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FORCED VIBRATIONS OF SANDWICH STRUCTURES

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FOREWORD

This report was prepared by the Department of Civil Engineering of Columbia University, New York, New York, under Contract No. AF-33(616)-7042. This contract was initiated under Project No. 7351, "Metallic Materials", Task No. 73521, "Behavior of Metals". The work was administered under the direction of the Materials Central, Directorate of Advanced Systems Technology, Wright Air Development Division, with Mr. D.M. Forney, Jr. acting as project engineer.

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ABSTRACT

In Part I of this report, a method is presented for the determination of the frequency response functions of the components of deformation and of stress in orthotropic sandwich plates. It applies to the case of simply supported rectangular plates loaded by dynamic pressure normal to their planes.

In Part II, a similar method is presented for orthotropic sandwich cylindrical shells. The boundaries of the shell are assumed as simply supported, and the dynamic pressure is normal to the middle surface. In both problems, the analysis takes into account the transverse shear deformation of the core and the material damping of core and facings. The results are presented in the form of expressions suitable for numerical evaluation.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

W. J. Trapp

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PART I: VIBRATIONS OF SANDWICH PLATES

Introduction

In order to predict the behavior of a structural element subject to the action of random loading (for instance random noise), it is necessary to determine the frequency response functions of various quantities, such as the components of deformation, the components of stress, etc. This paper deals with the frequency response functions for a rectangular sandwich plate. The loading is assumed in the form of a normal pressure, uniformly distributed over the surface of the plate, and being random in time. The frequency response function of any quantity S (deformation, stress) is the function $\bar{S}(i\omega)$, such that the quantity S corresponding to the loading $q = 1 \cdot \varepsilon^{i\omega t}$ is represented in the form

$$S = S(t) = \bar{S}(i\omega) \cdot e^{i\omega t}$$

The discussions of the plate and the coordinate system are shown in Fig. 1. The analysis takes into account the effect of transverse shear deformation of the core. The thickness of the facings is assumed to be small in comparison to the thickness of the core.

The basic theory of such plates has been developed in two papers [1], [2] by R.D. Mindlin in which the detailed discussion of the assumptions is presented.

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The materials of the core and of the facings are assumed to be orthotropic. The elastic constants of the core E_{xx}, E_{yy}, E_{xy}, G_x G_y, G_z are defined by the stress-strain relations

$$\begin{aligned} \sigma_{yx} &= 2G_x c_{yz}, \quad \sigma_{xz} = 2G_y \varepsilon_{xz}, \quad \sigma_{xy} = 2G_z \varepsilon_{xy} \\ \sigma_{xx} &= E_{xx}\varepsilon_{xx} + E_{xy}\varepsilon_{yy}, \quad \sigma_{yy} = E_{yy}\varepsilon_{yy} + E_{xy}\varepsilon_{xx} \end{aligned} \tag{2}$$

and the elastic constants of the facings E'_{xx}, E'_{yy}, E'_{xy}, G'_z by the relations

$$\begin{aligned} \sigma'_{xy} &= 2G_z'\varepsilon'_{xy} \\ \sigma'_{xx} = E_{xx}\varepsilon'_{xx} + E'_{xy}\varepsilon'_{yy}, &\quad \sigma'_{yy} = E'_{yy}\varepsilon'_{yy} + E'_{xy}\varepsilon'_{xx} \end{aligned} \tag{3}$$

The transverse shear deformations of the facings are neglected. If the materials of the core and of the facings are isotropic, then

$$G_x = G_y = G_z = G = E/2(1+\nu)$$

$$E_{xx} = E_{yy} = E/(1-\nu^2), \quad E_{xy} = E\nu/(1-\nu^2)$$

with similar equalities for the constants of the facings.

