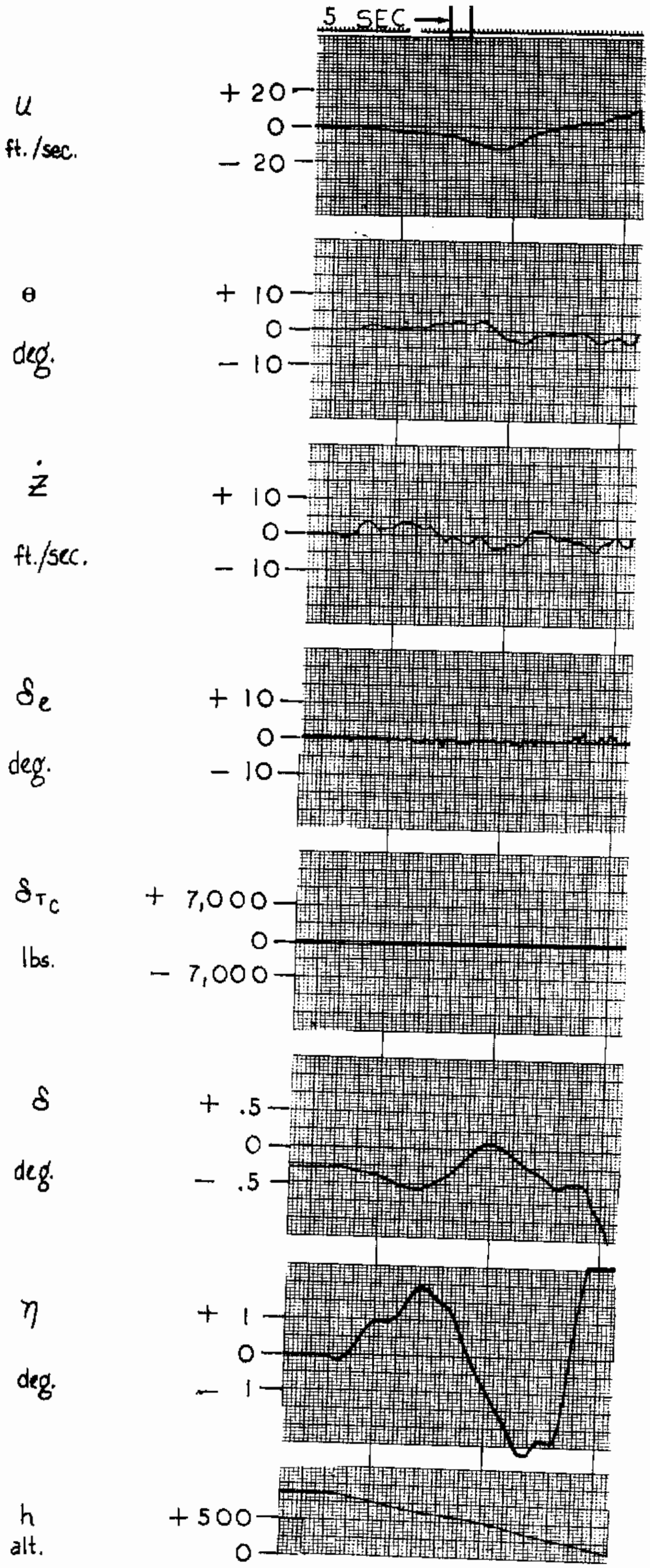
that was given to each pilot prior to the simulation and also a sample comment sheet.

Pilot Comments

After familiarizing himself with the simulation environment, while flying "basic" airframe, the pilot was requested to fly the basic airframe for recording and comment purposes. He was also asked to keep basic airframe response in mind as a reference for comparison to succeeding configurations. As expected the aircraft exhibited extremely undesirable response in the lateral directional mode. As shown in the η response in figure 4-2, pilots found tracking the localizer extremely difficult. The fact that both localizer and guide slope signals were simulated to originate from the same location may explain the large excursions in η as altitude decreased. Standard procedure is to originate localizer signal at the far end of the runway which would have added approximately one to two miles to the range. Note also the extreme sensitivity in β to small spoiler and rudder deflections.

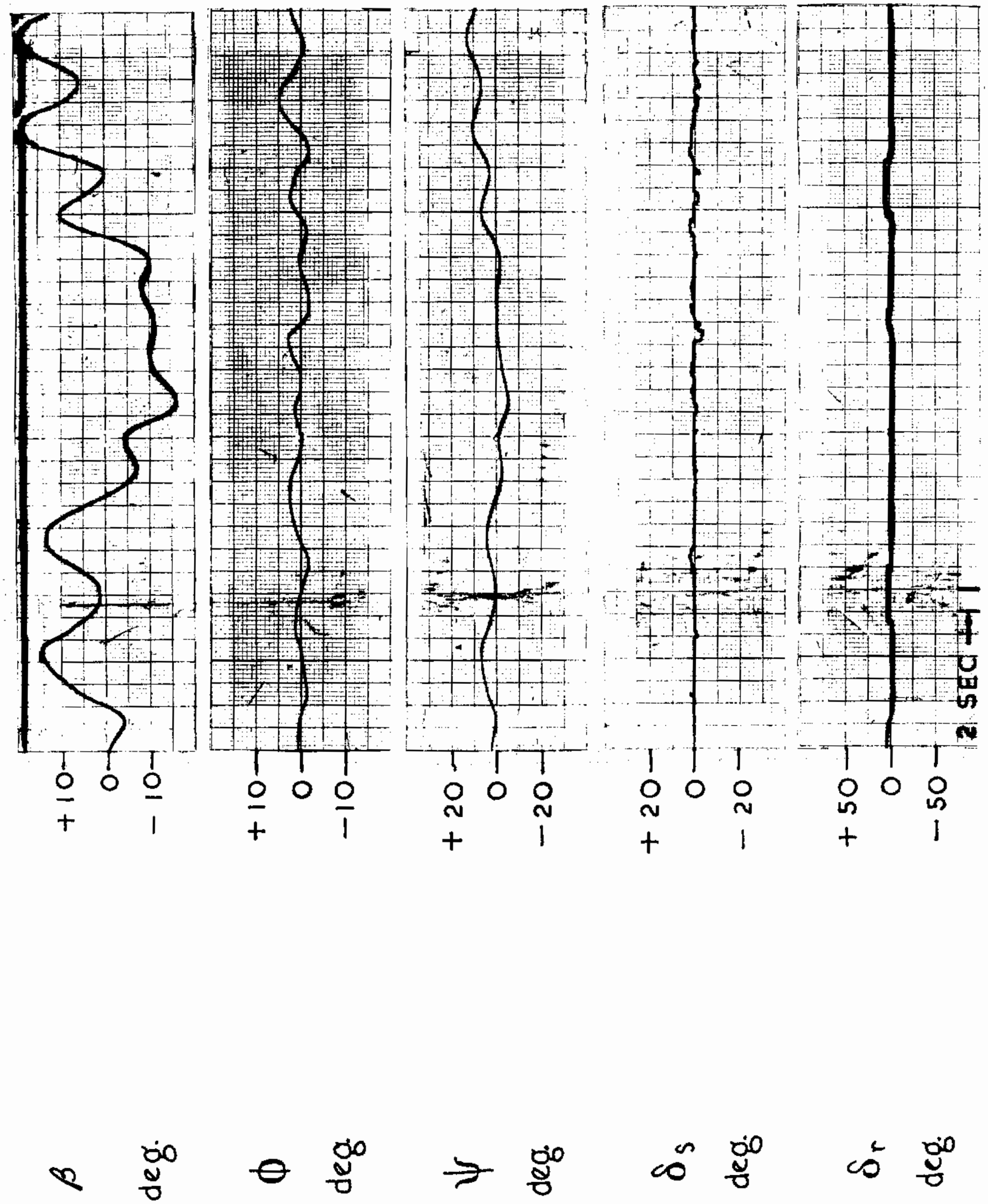
Case I

Case I, if you remember, considered control of pitch rate with longitudinal stick and angle of attack with throttle. For the configuration tested, pitch rate was proportional to stick displacement (Case III simulation tested command angle control). The feedback and feedforward gains are



Open Loop Response

Figure 4-2



Open Loop Response

Figure 4-2 (con't)

$$F = \begin{bmatrix} .00151 & -.0334 & -.5127 & -.0069 \\ -17.778 & 4712.569 & 109{,}151.606 & 780.663 \end{bmatrix}$$

$$G = \begin{bmatrix} -.1192 & .099 \\ -411.83 & -10.0 \end{bmatrix} \qquad (4\text{-}4)$$

The values shown in positions f_{12}, f_{13}, f_{22} and f_{23} are different than these given in Chapter 3. This discrepancy is a result of the command rate option employed in the configuration. As stated earlier the feedforward gains have been adjusted to compensate for the characteristics of the simulator controls.

Pilot Comments - Case I

All pilots agreed that this configuration was very sensitive in the pitch mode, with some pilots expressing critical comments on excessive overshoot. Ease in tracking the glide slope was rated from easy to extremely difficult. The objectionable comments on pitch response can be attributed to three factors.

First, the abrupt change in vehicle response. As stated, all pilots familiarized themselves with the simulation environment while flying basic aircraft dynamics. They were also given time to familiarize themselves with each configuration before flying it for comment purposes. Frequently, the time spent on the latter familiarization was much less than that spent on the former. Also, the command rate capability produces a pitch response alien to what the pilot would expect from previous flight experience.

Second, the one second delay which was installed in power response produced an overshoot in pitch response which would not have occurred had power response been instantaneous.

Third, and most important, pitch response to stick deflections was exponential. Although this would appear to be desirable, values for the time delay associated with the exponential response have never been related to handling qualities. For, this reason, a one second delay was chosen for not only pitch response, but also for the roll and yaw responses of Cases IV and V. For exponential response to practically realize its theoretical potential, a detailed handling qualities analysis should be undertaken to determine optimum values for time delays for each axis as a function of the aircraft task and capabilities.

Control of angle of attack solely by throttle was not consciously initiated by all pilots, though they were all aware of this capability. Two of the five pilots observed that the pitch mode was excited by the throttle. No comment was made by the other pilots. One pilot tried to capture and hold the glide slope purely with throttle and the time history of that response is shown in figure 4-3. As can be seen, pitch varies approximately 2 - 3° for a 5 ft/sec change in normal velocity which was enough for the pilot to capture the glide slope. The change in pitch was again due to the power delay. Due to the undesirable pitch up, speed had decreased by about 20 ft/sec which unless pitch attitude was corrected would have presented problems with stall. (Note: For the run shown in figure 4-3, pilot was concentrating solely on controlling longitudinal mode, thus explaining immediate divergence in η).

Case II

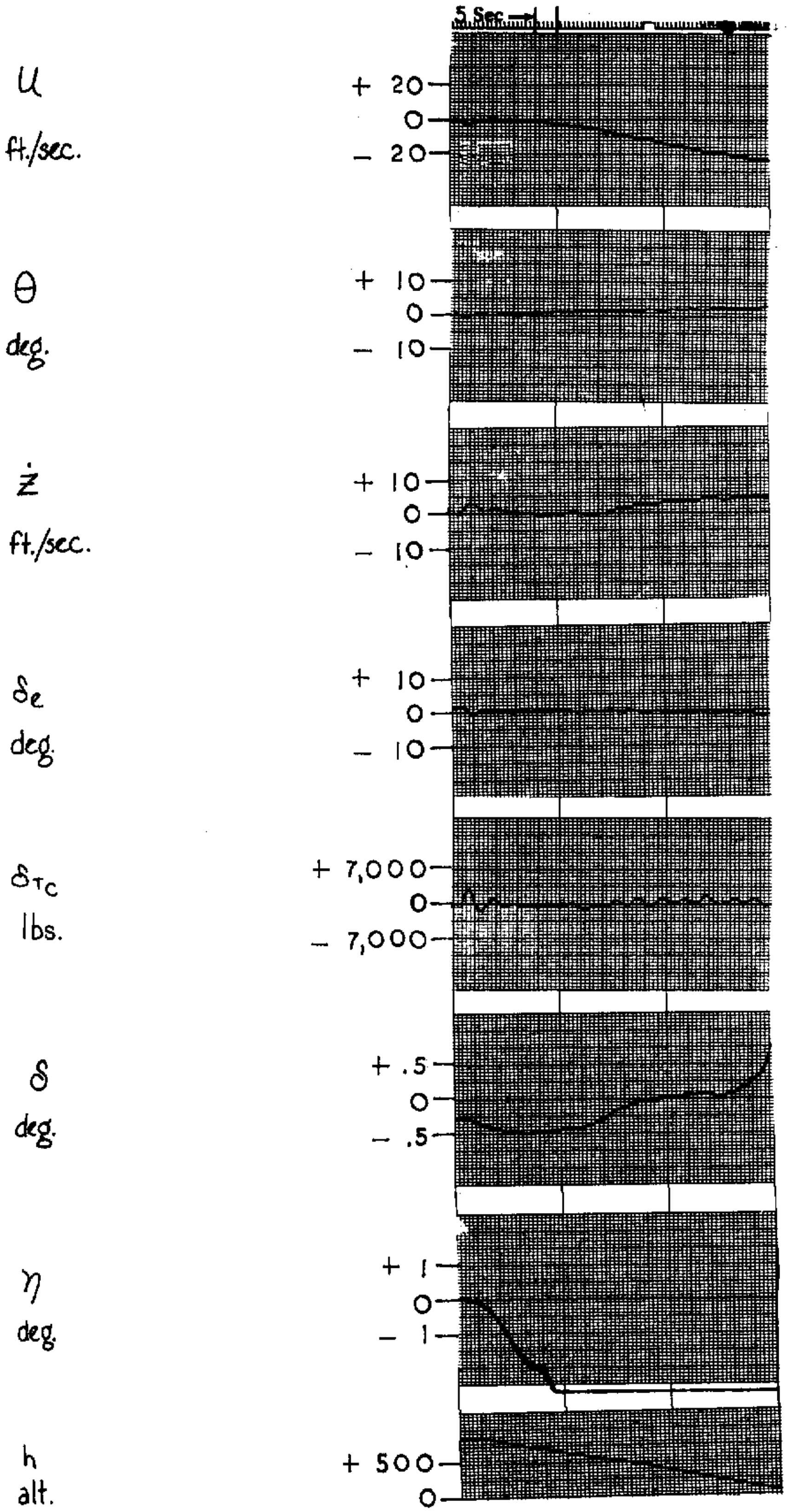
Case II again considered control of pitch rate with longitudinal stick but allowed the throttle to control flight path angle. Again the pilot was provided with the command pitch rate option and the discrepancies in elements f_{12}, f_{13}, f_{22} and f_{23} in the feedback matrix below result from this change.