The effect of damping is introduced by the use of the complex moduli of elasticity ([3] , p. 276)

$$\begin{aligned} \bar{G}_x &= G_x(1+i\mu_x), \quad \bar{G}_y = G_y(1+i\mu_y) \\ \bar{G}_z &= G_z(1+i\mu_z), \quad \bar{E}_{xx} = E_{xx}(1+i\eta_{xx}) \\ \bar{E}_{yy} &= E_{yy}(1+i\eta_{yy}), \quad \bar{E}_{xy} = E_{xy}(1+i\eta_{xy}) \end{aligned} \tag{4}$$

The coefficients μ_x, μ_y, μ_z are 2π times the specific damping $\Delta W_d/W_p$, where ΔW_d is the energy dissipated during one cycle in shear, and W_p is maximum shear strain energy. The coefficients η are related in a similar way to the dissipated and the strain energies in the cyclic tension - compression tests. In general, $\mu_x, \ldots\ldots, \eta_{xy}$ depend on the frequencies and amplitudes of cyclic stresses or deformations. In first approximation, it is assumed that the damping is frequency-independent. This assumption is in fair agreement with experimental results for various materials. It is also assumed that damping is independent of amplitude of applied stress. This assumption is less justified by the experimental evidence, since the damping of real materials usually increases with increasing amplitudes. However, without this assumption non-linear equations of the problem would be obtained. A proper selection of constant coefficients $\mu_x, \ldots\ldots, \eta_{xy}$, for an expected - not too wide - stress and strain range, may provide sufficient accuracy of the results of the analysis. The coefficients of damping and the complex moduli of elasticity of the facings will be denoted by μ_z', η_{xx}', η_{yy}', η_{xy}' and $\bar{G}_z'$, $\bar{E}_{xx}'$, $\bar{E}_{yy}'$, $\bar{E}_{xy}'$ respectively.

Basic Equations

The equations of motion of an elastic orthotropic sandwich plate are derived in [2], in the form

$$\frac{\partial M}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I\,\frac{\partial^2 \Psi_x}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M}{\partial y} - Q_y = I\,\frac{\partial^2 \Psi_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = M\,\frac{\partial^2 w}{\partial t^2}$$

where M_x, M_y, M_{xy} are bending and torsional moments, respectively; Q_x, Q_y are transverse shear forces; w, Ψ_x, Ψ_y are the components of deformation of the plate as shown in Fig. 2; q is the loading perpendicular to the plane of the plate; I is the moment of inertia of the mass of the plate with respect to its middle plane, M is the mass of the plate. The internal forces $M_x, \ldots, Q_y$ are taken per unit length of the cross-section, the loading q, moment I and mass M per unit area of the plate.

If the densities of the core and the facings are ρ and ρ' respectively, then

$$
\begin{aligned}
M &= \rho h + 2\rho' f \\
I &= \rho h^3/12 + \rho' f h^2/2
\end{aligned}
\tag{6}
$$

where h is the thickness of the core and f is the thickness of one facing.

The relations between the internal forces $M_x, \ldots, Q_y$ and the components of deformation w, ψ_x, ψ_y, as derived in [2], are

$$
\begin{aligned}
Q_x &= K_x \left(\psi_x + \frac{\partial w}{\partial x}\right), \qquad M_{xy} = H_{xy}\left(\frac{\partial \psi_y}{\partial x} + \frac{\partial \psi_x}{\partial y}\right) \\
Q_y &= K_y \left(\psi_y + \frac{\partial w}{\partial y}\right) \\
M_x &= D_x \frac{\partial \psi_x}{\partial x} + D_{xy} \frac{\partial \psi_y}{\partial y} \\
M_y &= D_y \frac{\partial \psi_y}{\partial y} + D_{xy} \frac{\partial \psi_x}{\partial x}
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
K_x &= \kappa^2 G_x h, \qquad K_y = \kappa^2 G_y h \\
D_x &= E_{xx} h^3/12 + E'_{xx} h^2 f/2 \\
D_y &= E_{yy} h^3/12 + E'_{yy} h^2 f/2 \\
D_{xy} &= E_{xy} h^3/12 + E'_{xy} h^2 f/2 \\
H_{xy} &= G_z h^3/12 + G'_z h^2 f/2
\end{aligned}
\tag{8}
$$

The equations (5), (7) and (8) are also valid for plates with damping if the moduli $E_{xx}, \ldots, G_z$, $E'_{xx}, \ldots, G'_z$ are replaced by the complex moduli $\bar{E}_{xx}, \ldots, \bar{G}_z$, $\bar{E}'_{xx}, \ldots, \bar{G}'_z$. As it follows from [2], the coefficient κ for sandwich plates is very close to 1 and, therefore, it will be neglected in further considerations.