$$F = \begin{bmatrix} .00151 & .7608 & .2814 & -.0069 \\ -.17.78 & -107.491.46 & -3052.426 & 780.66 \end{bmatrix}$$

$$G = \begin{bmatrix} -.1192 & -.099 \\ -411.83 & 10.0 \end{bmatrix} \qquad (4-5)$$

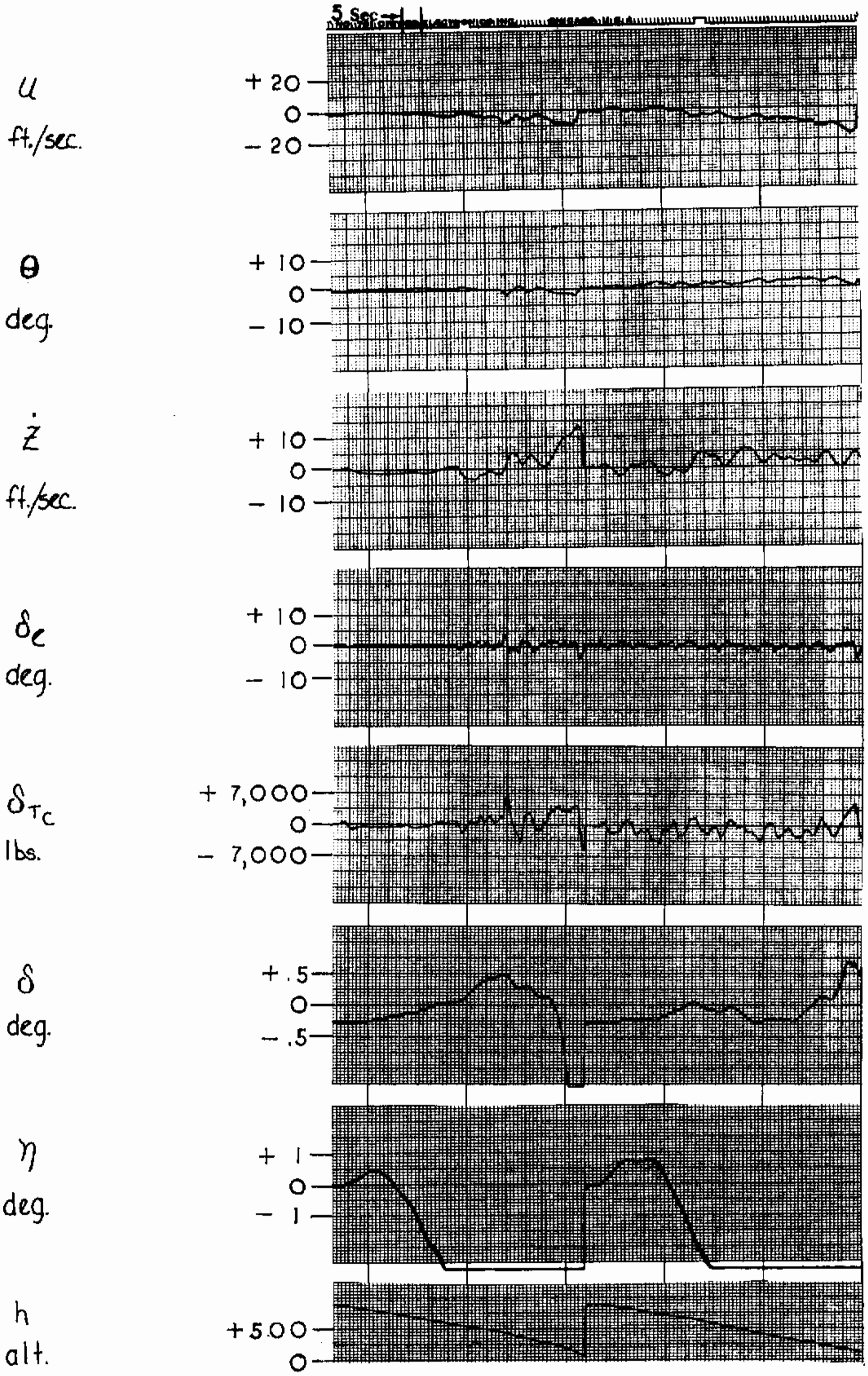
Pilot Comments

All pilots, save one, found tracking the glide slope an easy task. Consistent with the flight path angle-pitch decoupled configuration, the pilots found that they could not capture the glide slope through corrections in pitch attitude, i.e., changes in pitch attitude did not effect changes in flight path angle. Two runs are shown in figure 4-4, both of which are responses to tracking the glide slope solely with the throttle. Observe a maximum 3° change in pitch which produced a -15 ft/sec change in airspeed which if left



CASE I Response

Figure 4-3



CASE II Response

Figure 4-4

uncorrected would create stall problems.

As indicated in Chapter 3, the pilot is effectively controlling flight path through control of angle of attack, for a zero pitch attitude. This configuration, then, is identical to Case I and time history response is comparable. If, however, the pilot has given the plane a constant pitch attitude, or if he is varying pitch attitude with longitudinal stick inputs, throttle inputs will effect changes in angle of attack to produce pure flight path angle deviations. Again lack of concern for controlling η is evident.

Case III

Case III investigated decoupling pitch from aircraft forward velocity. For this case, pilots were presented with both pitch rate and pitch angle command ability. Feedback and feedforward matrices to produce pitch rate command ability are given in equation (4-6)

$$F = \begin{bmatrix} .01193 & -.3506 & .26 & .000003 \\ -1489.23 & 49{,}538.46 & 0 & -204.615 \end{bmatrix}$$

$$G = \begin{bmatrix} -.088 & -.0229 \\ 0 & 3230.76 \end{bmatrix} \tag{4-6}$$

and the feedback matrix to produce pitch angle command is given in equation (4-7). Feedforward matrix remains the same.

$$F = \begin{bmatrix} .01193 & .6605 & .6643 & .000003 \\ -1489.23 & 49{,}538.46 & 0 & -204.615 \end{bmatrix} \tag{4-7}$$

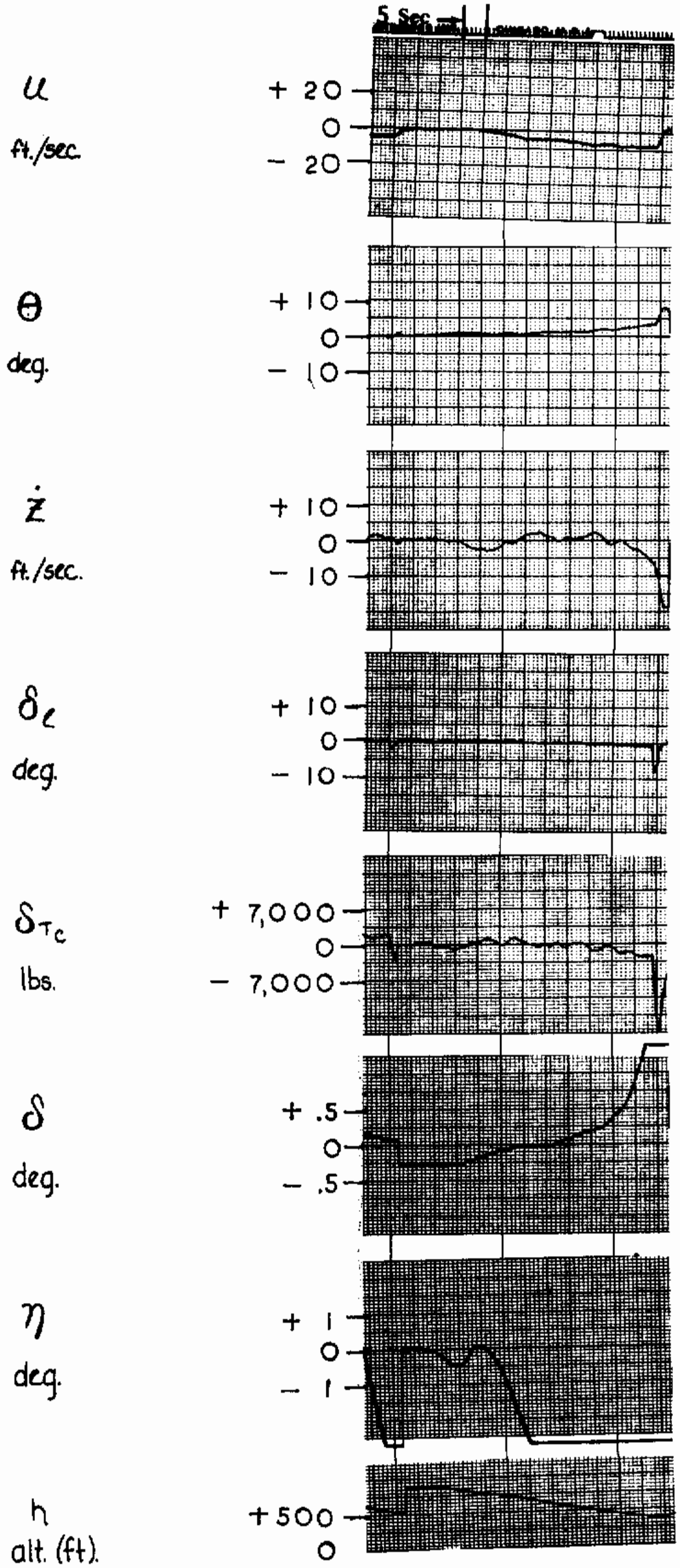
$$G = \begin{bmatrix} -.088 & -.0229 \\ 0 & 3230.76 \end{bmatrix} \qquad \text{(4-7 con't)}$$

Comparison of the F matrices in Equations (4-6) and (4-7) show that they differ only in elements f_{12} and f_{13}. These are the gains associated with the feedback of pitch and pitch rate to elevator. All other elements in the F matrices are identical as are the elements of the G matrices. The difference in the values of these elements and only these elements stems from the fact that the σ's which determine the transient response of single - input single - output subsystems affect only certain elements in the class of F matrices which decouple the system. A more detailed explanation is presented at the end of this chapter.

CASE III - Pilot Comments

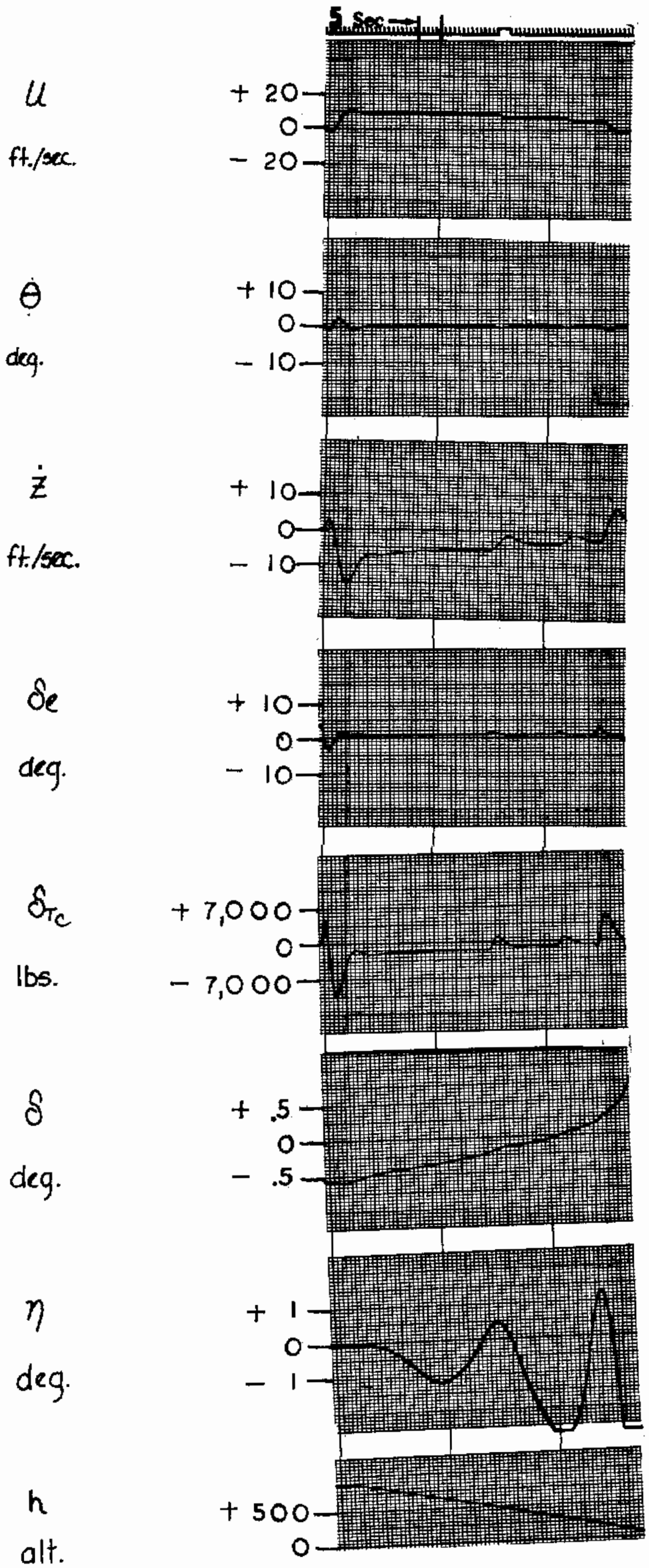
With respect to the pitch rate command configuration, the effect of the power delay was considerably more noticeable in this case. All pilots complained of excessive overshoot and little damping. In addition, pitch was excited with the throttle more readily and with slightly greater amplitudes than in Cases I and II. As can be observed in figure 4-5, there is a build up, although the pilot was controlling with only the throttle. This build-up again was due to the power delay. Since airspeed had reached a steady state value, this pitch variation resulted in a change of angle of attack and consequently vertical velocity which made it impossible to hold the glide slope with throttle only. At the expense of greater pilot workload, pitch deviation could have been corrected with the stick in order to regain glide slope tracking ability.

Unexpectedly, the pitch angle command configuration elicited much more favorable comments. Most probably these comments were due to the absense of overshoots in pitch to pulse stick inputs. Also pitch response more closely resembled standard open loop pitch response. Throttle effect on pitch was less noticeable. A sample run of this configuration for calm air is shown in figure 4-6. Pilot is controlling purely with throttle. Notice the slight perturbation in pitch due to the power delay for a throttle input. After speed reaches steady state pitch returns to zero.



CASE III Command Rate Response

 Figure 4-5



CASE III Command Angle Response

Figure 4-6

Case IV

In Case IV longitudinal SAS was removed and lateral SAS, designed to decouple roll from yaw, was implemented. Command rate and command angle configurations were tested in both roll and yaw axis and the respective feedback and feedforward matrices are given in equations (4-8) and (4-9).

$$F = \begin{bmatrix} .0012 & -.7689 & -.5885 & -.2236 & -.3461 \\ .0213 & .3907 & .096 & 4.175 & 4.522 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.43 & .02 \\ -.72 & .3828 \end{bmatrix} \qquad (4\text{-}8)$$

$$F = \begin{bmatrix} .00127 & 0 & -.1272 & 0 & -.2567 \\ .02129 & 0 & -.1384 & 0 & 2.8518 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.43 & .02 \\ -.76 & .3828 \end{bmatrix} \qquad (4\text{-}9)$$

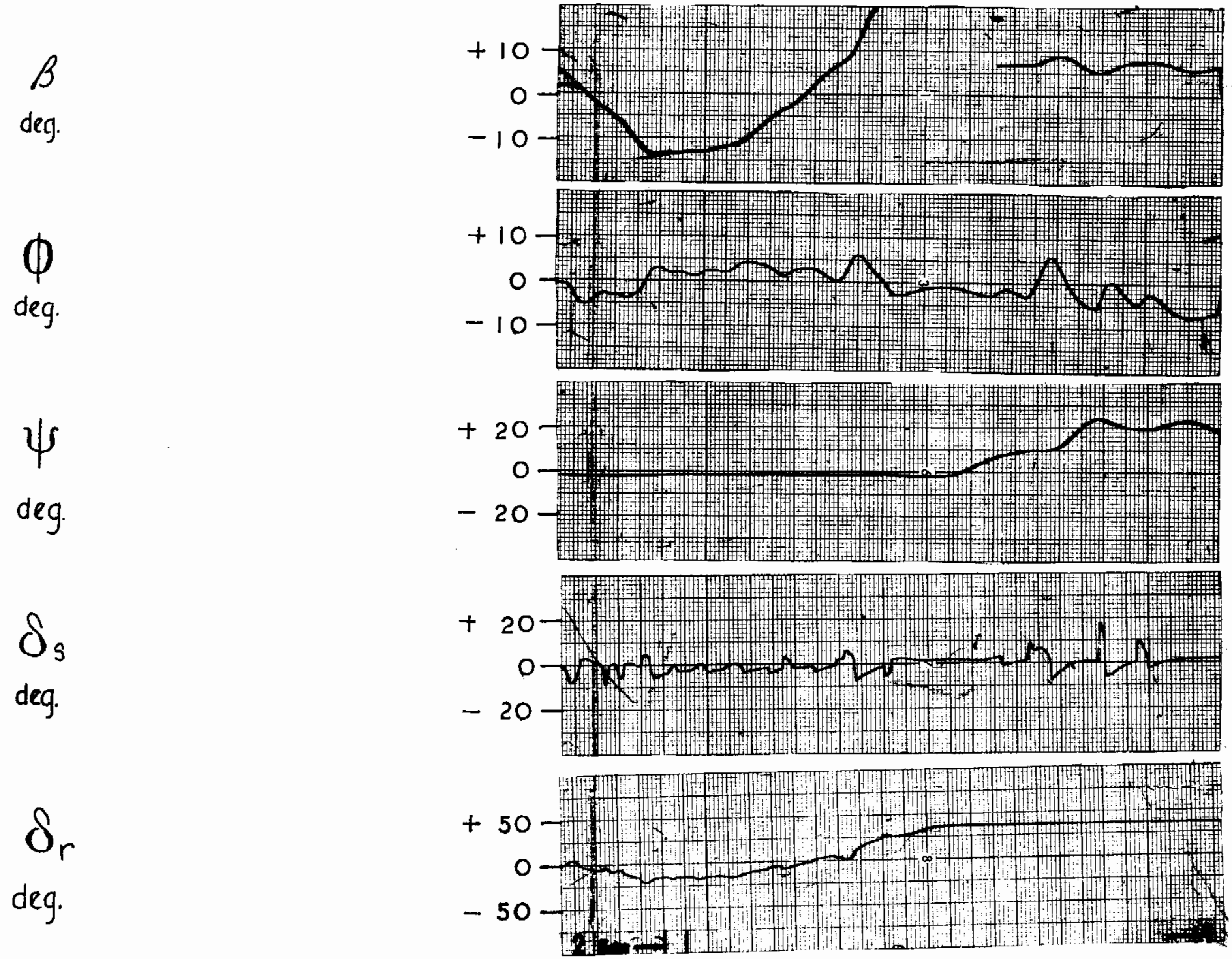
Case IV - Pilot Comments

Major difficulty was encountered in attempting to decouple roll response from yaw response. In both the roll and yaw axis, command angle and command rate configurations were investigated and in both configurations, rudder control was saturated. This saturation resulted from a build-up in sideslip velocity which accompanied any roll disturbance and the feedback of this sideslip velocity build-up to the rudder. In the command roll rate configuration, the time lag was increased with no beneficial result other than to proportionally delay the time to rudder saturation.

Two pilots were presented with the varying configurations of this case. As shown in figure 4-7, for the command angle configuration and figure 4-8 for the command rate configuration decoupling was maintained until feedback of sideslip velocity to the rudder caused it to saturate. At this point, pilots were unable to override rudder deflections and the aircraft became uncontrollable. There seemed to be little difference between either the command angle or command rate option. Physically speaking, the airplane would be expected to "slip" for a constant roll attitude and no yaw change. However, the resultant loss of control due to rudder saturation was not expected. An airplane that exhibited better lateral directional behavior might not experience the same result.

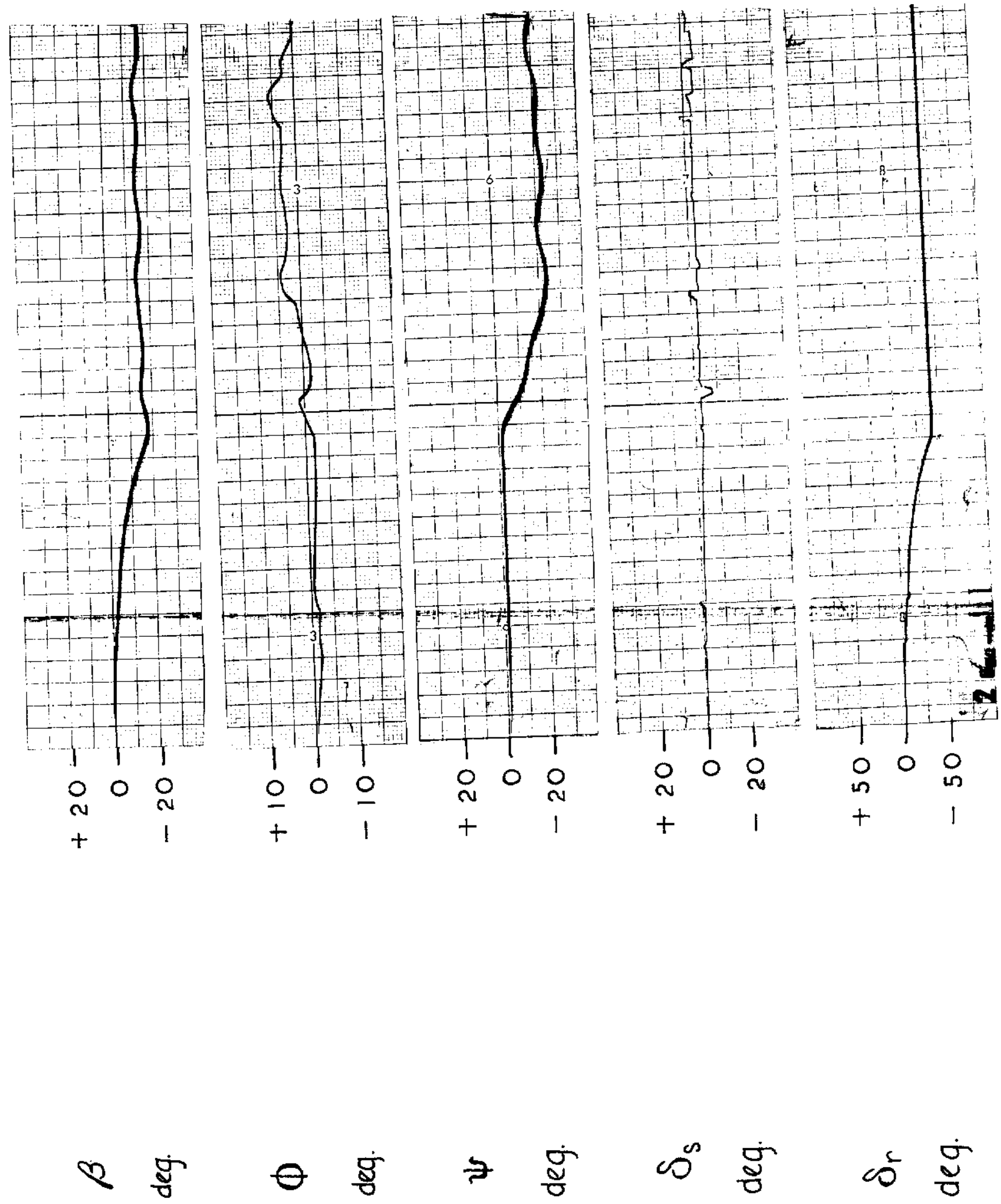
Case V

Case V investigated the highly disirable premise of decoupling roll from sideslip in order to achieve automatic turn coordination. The difficulties encountered in Case IV were thankfully absent from this case. Control authority was never exceeded. Both command roll angle and command roll rate configurations were investigated and their respective feedback and feedforward matrices are given in equations (4-10) and (4-11).



CASE IV Command Angle Response

Figure 4-7



CASEIV Command Rate Response

Figure 4-8

$$F = \begin{bmatrix} .0117 & -.4031 & -.5854 & -.0449 & -1.175 \\ -.174 & -6.44 & 0 & .84 & 20.0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.43 & -.02 \\ 0 & .3828 \end{bmatrix} \qquad (4\text{-}10)$$

$$F = \begin{bmatrix} .0117 & .3448 & -.1346 & -.0449 & -1.175 \\ -.174 & -6.44 & 0 & .84 & 20.0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.43 & -.02 \\ 0 & .3828 \end{bmatrix} \qquad (4\text{-}11)$$

Case V - Pilot Comments

The difficulties encountered in Case IV were absent from Case V. Control authority was never exceeded in decoupling sideslip velocity from roll. Both command roll angle and command roll rate configurations were investigated, producing significant differences in pilot comments.

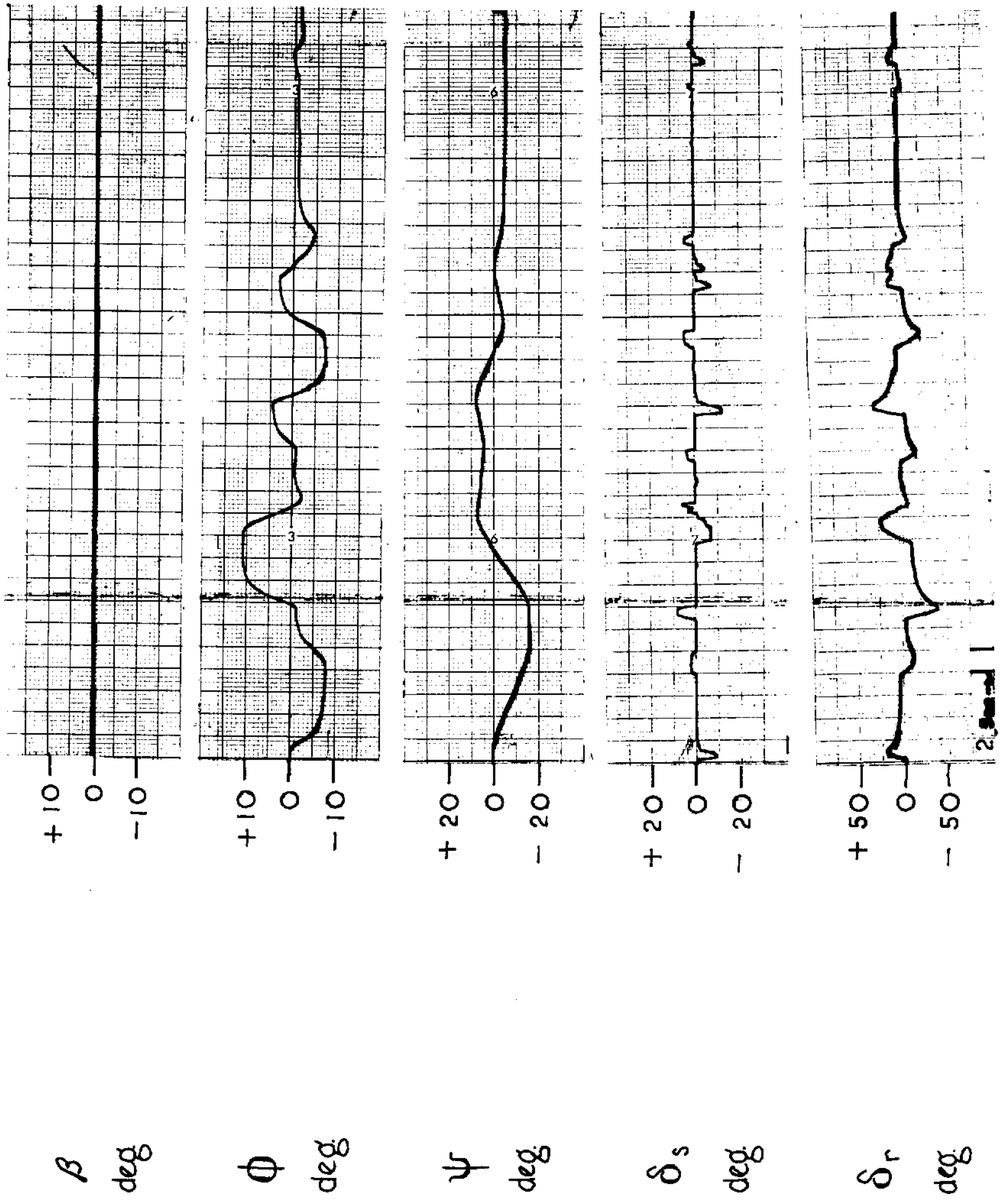
Four out of the five pilots preferred command angle to command rate configuration. This was due in part to the sensitivity of yaw to roll disturbances. The one dissenting pilot did specifically say that he enjoyed the capability of holding a roll angle after pulsing the stick. I expected a command rate preference, but the aircraft's low lateral stability could have influenced the pilot's choice.