Substituting the expressions (7) into equations (5), the following three equations for the components of deformation are obtained:

$$\bar{D}_x \frac{\partial^2 \psi_x}{\partial x^2} + \bar{D}_{xy} \frac{\partial^2 \psi_y}{\partial x \partial y} + \bar{H}_{xy} \frac{\partial^2 \psi_y}{\partial x \partial y} + \bar{H}_{xy} \frac{\partial^2 \psi_x}{\partial y^2} - \bar{K}_x \left(\psi_x + \frac{\partial w}{\partial x} \right) = I \frac{\partial^2 \psi_x}{\partial t^2}$$

$$\bar{D}_y \frac{\partial^2 \psi_y}{\partial y^2} + \bar{D}_{xy} \frac{\partial^2 \psi_x}{\partial x \partial y} + \bar{H}_{xy} \frac{\partial^2 \psi}{\partial x \partial y} + \bar{H}_{xy} \frac{\partial^2 \psi_y}{\partial x^2} - \bar{K}_y \left(\psi_y + \frac{\partial w}{\partial y} \right) = I \frac{\partial^2 \psi_y}{\partial t^2} \qquad (9)$$

$$\bar{K}_x \frac{\partial \psi_x}{\partial x} + \bar{K}_y \frac{\partial \psi_y}{\partial y} + \bar{K}_x \frac{\partial^2 w}{\partial x^2} + \bar{K}_y \frac{\partial^2 w}{\partial y^2} + q = M \frac{\partial^2 w}{\partial t^2}$$

The coefficients $\bar{D}_x, \ldots, \bar{K}_y$ are now complex, as they contain the complex moduli of elasticity in the form (4). Equations (9) determine an unique solution if on the boundary one quantity of each of the pairs

$$(M_i, \psi_i), \quad (M_{ij}, \psi_i), \quad (Q_i, w); \quad i = x, y, \quad j = x, y$$

are given. For the simply supported plate, the following boundary conditions will be assumed:

for $x = 0, a$, $M_x = 0$, $\psi_y = 0$, $w = 0$

for $y = 0, b$, $M_y = 0$, $\psi_x = 0$, $w = 0$

Determination of Dynamic Response

In order to determine the response of the plate to a loading with known space distribution $\bar{q}(x, y)$, being random in time, it is necessary to determine first the response of the plate to the loading

$$q(x,y,t) = \bar{q}(x,y)\,e^{i\omega t} \tag{11}$$

Employing Fourier series expansion, the loading $q(x,y,t)$ may be represented in the form

$$q(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} Q_{mn} \sin\alpha_m x \sin\beta_n y\, e^{i\omega t} \tag{12}$$

where

$$\alpha_m = m\pi/a \quad , \qquad \beta_n = n\pi/b$$
$$m = 1, 2, 3 \ldots \quad , \qquad n = 1, 2, 3, \ldots$$

with the coefficients Q_{mn} properly determined for each particular type of loading $\bar{q}(x,y)$. If the loading is in the form of the unit pressure uniformly distributed over the plate, i.e., $\bar{q}(x,y) = const =$ $= 1$, then

$$Q_{mn} = 16/mn\pi^2 \; ; \quad m = 1, 3, 5, \ldots , \; n = 1, 3, 5, \ldots \tag{13}$$

The solutions for Ψ_x, Ψ_y, w are assumed in the forms

$$\begin{aligned}
\Psi_x &= \sum_m\sum_n \Psi_{xmn} \cos\alpha_m x \sin\beta_n y\, e^{i\omega t} = \bar{\Psi}_x\, e^{i\omega t} \\
\Psi_y &= \sum_m\sum_n \Psi_{ymn} \sin\alpha_m x \cos\beta_n y\, e^{i\omega t} = \bar{\Psi}_y\, e^{i\omega t} \\
w &= \sum_m\sum_n W_{mn} \sin\alpha_m x \sin\beta_n y\, e^{i\omega t} = \bar{w}\, e^{i\omega t}
\end{aligned} \tag{14}$$

which satisfy the boundary conditions (10). Substituting (14) into equations (9) the following system of linear algebraic equations is obtained for Ψ_{xmn}, Ψ_{ymn}, W_{mn},