Again, four out of the five, with a different dissenter this time, rated the localizer from easy to tolerably difficult to track. Three pilots rated it easy. All five, however, and this was expected, noticed a large reduction in workload from the open loop case. A sample time history for calm air is given in figure 4-9.

As can be seen there was no change in side slip velocity for roll inputs. This frees the pilot from his usual task of applying rudder pedal to coordinate a turn, i.e., eliminate sideslip. Pilot still has control of heading as in standard operations through roll angle.

The primary concern of this configuration was to eliminate sideslip. Rudder pedals were designed in this case to "slip" the aircraft with no change in roll. The gains involved, however, were too small to produce significant changes in either sideslip or yaw. An increase in gain between rudder pedal and sideslip could increase but the practicality of "slipping" an aircraft with no change in roll is questionable.

It is necessary to add that the responses shown in figure 4-9 are for a calm air condition In gusty air, the effectiveness of the decoupling augmentation system is dependent on the speed of response of the control surfaces. It is to be expected, therefore, that the decoupled response will not be as pure as that shown in figure 4-9. However, the response with actuator dynamics included should be comparable to the longitudinal response with a throttle delay implemented.



CASE V Response

Figure 4-9

Sensitivity

In actual flight operations, it is desirable to minimize the number of measured values which are to be fed back to the controls. This is done for two reasons, both of which are self-explanatory. The first reason is cost and the second reason is that a reduction in the number of feedback variables reduces the concern caused by errors in measurements of feedback variables. The analysis of the effects of parameter variations on the response of a closed loop system is standardly referred to as sensitivity analysis. There has been a great amount of work done on sensitivity analysis techniques, applied to both classical and modern control, and their relative advantages and disadvantages. A precise analysis, as applied to this problem, would entail another thesis and for this reason I limited myself to the basic sensitivity analysis that follows. Case III has been chosen as the sample configuration.

Minimizing Feedbacks

In Case III, pitch response was decoupled from forward speed response. The transfer functions that determined pitch response to longitudinal stick inputs and speed response to throttle inputs were given in equations (3-14) and (3-15). Values for σ_{11}, σ_{12}, and σ_{21} can be chosen by the designer thus determining specific feedback gains from a class of feedback gains. For Case III, the class of feedback gains is given by

$$F = -\begin{bmatrix} -.00104 & .3506 & .75126 & -.00003 \\ -49.23077 & -49{,}538.46 & 0 & 204.615 \end{bmatrix}$$

$$+\sigma_{11}\begin{bmatrix} 0 & 0 & -1.011 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+\sigma_{12}\begin{bmatrix} 0 & -1.011 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+\sigma_{21}\begin{bmatrix} -.01089 & 0 & 0 & 0 \\ 1538.46 & 0 & 0 & 0 \end{bmatrix} \quad (4\text{-}4)$$

or

$$F = \begin{bmatrix} (.00104 - .01089\sigma_{21}) & (-.3506 - 1.011\sigma_{12}) & (-.75126 - 1.011\sigma_{11}) & .00003 \\ (49.23 + 1538.46\sigma_{21}) & 49{,}538.46 & 0 & -204.615 \end{bmatrix} \quad (4\text{-}5)$$

Equation (4-5) shows that all the states are fed back to both the elevator and thrust except for pitch rate to thrust since element $f_{23} = 0$. The gain represented by element f_{14}, which is associated with normal velocity feedback to elevator, is very small. Both an analog and a digital analysis, showed that ignoring this gain had absolutely no effect on decoupling or transient response. Of the remaining six gains, four can be specified by the designer. He has no power over the gains associated with feedback of pitch rate and normal velocity to thrust.

As can be seen in equation (3-14), parameters σ_{11} and σ_{12} determine pitch transient response. The object of this section is to minimize the number of feedbacks. Feedback of pitch and pitch rate to elevator can be eliminated by the proper choice of σ_{11} and σ_{12}. Eliminating pitch feedback to elevator implies setting element $f_{12} = 0$ i.e.,

$$-.3506 - 1.011\ \sigma_{12} = 0 \qquad (4\text{-}6)$$

or that $\sigma_{12} = -.3468$. Eliminating pitch rate feedback to elevator implies setting element $f_{13} = 0$ i.e.,

$$-.75126 - 1.011\ \sigma_{11} = 0 \qquad (4\text{-}7)$$

or that $\sigma_{11} = -.7512$. Substituting these values into equation (3-14),

$$\frac{\dot{\theta}}{\delta_{e_{stick}}} = \frac{\lambda_1 s}{s^2 + .7512s + .3468} \qquad (4\text{-}8)$$

Thus, $\omega_\theta^2 = .3468 \dfrac{\text{rad}^2}{\text{sec}^2}$ or $\omega_\theta = .589 \dfrac{\text{rad}}{\text{sec}}$

and $2\zeta\omega_\theta = .7512$ or $\zeta = .6377$.

Though, the frequency is low and the damping ratio is less than optimum, both satisfy the specifications given in reference 47. Elimination of the pitch and pitch rate feedback to elevator is possible, therefore provided the designer can accept the corresponding transient response.

It can, also, be seen from the above analysis that eliminating the above feedbacks does not affect decoupling.

Only the pitch transient response is affected. Likewise, any changes in elements f_{12} and f_{13}, i.e., the gains associated with pitch and pitch rate feedback, whether they are produced by the designer, or caused by errors in state variable measurement, will not affect decoupling. Pitch and pitch rate feedback to elevator can only be considered critical for maintaining transient response, not for decoupling.

From Equation (4-5), observe that feedback of forward speed can be eliminated from either thrust or elevator but not both. Obviously, since σ_{21} = 1/time delay, the only practical consideration would be to consider the case of eliminating forward speed feedback to elevator. This necessitates

$$-.01089\ \sigma_{21} = -.00104$$
$$\sigma_{21} = .0955$$

Since σ_{21} is positive, the system is unstable. This instability only applies if σ_{21} coefficient also multiplies the second row of its product matrix to obtain the total F. Loss or error in measurement of forward speed for feedback to elevator is another matter, to be considered in the next section.

Loss and Errors in Measurements of Critical Feedback Variables

The digital computer program of Appendix D was used to determine the effect of the loss of a critical feedback variable or of errors in measurement of a critical feedback variable in maintaining decoupled response. Each feedback gain, individually, was zeroed and the response to step stick and step throttle inputs was analyzed. It was found in the preceeding section that loss of, or errors in measurement of, feedback of pitch and pitch rate to elevator did not affect decoupling. Of the remaining four feedback variables, i.e., u to elevator and thrust, and θ and $\dot{z}$ to thrust, only the loss of, or errors in measurement of, $\dot{z}$ for feedback to thrust affected decoupled system response to <u>both</u> stick and throttle inputs. With no $\dot{z}$ feedback to thrust for a step stick input, both decoupling capability and exponential pitch response capability were effectively lost. Compared to open loop response, pitch response appeared to be less sensitive to stick inputs; however, pitch rate continued to build instead of oscillate. A simulation was run for this case and the pilot found it extremely difficult to control the aircraft. For step throttle inputs, again both decoupling and exponential speed response were affected but not as severely as for stick inputs. Since for this case, $\dot{z}$ is the "uncontrolled" variable, it is easy to see that since it is excited by both controls when the system is decoupled, loss of its feedback would most critically affect the decoupled response.

Independent loss of u feedback to elevator and thrust affected decoupled response to a throttle input, but did not affect decoupled response for a stick input. For loss of u to elevator, speed exponential response to throttle input was not affected but along with a steady state speed response of 2.1 ft/sec, occurred a steady state.023 rad/sec pitch rate.

Loss of u to thrust, created a much more severe problem with all variables continuing to build up for a step throttle input. Simulation, however, showed that the configuration was still controllable.

An even more severe problem in control resulted from loss of θ to thrust. For a stick input, all decoupling and exponential pitch response capability was lost. Response was comparable to that caused by a loss of $\dot{z}$ to thrust for a step stick input.

The effect of errors in measurement was analyzed by varying the feedback gains by 10% both independently and in combinations. Except for the θ to thrust variation, independent variations on feedback gains had an insignificant effect on either decoupling or exponential response capability. For the θ to thrust case, decoupling is affected, but exponential pitch response to a step stick input, while not holding a constant steady state pitch rate decreased slightly rather than build-up. This, then, is comparable to open loop response with less sensitivity to stick inputs.

For combinations of 10% variations of all feedback gains, decoupling was affected for all step stick inputs. For step throttle inputs, there was little or no effect on either decoupling or exponential speed response capability. Response seemed to be less sensitive to positive variations in pitch than negative. All responses to throttle input were less sensitive in pitch response than the open loop case.

The low sensitivity encountered for step throttle inputs was probably due to the small gains implemented between throttle and magnitude of speed response. A positive one inch throttle deflection produced only a 2.1 ft/sec steady state increase in airspeed which even in the open loop case would not produce much of a pitch change. The gains were kept small since we were only considering perturbations to trim landing configuration. Had the gains been higher, decoupling would be more severely affected, producing much larger pitch responses to throttle inputs, which would lead to loss of exponential speed response.

The obvious conclusion is that the degree of decoupling maintained is dependent on the accuracy of the measurements, the most critical being $\dot{z}$ to both elevator and thrust, and θ to thrust.

Sensitivity in Implementation

Although the feedback and feedforward gains are determined for a specific trim condition, practically speaking, the pilot is rarely able to obtain the exact trim conditions for which the gains were designed. The dimensional stability derivatives used in the model are a function of speed, angle of attack, weight, and air density for a particular aircraft. The model used in this analysis, then, was based on a particular value for each of these parameters. If, in reality, one or more of these values are not met, the model will not accurately represent the aircraft dynamics. This section will treat the effect of model discrepancies on the ability to decouple the system.

Referring to the equations of motion in Appendix A and Table A-4, the non-dimensional form of the dimensional derivatives used in the model can be equated, as for example:

$$X_u = \frac{\rho S U_0}{m} (-C_D - C_{D_u})$$

where ρ = air density, S = wing area, m = mass of aircraft. If the aircraft is slightly off trim, X_u will vary and a new value $\bar{X}_u$ can be represented as

$$\bar{X}_u = \frac{\bar{\rho} S \bar{U}_0}{\bar{m}} (-\bar{C}_D - \bar{C}_{D_u})$$

For a small change in altitude, it can be assumed $\rho = \bar{\rho}$. Also, the mass will not change significantly, so $m = \bar{m}$. Immersed wing area S may be assumed to be the same. The drag coefficient C_D due to u are functions of mach number and angle of attack. Because of the low speed flight configuration, mach number effects are limited. If as stated in Reference 16, the pilot desires to fly a constant angle of attack, the variations due to angle of attack can be assumed small.

Looking at the ratio of X_u to $\bar{X}_u$, therefore,

$$\frac{\bar{X}_u}{X_u} = \frac{\bar{U}_0}{U_0}$$

Thus, based on our assumptions, the new derivative is purely a function of the ratio of trim speed changes and the old derivatives.

The original model trim speed was 60 knots. Stall speed for this configuration is approximately 54 knots. Let the new trim speed be 55 knots and therefore:

$$\bar{X}_u = .91\ X_u$$

A similar analysis may be applied to the other derivatives. The new model then becomes

$$\dot{x} = \begin{bmatrix} -.02912 & -32.2 & 0 & .12103 \\ 0 & 0 & 1 & 0 \\ .00125 & 0 & -.676 & -.00127 \\ .0182 & 4.2 & 89.829 & -.273 \end{bmatrix} x$$

$$+ \begin{bmatrix} 0 & .00055 \\ 0 & 0 \\ -.831 & -.000006 \\ 2.52 & .00073 \end{bmatrix} u$$

For Case III

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} x$$

Applying the feedback and feedforward gains designed for the original trim case and utilizing the computer program of Appendix D, time history response shows that the decoupling while slightly affected can still be maintained.

For a step throttle input, speed reaches commanded steady state value in 8 seconds with a .002 rad, change in pitch.

Decoupling is affected slightly as evidenced by response to a step longitudinal stick input. For the command rate configuration, pitch rate reaches its commanded steady state value in 4 seconds. At that point pitch has been altered by .224 rad. Speed, however, has also changed by -.813 ft/sec.

In the open loop case, for a step stick input, speed changes approximately -1.5 ft/sec for a .2 rad change in pitch. A 50% reduction in open loop speed response is not overwhelming. However, the commanded pitch rate ability has been maintained which significantly affects the speed response. For example, in the open loop case, for step stick input, a one radian pitch change occurs in approximately 2 seconds with speed varying by 14 ft/sec and increasing with negative pitch changes. Adding the decoupling SAS slows pitch response and consequently speed build-up.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The advantages of the use of STOL aircraft for alleviating some of today's air traffic congestion were mentioned in Chapter 1. These advantages have never been demonstrated due in part to a tardiness in specifying handling qualities and also due to control problems that are unique to STOL aircraft. Some of these control problems are produced by undesirable coupling in the longitudinal and lateral modes which, in the past, pilots have been expected to compensate for. The approach taken in this analysis is to eliminate the coupling as much as possible through an application of Gilbert's decoupling technique which was presented in Chapter 2. This technique provides the designer with a class of feedback and feedforward gains whose values are a function of the time response of the system. Chapter 3 dealt with determining the parameters that produced satisfactory time response based on transient response criteria.[41] This digital analysis was then extended to an analog simulation in an effort to obtain pilots' reactions to several decoupled configurations.

The results, while not the panacea originally expected, can, at worst, be labeled promising. Heading the advantages column is the introduction of a simplified method for the design of stability augmentation systems. In both the longitudinal and lateral-directional modes, the critical attitude loops, i.e., pitch and roll, can be closed without the fear of outer loop interference which is normally encountered in classical design. Attitude response can be much more easily specified given the single-input, single-output transfer functions which are provided by the decoupling program. For second order attitude response, it was found that transient response criteria for STOL aircraft in a landing configuration could be satisfied by specifying damping ratios and frequencies that have classically been equated with "good" response. This specification was done without the tedia of root locus or Bode plot procedures and also without the hazards of choosing a representative performance index used in modern control analysis. And not to be emphasized too much, the design was accomplished without the complexity resulting from multiple nested loop closures.

Also encountered in the design analysis was the potential for specifying exponential attitude response. Although exponential or command rate response was panned by four of the five pilots, I believe that the potential can still be realized. Possible explanations for the pilots' critical comments were given in Chapter 4. As stated, the most important consideration involving exponential response is the determination of the relationship between the time delay parameter and handling qualities. The replacement of higher order response with exponential response seems to be an obvious improvement.

However, new difficulties usually ride the tails of new technology and these difficulties will have to be further investigated before achieving exponential response for aircraft.

Although dissatisfied with the command rate capability, two pilots did indicate preference for some type of attitude hold ability. One pilot suggested a trim button and another a friction stick. In the command angle configuration, this would enable the pilot to hold attitude without constant pressure on the stick. These suggestions further verify the desirability of the command rate option which did have attitude hold capability, but excessive sensitivity overshadowed its usefulness.

A second advantage, when it was capitalized on, was decoupled response. For the task chosen, i.e., capturing and tracking the glide slope and localizer, it appeared that the most practical configuration was Case II, flight path decoupled from pitch response. As indicated in Chapter 4, the pilot could capture and track the glide slope solely through throttle input producing minimal changes in pitch and reasonable changes in relative airspeed. Pilots are reluctant to fly "hands-off" and consequently any undesirable changes in pitch could be corrected with the stick. Case I, angle of attack decoupled from pitch response, and Case III, airspeed decoupled from pitch response, also demonstrated that the glide slope could be captured and tracked solely through throttle inputs.

If maintaining constant airspeed was critical, Case III decoupling might be preferable to Case II or Case I. In this instance, the decoupled system is comparable to an autothrottle system. However, combined with the ability to specify pitch response and also to make exponential changes in airspeed through throttle input with no pitch interaction makes Case III decoupling a much more powerful technique than a pure autothrottle application.

In the lateral directional mode, the elimination of side slip due to roll inputs which was analyzed in Case V appears to be highly desirable. In flight, it is equivalent to automatic turn coordination for small roll angle inputs. Although pilots still did not rate ease in tracking localizer any better than tolerably difficult, they all did recognize a reduction in workload.

In Case IV, a serious problem was encountered with control saturation. It was attempted to decouple roll response from yaw response which, practically speaking, translated into trying to "slip" the aircraft with a stick input with no yaw interaction or "slip" the aircraft with a rudder pedal input with no roll interaction. The problem resulted from extreme side slip disturbances to stick and rudder pedal inputs which when fed back to the rudder in an effort to maintain decoupling produced rudder saturation. Since there were no constraints imposed on control authority in the decoupling procedure, it was hoped that aircraft control authority would be sufficient to achieve and maintain decoupling. In this case it was not. The conclusion that can be readily drawn is that for decoupling theory to produce a tractable method for designing and implementing a stability augmentation system, it must take into account the control authority of the system to which it is applied. Of the five cases tested here, only one displayed a control saturation problem. The point, however, is that there is a control saturation potential in aircraft systems and the theory, at present, provides no way to handle it.

Sensitivity reared its ugly head as it often does in modern control applications and also, though usually not as frequently, in classical control applications. For the case analyzed, 50% of the feedback gains appeared to be very sensitive to errors in measurement. Of particular importance are errors in measurement of the "uncontrolled" variable. However, the increasing use of modern optimal control in aircraft systems which usually requires a large number of feedback gains, can only lead to the development and production of more accurate measuring devices.

Model errors did not appear to have much effect on decoupled response for accurate measurements of all feedback variables. However, since decoupled response would be even more attractive for maneuvering purposes, it might be possible to implement an adaptive controller which compensated for model errors. Another possibility, which would eliminate the need for analyzing multiple trim conditions and their corresponding control laws, might be an on-line identification process. Given the ability to update the model from a short time history response, control laws could be automatically implemented. The parameters which determined the nature of the closed loop response would have to be specified previously as a function of the trim configuration.

To summarize, decoupling theory applied to aircraft systems, could become a powerful aid in designing stability augmentation systems. Given values for the parameters which produce good handling qualities in the single input, single output subsystems, it is, then, a simple task to determine feedback gains. The analysis, presented here, indicates that second order attitude response parameters are simply those which produce good second order response for any system. It has been shown that, decoupling effects aside, it is possible to improve the handling qualities of the aircraft. It has also been shown that decoupled response capability does decrease the pilot's workload. In the example presented, it made tracking an ILS signal easier. It is to be emphasized that the choice of what variables to decouple is dependent on the task. Different variables would be decoupled in a tracking task than in a gun fire control task. Finally, the results presented in this analysis are not limited to STOL aircraft applications but are also directly applicable to VTOL and conventional aircraft.

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Appendix A

Equations of Motion

The equations of motion for a typical STOL aircraft in a landing configuration are given in numerical form in Chapter 3 and are based on the following assumptions:[43]

1. The airframe is assumed to be a rigid body.

2. The earth is assumed to be fixed in space, and the earth's atmosphere is assumed to be fixed with respect to the earth.

3. The mass of the airplane is assumed to remain constant.

4. With the origin at the c.g. of the airplane, the x axis oriented parallel to the relative wind, and the y axis out the right wing, with the z axis pointed down (stability axis), the xz plane is assumed to be a plane of symmetry.

5. Flight is along a straight path during which the linear velocity vector measured relative to fixed space is invariant and the angular velocity is constant or zero (steady flight).

6. The disturbances from the steady flight condition are assumed to be small enough so that products and squares of the changes in velocities are negligible in comparison with the changes themselves. Also, the disturbance angles are assumed to be small enough that sine and cosine approximations are valid. Products of these angles are also approximately zero and can be neglected. And, since the disturbances are small,

the change in air density encountered by the airplane during any disturbance can be considered zero.

7. The aerodynamic forces and moments acting on the airplane are dependent only on the velocities of the airplane relative to the air mass.

The trim conditions for the landing configuration are 60 knots (100 ft/sec) airspeed, 732 ft/min sink rate, with a 98° flap deflection. Initial angle of attack is 7½°.

In general form, the linearized perturbation equations about this trim condition, in stability axes, are:

$$\dot{u} = X_u u - g\theta \cos \theta_o + X_{\dot{\theta}}\dot{\theta} + X_{\dot{z}}\dot{z} + X_{\delta_e}\delta_e + X_{\delta_T}\delta_T$$

$$\ddot{\theta} = M_u u + M_{\dot{\theta}}\dot{\theta} + M_{\dot{z}}\dot{z} + M_{\delta_e}\delta_e + M_{\delta_T}\delta_T \qquad \text{(A-1)}$$

$$\ddot{z} = Z_u u + g\theta \sin \theta_o + (U_o + Z_\theta)\dot{\theta}\; Z_{\dot{z}} + Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T$$

$$\dot{v} = Y_v v + g\phi \cos \theta_o + Y_p p\; g\psi \sin \theta_o$$

$$+ (Y_r - U_o)r + Y_{\delta_s}\delta_s + Y_{\delta_r}\delta_r$$

$$\ddot{\phi} = L_v v + L_p p + L_r r + L_{\delta_s}\delta_s + L_{\delta_r}\delta_r \qquad \text{(A-2)}$$

$$\ddot{\psi} = N_v v + N_p p + N_r r + N_{\delta_s}\delta_s + N_{\delta_r}\delta_r$$

where all variables have been defined in the symbols table. Note that as a result of the assumptions made, Equations (A-1) and (A-2) are decoupled. Table A-1 presents some typical STOL geometric data taken from reference 16. (See NASATN 2231 Table I p.22)

Table A-1

Geometric Data For Typical STOL Aircraft

Wing	
Area, sq ft	889
Span, ft	76.1
Mean aerodynamic chord (reference), ft	12.15
Incidence root, from fuselage reference line, deg	3
Twist, deg	0
Dihedral, deg	4
Airfoil section with cambered leading edge from internal nacelle to wing tip	63A416
Aspect ratio	6.25
Taper ratio	0.507
Flap deflection (maximum), deg	Internal 98; external 65
Flap chord (percent wing chord)	38.5
Spoiler spanwise location	From 56 to 97 percent of span
Spoiler deflection, deg	45
Spoiler chord, percent chord	7
Horizontal tail	
Total area, sq ft	320
Span, ft	32.8
Mean aerodynamic chord, ft	9.92
Airfoil section	63A212 inverted with cambered leading edge
Elevator area, sq ft	119
Elevator deflection, deg	
Maximum trailing edge up	-35
Maximum trailing edge down	+25
Stabilizer deflection, deg	+1 to +9 to fuselage ref. (leading edge up)
Vertical tail	
Total area, sq ft	219
Span, ft	17.9
Mean aerodynamic chord, ft	13.1
Airfoil section (modified)	63A013
Rudder area, sq ft	82.6
Rudder deflection, deg	
First rudder	±20
Second rudder	±40
Moment of inertia (approximate for 38,500 lb gross weight)	
I_{xx}, slug-ft^2	225,000
I_{yy}, slug-ft^2	140,000
I_{zz}, slug-ft^2	400,000

Table A-2 gives values for the dimensional stability derivatives shown in Equation (A-1). θ_o is taken to be the initial flight path angle of the aircraft and equal to $-7\frac{1}{2}^\circ$ = -.13 rad. Values were obtained from references 16, 17, and 44.

Table A-3 shows the frequency and damping ratio associated with the longitudinal and lateral modes of the model.

Table A-3

Open Loop Dynamics

Frequency - short period	ω_{sp}	.59 rad/sec
Damping ratio - short period	ζ_{sp}	.9 sec $^{-1}$
Period - short period	T_{sp}	10.6 secs
Frequency - phugoid	ω_{ph}	.19 rad/sec
Damping ratio - phugoid	ζ_{ph}	.015 sec
Period - phugoid	T_{ph}	35 secs
Frequency - dutch roll	ω_d	.76 rad/sec
Damping ratio - dutch roll	ζ_d	.12 sec $^{-1}$
Period - dutch roll	T_d	8.26 secs

It is interesting to note that lateral control is achieved through rudder and spoiler, differential propeller pitch and aileron interconnects. Since I was unable to obtain data which presented numerically the relative displacement of these controls as a function of lateral stick movement, I assumed that spoilers were the most effective control of the three. (See Reference 16). It is for this reason that only maximum spoiler displacements were cited as control constraints in Chapter 3.

Table A-2

Values of Derivatives Used in Simulation

Derivative	Dimension	Value
X_u	1/sec	-.032
$X_{\dot{\theta}}$	ft/sec rad	0
$X_{\dot{z}}$	1/sec	.133
X_{δ_e}	ft/sec^{2}rad	0
X_{δ_T}	ft/sec^{2}lbs	.00065
M_u	1/sec ft	.00137
$M_{\dot{\theta}}$	1/sec rad	-.743
$M_{\dot{z}}$	1/sec ft	-.0014
M_{δ_e}	1/sec^{2}rad	-.989
M_{δ_T}	1/sec^{2}lbs	-.000007
Z_u	1/sec	-.02
$Z_{\dot{\theta}}$	ft/sec rad	-3.5
$Z_{\dot{z}}$	1/sec	-.3
Z_{δ_e}	1/sec^{2}rad	3.0
Z_{δ_T}	1/sec^{2}lb	-.00087
Y_v	1/sec rad	-.0013
Y_p	1/rad	0
Y_r	1/rad	0
Y_{δ_s}	1/sec rad	0
Y_{δ_r}	1/sec rad	5.0
L_v	rad/ft-sec	-.00322

Table A-2 (cont.)

Derivative	Dimension	Value
L_p	1/sec rad	-.82
L_r	1/sec rad	.139
L_{δ_s}	$1/sec^2$ rad	1.337
L_{δ_r}	$1/sec^2$ rad	.0716
N_v	rad/ft sec	.0054
N_p	1/sec rad	-.05
N_r	1/sec rad	-.33
N_{δ_s}	$1/sec^2$ rad	-.1251
N_{δ_r}	$1/sec^2$ rad	-.2462

Table A-4

Dimensional and Non-Dimensional Derivatives

Quantity	Non-Dimensional Form
X_u	$\frac{\rho SU}{m} (-C_D - C_{D_u})$
$X_{\dot{z}}$	$\frac{\rho SU}{m} (-C_L - C_{D\alpha})$
X_δ	$\frac{\rho SU^2}{2m} (-C_{D_\delta})$
M_u	$\frac{\rho SU_c}{Iy} (C_m + C_{m_u})$
$M_{\dot{\theta}}$	$\frac{\rho SU_c{}^2}{4Iy} (C_{m_q})$
$M_{\dot{z}}$	$\frac{\rho SU_c}{2Iy} (C_{m_\alpha})$
Z_u	$\frac{\rho SU}{m} (-C_L - C_{L_u})$
$Z_{\dot{\theta}}$	$\frac{\rho SU_c}{4m} (-C_{L_q})$
$Z_{\dot{z}}$	$\frac{\rho SU}{2m} (-C_{L_\alpha} - C_D)$

Appendix B

Gust Model

Atmospheric turbulence is a continuous vector random process that varies in three space dimensions and also in time. Obviously, any mathematical model that could be found to describe all that is known about atmospheric turbulence would be extremely complicated. It is necessary then, to make some simplifying assumptions, which can be found in Reference 45.

Assumption 1. Turbulence is isotropic, i.e., the statistical properties of the turbulence are invariant with axis translation, rotation, and reflection. This allows us to describe the turbulence relative to the airplane body-axis system.

Assumption 2. There is no cross correlation among the three turbulent velocity components.

Assumption 3. Taylor's hypothesis that time variations are statistically equivalent to distance variations in traversing the turbulence field is true.

Assumption 4. The turbulent velocity field is assumed to be a zero mean Gaussian (Normal) random process.

Gust Spectra

The Dryden form of the power spectral density of the turbulence will be used to describe the frequency distribution. The Dryden form of the power spectral density approaches a constant value asymptotically according to the -2 power of frequency for high spatial frequencies, as shown in Equation (B-1):

$$\Phi_{u_g}(\Omega) = \sigma_u^{\,2} \; \frac{2L_u}{\pi} \; \frac{1}{1 + (L_u\Omega)^2}$$

$$\Phi_{v_g}(\Omega) = \sigma_v^{\,2} \; \frac{L_v}{\pi} \; \frac{1 + 3\,(L_v\,\Omega)^2}{\left[1 + (L_v\,\Omega)^2\right]^2}$$

$$\Phi_{w_g}(\Omega) = \sigma_w^{\,2} \; \frac{L_w}{\pi} \; \frac{1 + 3(L_w \quad)^2}{\left[1 + (L_w\,\Omega)^2\right]^2} \qquad \text{(B-1)}$$

where the subscripts u_g, v_g, and w_g refer to the longitudinal, lateral, and vertical gust components, the mean square σ's are given by Equation (B-2)

$$\sigma_1^{\,2} = \int_o^\infty {}_i\,(\Omega)\; d\Omega_i \qquad i = u,\ v,\ \text{or } w \qquad \text{(B-2)}$$

and the scales for clear air turbulence are

$$L_w = h \text{ feet}$$

$$L_u = L_v = 145\ h^{\ 1/3} \qquad \text{(B-3)}$$

for an altitude h less than 1750 feet.

The root mean squate intensity σ_w for clear air turbulence is defined in Reference 45. The corresponding intensities may be obtained from:

$$\frac{\sigma_u^{\,2}}{L_u} = \frac{\sigma_v^{\,2}}{L_v} = \frac{\sigma^2_{\ w}}{L_w} \qquad \text{(B-4)}$$

The gust model used will be a slight corruption of the model presented above since only the w_g and v_g components will be simulated and the power spectral density function of the u_g component will be used for both w_g and v_g. Based on these assumptions and realizing that

$$\Phi_{u_g} = \left|T_{u_g}\right|^2 \tag{B-5}$$

the model takes the form

$$T_{v_g} = \sigma_v\sqrt{\frac{2L_v}{\pi U_0}}\left[\frac{1}{1+L_v/U_0 s}\right] = \frac{2.707\sigma_v}{1+11.5s}$$

$$T_{w_g} = \sigma_w\sqrt{\frac{2L_w}{\pi U_0}}\left[\frac{1}{1+L_w/U_0 s}\right] = \frac{1.78\sigma_w}{1+5.0s} \tag{B-6}$$

where U_0 = 100 ft/sec,and L_v and L_w, obtained from Eqn. (B-3) with h=100 ft.,have been substituted in. The filters shown in Eqn. (B-6) are used to shape the output of the random noise generators to obtain the desired gust response.

Schematic for the gust model is given in Figure B-1.

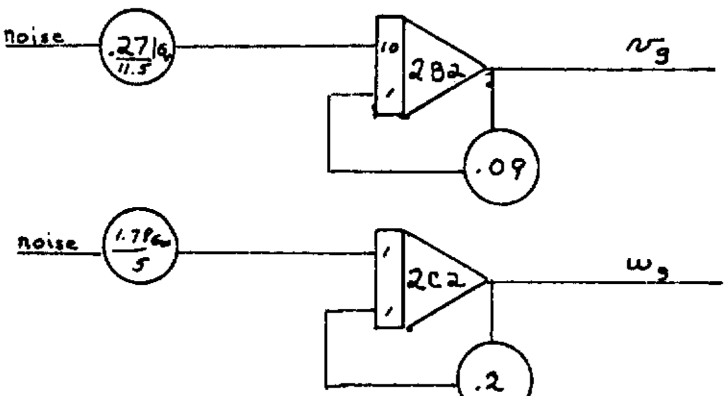


Figure B-1 Gust Model Schematic

Appendix C

The Gilbert-Pivichny Decoupling Program

The program is written in Fortran IV and was run on both a CDC 6500 and 6600. The subroutines and their functions are listed in Table C-1 and a flow chart is given in Figure C-1. The only input to the program is the order of the states and of the controls and the A, B, and C matrices given in Equations (2-1) and (2-2). This data is then used in the calculations of d_i and D_i which together with $|D|$ are printed. If $|D| = 0$, program stops; otherwise, it proceeds to calculate A^*, $\bar{A}$, $\bar{B}$, $\bar{C}$ and prints them. The system is then checked for controllability and if n_c the rank of H when

$$H = \bar{B}, \overline{AB}, A^2\bar{B}, \ldots A^{n-1}B$$

is less than n, system is divided into the controllable part represented by $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ which are printed.

Next, the canonically decoupled representation is calculated from $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\hat{A}$, $\hat{B}$, and $\hat{C}$ together with p_i are printed.

After the G_i matrices are calculated, the coefficient of the characteristic equation for each A_i matrix, $i = 1, \ldots m + 1$ are calculated using the Leverrier algorithm. The characteristic polynomial is then factored using subroutine PRQD and the J_k^i matrices are calculated and printed.

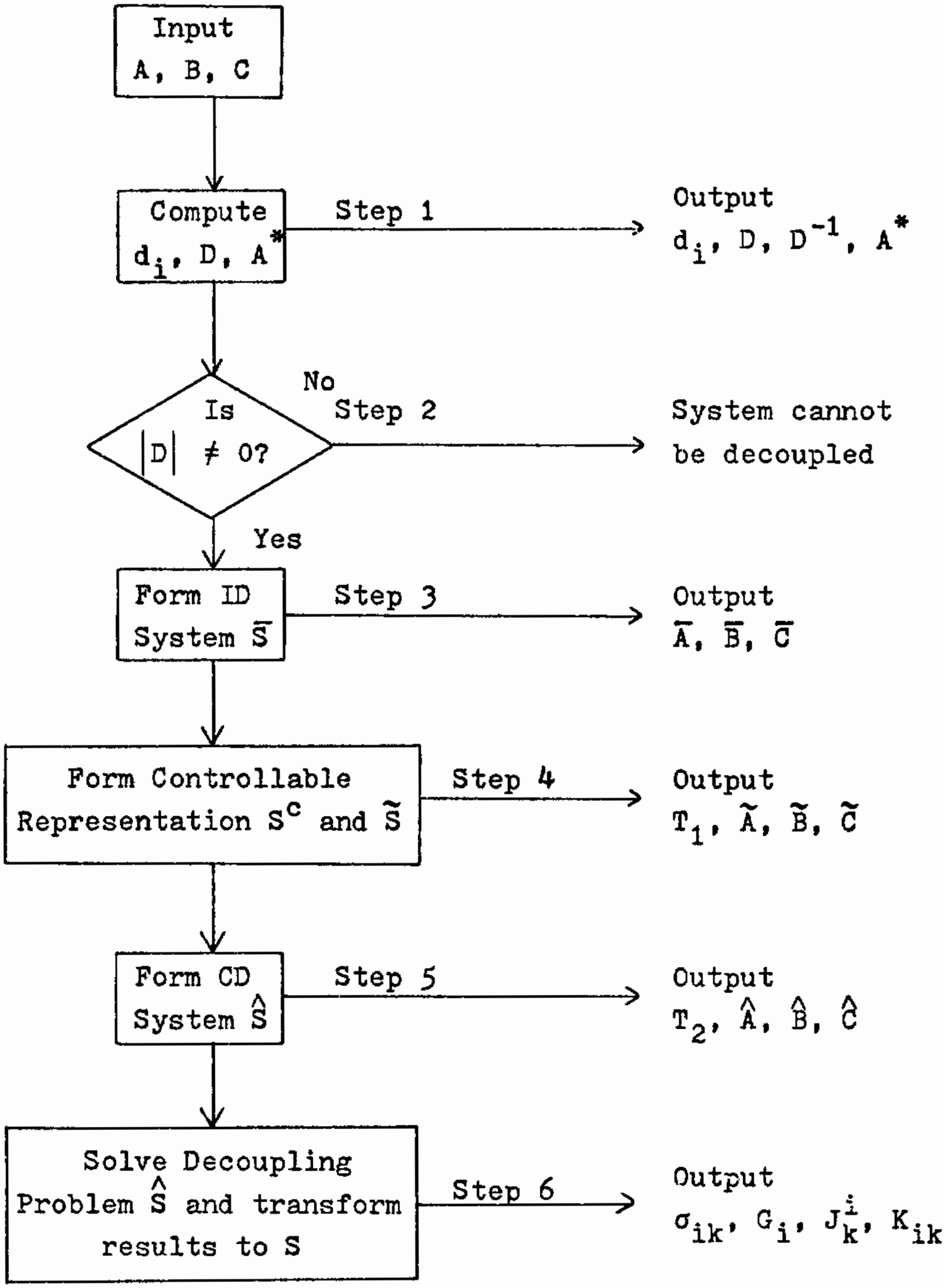
The above summary was condensed from "A Computer Program for the Design of Multivariable Decoupled Feedback

Systems" which was written by John Pivichny, who at the time was in the Computer, Information, and Control Engineering Program at the University of Michigan, Ann Arbor, Michigan.

Table C-1

Subroutines

Subroutine	Function
ARRAY	vector storage
MCPY	matrix copy
RCPY	copy row of matrix to vector
GMPRD	product of general matrix
MINV	matrix inversion
SCLA	matrix clear and add scalar
GMTRA	transpose of general matrix
RADD	add row to row
MFGR	matrix factor and rank determine
DCLA	replace diagonal with scalar
XCPY	copy submatrix from given matrix
SRMA	scalar multiply row and add
LOC	location in compressed stored material
GMSUB	subtract two matrices
CADD	add columns
DCPY	copy diagonal of matrix to vector
PRQD	roots of real polynomial



Flow Diagram for Main Steps in Decoupling Program[38]

Figure C-1

Appendix D

Transient Response Analysis

A computer program was written to analyze the response of the closed loop system to step inputs. The approach taken was to augment the state vector x with the control vector u so that the system would be represented by the linear, time fixed coefficient equation

$$\dot{x} = Ax \tag{D-1}$$

The solution, for constant A, is

$$x\ (t) = e^{A(t-\tau)}\ x\ (\tau) \tag{D-2}$$

where e^{At} can be represented by the infinite series[46]

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \ldots. \tag{D-3}$$

If we set all initial conditions equal to zero, save the control inputs, Equation (D-2) becomes

$$x = e^{At}\ x\ (0) \tag{D-4}$$

Neglecting all powers greater than two, Equation (D-3) is

$$e^{At} = I + At + \frac{A^2t^2}{2!} \tag{D-5}$$

Since the program is relatively short, a listing will follow.

```
C       PROGRAM TO OBTAIN STEP INPUT RESPONSE
C       FOR LONGITUDINAL DECOUPLED CLOSED LOOP
C       SYSTEM.
C
C       INPUTS
C          NXN MATRIX A, NXM MATRIX B,
C          MXN MATRIX C OBTAINED FROM EQUATIONS
C          DX/DT=AX + BU AND Y=CX
C          FEEDBACK NXM MATRIX F AND FEEDFORWARD
C          MXM MATRIX G OBTAINED FROM EQUATIONS
C          U=FX+GV
C          N = NUMBER OF STATE VARIABLES
C          M = NUMBER OF CONTROLS
C          X = INITIAL CONDITIONS ON X AND V
C
        DIMENSION A(10,10),B(10,10),C(10,10),X(10),YY(10),PHI(10,10)
        DIMENSION AS(10,10),AR(10,10), U(10),F(10,10),G(10,10)
        DIMENSION BF(10,10),BG(10,10),AC(10,10)
      5 FORMAT(2I3)
      6 FORMAT(3X,2HN=,I2,3X,2HM=,I2)
C       INPUT N, M, A, B, C, F, G, INITIAL X
     10 READ(2,5) N,M
        WRITE(3,96)
     96 FORMAT(10X,17HLONGITUDINAL CASE)
        WRITE(3,6)N,M
        NM=N+M
        N2=N
     15 FORMAT(6(F13.6,2X))
        DO 20 I=1,N
     20 READ(2,15) (A(I,J),J=1,N)
        DO 21 I=1,N
     21 READ(2,15)(B(I,J),J=1,M)
        DO 22 I=1,M
     22 READ(2,15)(C(I,J),J=1,N)
        DO 23 I=1,M
     23 READ(2,15)(F(I,J),J=1,N)
        DO 24 I=1,M
     24 READ(2,15)(G(I,J),J=1,M)
        READ(2,159)(X(I),I=1,NM)
    159 FORMAT(7(F8.2,2X))
C       PRINT A, B, C, F, G
     25 FORMAT(10X,8HA MATRIX)
        WRITE(3,25)
        DO 26 I=1,N
     26 WRITE(3,15) (A(I,J),J=1,N)
     27 FORMAT(10X,8HB MATRIX)
        WRITE(3,27)
        DO 28 I=1,N
     28 WRITE(3,15)(B(I,J),J=1,M)
     29 FORMAT(10X,8HC MATRIX)
```

```
      WRITE(3,29)
      DO 30 I=1,M
   30 WRITE(3,15)(C(I,J),J=1,N)
   31 FORMAT(10X,8HF MATRIX)
      WRITE(3,31)
      DO 32 I=1,M
   32 WRITE(3,15)(F(I,J),J=1,N)
   33 FORMAT(10X,8HG MATRIX)
      WRITE(3,33)
      DO 34 I=1,M
   34 WRITE(3,15) (G(I,J),J=1,M)
C     CALCULATE BF PRODUCT MATRIX USED IN
C     CLOSED LOOP EQUATION DX/DT=(A+BF)X+BGV
      CALL MATMU(B,F,BF,N,N,M)
      WRITE(3,351)
  351 FORMAT(10X,9HBF MATRIX)
      DO 350 I=1,N
  350 WRITE(3,15)(BF(I,J),J=1,N)
C     ADD A+BF
      CALL MATAD(A,BF,AC,N,N)
C     CALCULATE BG PRODUCT MATRIX USED IN EQN
C     GIVEN ABOVE
      CALL MATMU(B,G,BG,N,M,M)
  349 FORMAT(10X,20HCLOSED LOOP DYNAMICS)
      WRITE(3,349)
      WRITE(3,25)
      DO 279 I=1,N
C     PRINT CLOSED EQNS OF MOTION
  279 WRITE(3,15) (AC(I,J),J=1,N)
      WRITE(3,27)
      DO 299 I=1,N
  299 WRITE(3,15) (BG(I,J),J=1,M)
      DO 352 I=1,N
      DO 352 J=1,N
  352 A(I,J)=AC(I,J)
      DO 300 I=1,N
      DO 300 J=1,M
      N1=N+J
  300 A(I,N1)=BG(I,J)
      DO 301 I=1,N
      DO 301 J=1,M
      N1=N+J
  301 F(I,N1)=G(I,J)
      N=N+M
      DO 302 I=1,N
      DO 302 J=1,M
      N1=N2+J
  302 A(N1,I)=0.0
      WRITE(3,303)
  303 FORMAT(20X,10HPHI MATRIX)
```

```
C     CALCULATE PHI MATRIX USED IN
C     X=EXP(-PHI.T) SOLUTION
      IND=0
      DO 3 I=1,N
      XX(I)=0.0
      DO 3 J=1,N
      PHI(I,J)=0.0
      AS(I,J)=0.0
    3 AR(I,J)=0.0
      DO 12 I=1,N
      J=I
      AS(I,J)=1.0
   12 PHI(I,J)=1.0
      S=1.0
      CO=1.0
      DT=.01
      DO 14 KO=1,20
      CALL MATMU(A,AS,AR,N,N,N)
      DO 16 I=1,N
      DO 16 J=1,N
   16 AS(I,J)=AR(I,J)
      CO=(1.0/S)*CO
      DO 13 I=1,N
      DO 13 J=1,N
   13 PHI(I,J)=PHI(I,J)+AS(I,J)*(DT**S)*CO
   14 S=S+1.0
C     PRINT PHI MATRIX
      DO 200 I=1,N
  200 WRITE(3,101)(PHI(I,J),J=1,N)
  101 FORMAT(1X,7E15.5)
      WRITE(3,90)
   90 FORMAT(5X,4HX1=U)
      WRITE(3,91)
   91 FORMAT(5X,8HX2=THETA)
      WRITE(3,92)
   92 FORMAT(5X,12HX3=THETA-DOT)
      WRITE(3,93)
   93 FORMAT(5X,8HX4=Z-DOT)
      WRITE(3,94)
   94 FORMAT(5X,14HV1=STICK INPUT)
      WRITE(3,95)
   95 FORMAT(5X,17HV2=THROTTLE INPUT)
      WRITE(3,97)
   97 FORMAT(5X,22HU1=ELEVATOR DEFLECTION)
      WRITE(3,98)
   98 FORMAT(5X,17HU2=THRUST CHANGES)
C     PRINT LONGITUDINAL OUTPUT COLUMN HEADINGS
      WRITE(3,102)
  102 FORMAT(5X,1HT,7X,2HX1,8X,2HX2,8X,2HX3,8X,2HX4,8X,2HV1,8X,2HV2,
     1 8X,2HU1,8X,2HU2)
```

```
      T=0.0
C     PRINT INITIAL CONDITIONS
      WRITE(3,100)T,(X(I),I=1,NM)
  100 FORMAT(5X,F4.1,11F10.3)
      N2=N2+2
C     CALCULATE VALUES OF X FOR DT=.01
      DO 59 K=1,2000
      T=T+DT
      DO 109 I=1,N
      SUM=0.0
      DO 11 J=1,N
   11 SUM=SUM+PHI(I,J)*X(J)
  109 XX(I)=SUM
      DO 9 I=1,N
    9 X(I)=XX(I)
      DO 259 I=1,M
  259 U(I)=0.0
C     CALCULATE CONTROL SURFACE RESPONSE
      DO 238 I=1,M
      DO 238 J=1,N
  238 U(I)=U(I)+F (I,J)*X(J)
      IND=IND+1
      IF(IND-10)59,309,59
  309 IND=0.0
      DO 249 I=1,M
      N1=N+I
  249 X(N1)=U(I)
      N3=N1+1
      N4=N1+2
      X(N3)=0.0
      X(N4)=0.0
C     CALCULATE TIME DERIVATIVES OF X
      DO 250 L=1,N2
      X(N4)=X(N4)+A(5,L)*X(L)
  250 X(N3)=X(N3)+A(3,L)*X(L)
C     PRINT TIME HISTORY
      WRITE(3,100)T,(X(I),I=1,N3)
      CALL DATSW(1,I)
      GO TO (60,59),I
   59 CONTINUE
   60 GO TO 10
      END
      SUBROUTINE MATAD(A,B,C,N,M)
      DIMENSION A(10,10),B(10,10),C(10,10)
      DO 1 I=1,N
      DO 1 J=1,N
    1 C(I,J)=0.0
      DO 2 I=1,N
      DO 2 J=1,M
    2 C(I,J)=A(I,J)+B(I,J)
      RETURN
      END
```

```
      SUBROUTINE MATMU (A,B,C,N,M,L)
      DIMENSION A(10,10),B(10,10),C(10,10)
      DO 1 I=1,N
      DO 1 J=1,N
    1 C(I,J)=0.0
      DO 2 I=1,N
      DO 2 J=1,M
      DO 2 K=1,L
    2 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
```

Appendix E

Simulation Equations

The longitudinal and lateral directional scaled aircraft equations of motion are given in Equations (E-1) and (E-2), respectively. The scaling factors for all simulation parameters are given in Table E-1.

$$\dot{\bar{u}} = -.032\ \bar{u} - .32\theta + .013(\dot{\bar{z}} - \dot{\bar{z}}_g) + .09\ \bar{\delta}_T$$

$$\ddot{\theta} = .137\ \bar{\ } - .743\dot{\theta} - .014\ (\dot{\bar{z}} + \dot{\bar{z}}_g) - .989\delta_e - .098\bar{\delta}_T$$

$$\ddot{\bar{z}} = -.2\ \bar{\ } - .42\theta - 9.65\dot{\theta} - .3(\dot{\bar{z}} + \dot{\bar{z}}_g) + .3\delta_e - 1.218\bar{\delta}_T \qquad \text{(E-1)}$$

$$\dot{\bar{v}} = -.13\bar{v} + .161\phi - .0161\psi - .5\dot{\psi} - .025\delta_r$$

$$\ddot{\phi} = -.644\bar{v} - .82\dot{\phi} + .139\dot{\psi} + 1.337\delta_s - .0716\delta_r$$

$$\ddot{\psi} = 1.08\bar{v} - .05\dot{\phi} - .33\dot{\psi} - .125\delta_s - .246\delta_r \qquad \text{(E-2)}$$

The scaled feedback and feedforward gains are given in Table E-2 for the longitudinal and Table E-3 for the lateral-directional cases and the respective schematics in Figures E-1 and E-2.

Table E-1

Scaling Factors

Unscaled Parameter	Scaled Parameter	Scaling Factor
u	$\bar{u}$	1/100
$\dot{z}$	$\bar{\dot{z}}$	1/10
δ_T	$\bar{\delta}_T$	1/14000
v	$\bar{v}$	1/200
R^{-1}	$\bar{R}^{-1}$	385
$\dot{d}$	$\bar{\dot{d}}$	1/200
d	$\bar{d}$	1/2000
$\dot{d}_s$	$\bar{\dot{d}}_s$	1/200
d_s	$\bar{d}_s$	1/200
$\dot{h}$	$\bar{\dot{h}}$	1/200
h	$\bar{h}$	1/1000

F_{ij}	F_{11}	F_{12}	F_{13}	F_{14}	F_{21}	F_{22}	F_{23}	F_{24}	G_{11}	G_{12}	G_{21}	G_{22}
Pot	1D1	1D2	1D3	1D7	1D5	1D4	1D8	1D6	1A6	1A7	1D9	1A8
Case												
I	.151	.0334	.0513 x10	.069	.1269	.3366	.7797 x10	.5576	.1192	.99	.0294	.99 x10
II	.151	.7608	.0281 x10	.069	.1269	.7678 x10	.218	.5576	.1192	.99	.0294	.99 x10
III * **	.1193 x10	.66 .3065	.66 .026 x10	.003 $x10^{-1}$	.10637 x100	.3538 x10	0	.1462	.1192	.99	0	.99

* Command Angle

** Command Rate

Feedback and Feedforward Pot Settings - Longitudinal

Table E-2

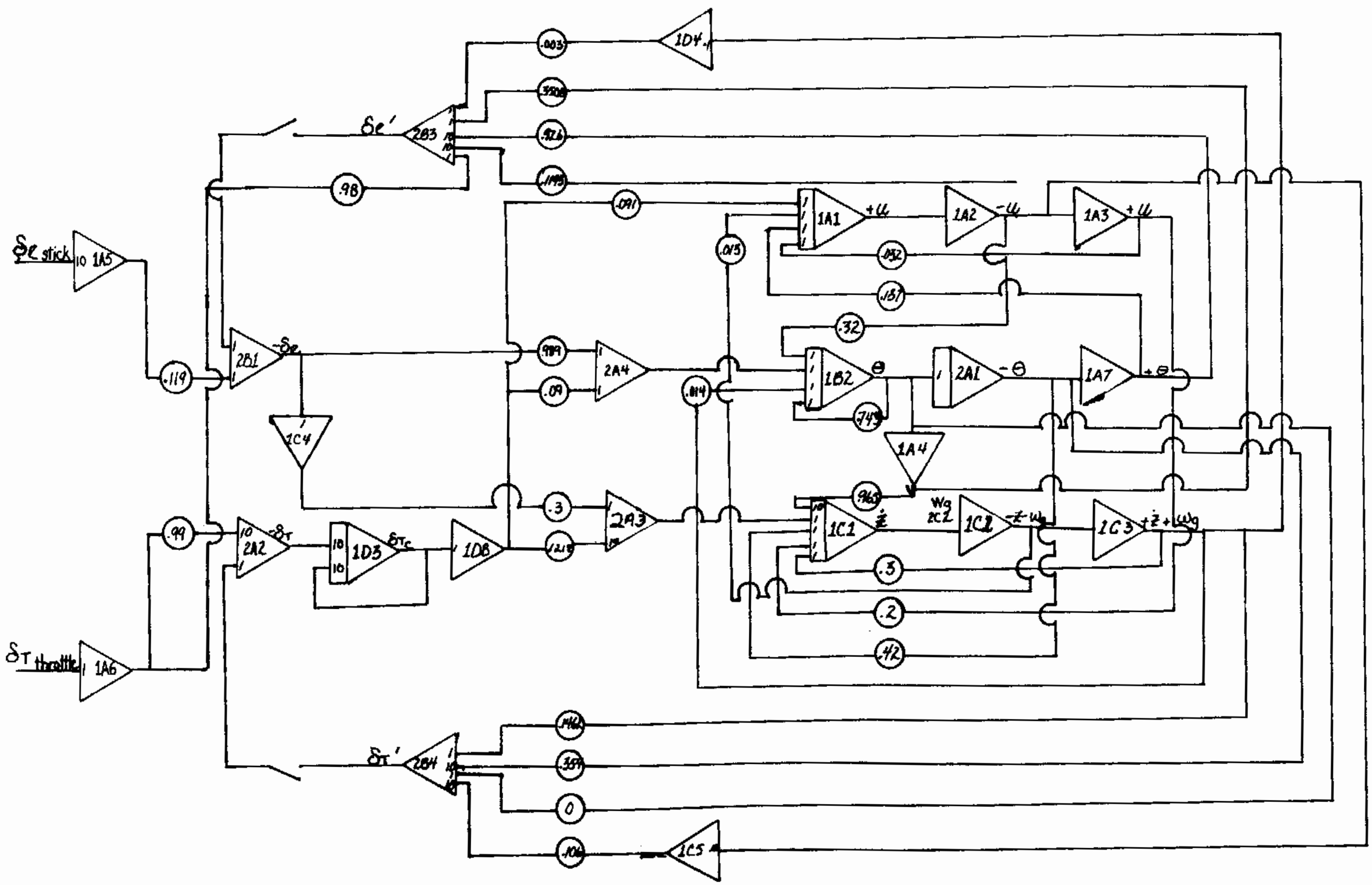
F_{ij}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{21}	F_{22}	F_{23}	F_{24}	F_{25}	G_{11}	G_{12}	G_{21}	G_{22}
Pot	1D7	1D8	1D9	1C10	1C9	1D1	2A4	1D3	1D4	1D5	1A7	1C8	1D6	1A6
Case														
IV*	.24	.7688	.5885	.2236	.346	.4258 x10	.3907	.096	.4175 x10	.45 x10	.143 x10	.02	.72	.3828
**	.24	0	.1272	0	.2567	.4528 x10	0	.1384	0	2.851 x10	.143 x10	.02	.72	.3828
V*	.262 x10	.4031	.5854	.0449	.1175 x10	.3994 x100	.644 x10	0	.84	.20 x100	.143 x10	.62	0	.3828
**	.262 x10	.2449	.1346	.0449	.1175 x10	.3994 x100	.644 x10	0	.84	.20 x1000	.143 x10	.02	0	.3828

* Command Angle

** Command Rate

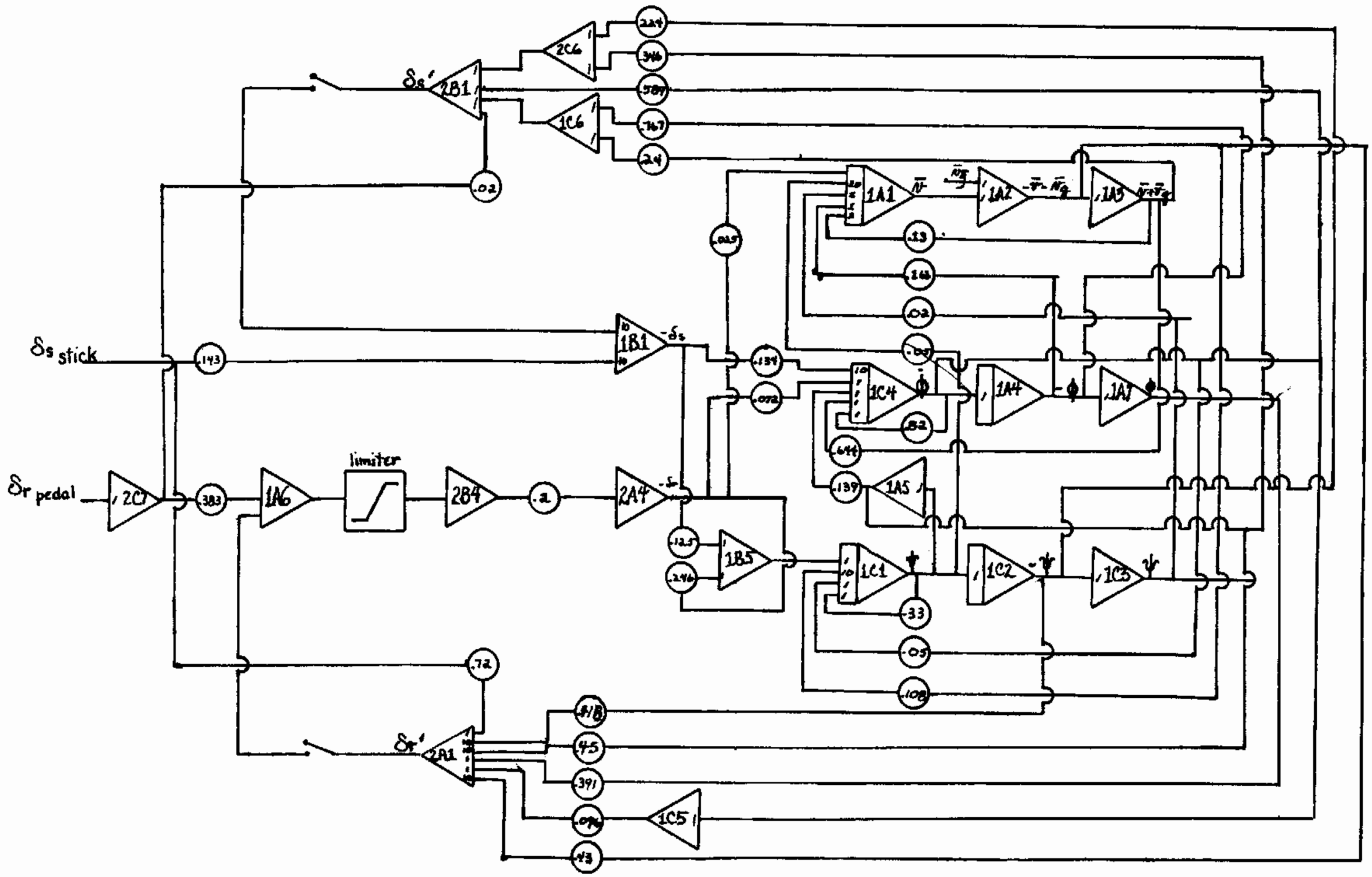
Feedback and Feedforward Pot Settings - Lateral

Table E-3



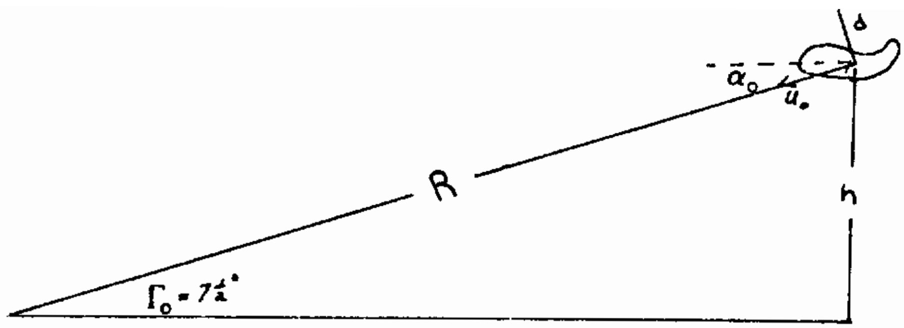
Analog Schematic for Longitudinal Dynamics With Feedback Augmentation for CASE III

Figure E-1



Analog Schematic For Lateral Dynamics With Feedback Augmentation For CASE IV

Figure E-2



Glide Slope

Figure E-3

Beam Model

The range R, shown in Figure E-3 is obtained from

$$R = R_o - U_o\, t \tag{E-3}$$

where R_o is the initial range. To obtain range changes, the following derivation was applied.

$$\frac{1}{R} = \frac{1}{(R_o - U_o t)} \tag{E-4}$$

Differentiating Equation (E-4),

$$\frac{d\,R^{-1}}{dt} = \frac{U_o}{(R_o - U_o t)^2} = U_o (R^{-1})^2 \qquad \text{(E-5)}$$

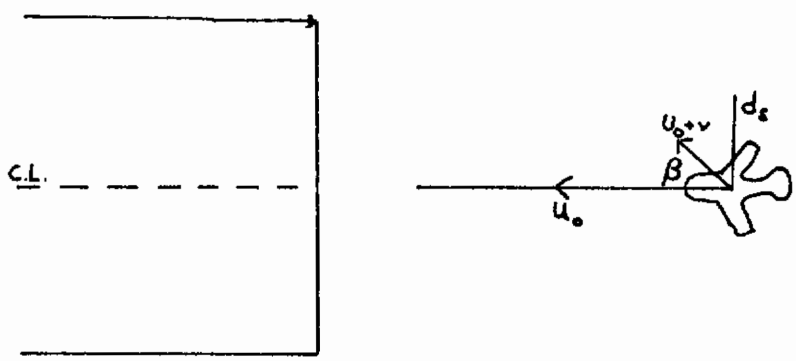
For $\Gamma_o = 7\frac{1}{2}^\circ$ at an initial altitude of 1000 feet,
$R_o = 7692$ feet.

The normal deviation from the glide slope, d in feet, is obtained by integrating

$$\dot{d} = \gamma(U_o + u) \qquad \text{(E-6)}$$

The deviation from the glide slope, δ in degrees, which was displayed to the pilot, is given by

$$\delta = 57.3d \cdot R^{-1} \qquad \text{(E-7)}$$



Localizer

Figure(E-4)

Localizer deviation d_s in feet (see Figure E-4), is obtained by integrating

$$\dot{d}_s = (U_o + v) \cdot (\psi + \beta) \qquad \text{(E-8)}$$

Localizer deviation, η in degrees, which was displayed to the pilot is given by

$$\eta = 57.3 d_s \cdot R^{-1} \qquad \text{(E-9)}$$

The schematic implemented to obtain both δ and η is given in Figure E-5.

Sink rate, $\dot{h}$, is given by

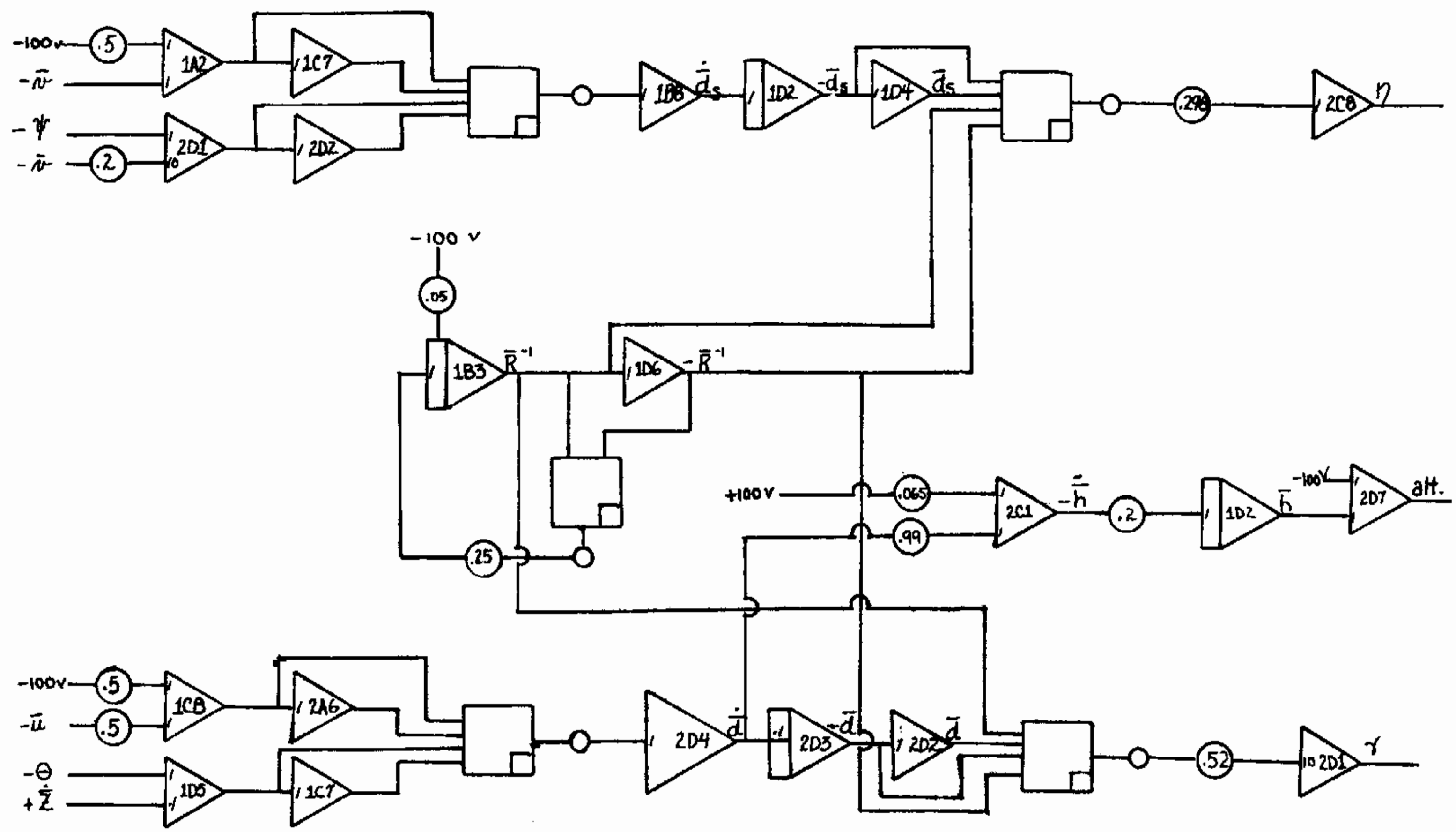
$$\dot{h} = -\Gamma_o U_o - d \cos \Gamma_o \qquad \text{(E-10)}$$

Altitude, h, is obtained from

$$h = 1000 - \int_0^{T} \dot{h} \, dt \qquad \text{(E-11)}$$

Schematic is given in Figure E-5.

I would like to thank Robert Lemble of the Flight Dynamics Laboratory for providing pilot comments on the mechanics of the simulator and for his help in the scavenging and assembly of the simulator parts, both of which were much needed.



Analog Schematic For Beam Mode 1, Altitude and Altitude Rate

Figure E-5

Appendix F

Briefing Guide and Rating Information For Handling Qualities Experiments

Objectives

General

Aircraft Role - STOL transport

Flight Segment - Approach to flare

Parameter or variable of primary interest: Deviation from ILS beam

Mission description:

General statement of the required operations for the flight segment of interest in context of aircraft role.

1. What is pilot-vehicle combination required to accomplish?

The pilot is requested to track the ILS beam from an altitude of 1000 feet to flare altitude 50 feet. No specific accuracy is required. $7\frac{1}{2}^\circ$ glide slope.

2. What are the conditions under which these required operations are to be carried out (i.e., aircraft state, environment, and cockpit interface)?

Approach configuration (98° flaps, 732 ft/min initial sink rate, 100 fps forward speed (60 knts)). Light to moderate turbulence, no crosswind. No consideration has been given to optimum position of instruments.

What is provided?

Test vehicle: (aircraft, spacecraft, simulator, etc.)

Fixed base simulator of STOL aircraft

cockpit control system not consistent with any

particular aircraft.

Aircraft State:

Normal _X_ Emergency ____ Identify failure:

Configuration:

Approach configuration, 98° flaps, 60 knots,

732 ft/min sink rate.

Gross weight: ≈ 38,000 lb.

Mass distribution: CG at 30.8% of MAC

Changes in aircraft state: None

Configuration changes (during task or designated phase)

None

Transient failures (unanticipated failures to be introduced into task for pilot reaction to and correction of resulting disturbance)

None

Cockpit interface:

(Brief description of important items noting unusual or detracting or limiting characteristics in particular)

Variables displayed

1. course deviation
2. sink rate
3. velocity (airspeed)
4. pitch
5. roll
6. yaw
7. sideslip

Scope

Task related only

Nature of task provided - Short term

Critical task is: to keep both longitudinal and lateral deviation from ILS beam as small as possible.

Are simulated disturbances provided or to be supplied by pilot?

Turbulence is to be simulated. Pilot may also induce disturbances.

Provision for familiarization:

According to individual pilot.

Measured performance:

Is performance to be measured?

No

Rating Information

What is to be rated?

__X__ Handling Qualities

__X__ Task - ILS tracking

____ Flight Segment - Approach

Principal controls:

Primary controls:

Longitudinal and lateral stick (side)

Rudder pedal

Throttle

Secondary controls:

None

Selectors:

Not provided - configuration unchanged.

Cues and disturbances provided:

Motion:

None

Visual:

Instruments: Conventional ____ Novel _X_

Full panel ____ Part panel _X_

(Brief statement of instruments or characteristics which may affect the evaluation)

Position of instruments may present scanning problem.

Comment Card Run ____________

In commenting, pilots are requested to compare, when possible, the response of the aircraft with SAS decoupling to the response of basic aircraft. Effects of simulation set-up should be included in any comments.

1. Ease of achieving a desired bank angle

a. Achieve roll rate

Easy

Tolerable difficulty

Extremely difficult

b. Attain desired bank angle

Easy

Tolerable difficulty

Extremely difficult

c. Overshoot

None

Slight

Moderate

Excessive

d. Is lead required?

Additional comments ____

2. Oscillatory characteristics*

a. Magnitude and mode

1) Pitch

a) Amplitude

b) Frequency

c) Damping

d) How excited

(1) stick

(2) throttle

e) How controlled

(1) stick

(2) throttle

f) *Ease of control

Additional comments

2) Roll

a) Amplitude

b) Frequency

c) Damping

d) How excited

(1) stick

(2) rudder pedal

e) How controlled

(1) stick

(2) rudder pedal

f) *Ease of control

Additional comments

*Pilot may be as specific or general as he desires in comments on amplitude, frequency, damping, case of control.

3) Yaw

a) Amplitude

b) Frequency

c) Damping

d) How excited

(1) stick

(2) rudder pedal

e) How controlled

(1) stick

(2) rudder pedal

f) Ease of control

Additional comments

3. ILS beam tracking ability

(ability to make small corrections)

a. Localizer

Tolerable difficulty

Extremely difficult

b. Glide slope

Easy

Tolerable difficulty

Extremely difficult

c. How is airspeed affected in tracking task?

Additional comments

4. Comparison of workload SAS to without SAS

less workload

same workload

greater workload

Additional comments

5. Did you consider the turbulence

light

moderate

heavy

6. What are the most objectionable features of the configuration just completed?

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13. ABSTRACT

Tactical aircraft with STOL capability exhibit undesirable coupled response during the landing phase of flight. A simplified method for designing a stability augmentation system which eliminates the coupling effects is demonstrated. The method is based on Gilbert's decoupling theory which utilizes a feedback control law to obtain a set of single input, single output subsystems. The augmentation system can be designed to provide either command rate or command angle authority in the three rotational axes. Analyses is facilitated through the use of two computer programs, the first of which determines the class of control laws which will decouple a system. The second computer program determines, through transient response analysis, the values of the transfer function parameters required to satisfy response criteria. The results of a piloted simulation which analyzed several decoupled configurations is also presented. (U)

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