$$(\bar{D}_x \alpha_m^2 + \bar{H}_{xy} \beta_n^2 + \bar{K}_x - I\omega^2)\, \Psi_{xmn} + (\bar{D}_{xy} \alpha_m \beta_n +$$
$$+ \bar{H}_{xy} \alpha_m \beta)\, \Psi_{ymn} + \bar{K}_x \alpha_m W_{mn} = 0$$
$$(\bar{D}_{xy} \alpha_m \beta_n + \bar{H}_{xy} \alpha_m \beta_n)\, \Psi_{xmn} + (\bar{D}_y \beta_n^2 + \bar{H}_{xy} \alpha_m^2 + \tag{15}$$
$$+ \bar{K}_y - I\omega^2)\, \Psi_{ymn} + \bar{K}_y \beta_n W_{mn} = 0$$
$$\bar{K}_x \alpha_m \Psi_{xmn} + \bar{K}_y \beta_n \Psi_{ymn} + (\bar{K}_x \alpha_m^2 + \bar{K}_y \beta_n^2 - M\omega^2) W_{mn} =$$
$$= Q_{mn}$$

The coefficients of these equations are complex, and the solutions are, in general, also complex functions of ω or $i\omega$: $\Psi_{ymn}(i\omega)$, $\Psi_{ymn}(i\omega)$, $W_{mn}(i\omega)$ It is easy to check that the solution for W_{mn} is

$$W_{mn} = Q_{mn} \left[\frac{\mathcal{E} I\omega^2 + \mathcal{F}}{I^2\omega^4 + \mathcal{A} I\omega^2 + \mathcal{B}} + \mathcal{G} - M\omega^2 \right]^{-1} \tag{16}$$

The functions Ψ_{xmn} and Ψ_{ymn} can be expressed in terms of W_{mn} in the following way

$$\Psi_{xmn} = W_{mn} \frac{\bar{K}_x \alpha_m I\omega^2 + \mathcal{C}}{I^2\omega^4 + \mathcal{A} I\omega^2 + \mathcal{B}} \tag{17}$$

$$\Psi_{ymn} = W_{mn} \frac{\bar{K}_y \beta_n I\omega^2 + \mathcal{D}}{I^2\omega^4 + \mathcal{A} I\omega^2 + \mathcal{B}} \tag{18}$$

The notations in the expressions (16), (17), and (18) are:

$$\left.\begin{aligned}
\mathcal{A} &= -(\bar{D}_x \alpha_m^2 + \bar{D}_y \beta_n^2 + \bar{H}_{xy} \alpha_m^2 + \bar{H}_{xy} \beta_n^2 + \bar{K}_x + \bar{K}_y) \\
\mathcal{B} &= (\bar{D}_x \alpha_m^2 + \bar{H}_{xy} \beta_n^2 + K_x)(\bar{D}_y \beta_n^2 + \bar{H}_{xy} \alpha_m^2 + \bar{K}_y) - \\
&\quad - (\bar{D}_{xy} + \bar{H}_{xy})^2 \alpha_m^2 \beta_n^2 \\
&\;\vdots
\end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned}\mathcal{C} &= -\bar{K}_x\,\alpha_m\left(\bar{D}_y\beta_n^2 + \bar{H}_{xy}\alpha_m^2 + \bar{K}_y\right) + \bar{K}_y\,\beta_n\left(\bar{D}_{xy}\alpha_m\beta_n + \bar{H}_{xy}\alpha_m\beta_n\right)\\ \mathcal{D} &= -\bar{K}_y\beta_n\left(\bar{D}_x\alpha_m^2 + \bar{H}_{xy}\beta_n^2 + \bar{K}_x\right) + \bar{K}_x\,\alpha_m\left(\bar{D}_{xy}\alpha_m\beta_n + H_{xy}\alpha_m\beta_n\right)\end{aligned}\right\}$$

$$\begin{aligned}\mathcal{E} &= \bar{K}_x^2\alpha_m + \bar{K}_y^2\beta_n^2\\ \mathcal{F} &= -\bar{H}_{xy}\bar{K}_x^2\alpha_m^4 - \bar{H}_{xy}\bar{K}_y^2\beta_n^4 + \left(-\bar{D}_y\bar{K}_x^2 + 2\bar{D}_{xy}\bar{K}_x\bar{K}_y + \right.\\ &\quad \left. + 2\bar{H}_{xy}\bar{K}_x\bar{K}_y + \bar{D}_x\bar{K}_y^2\right)\alpha_m^2\beta_n^2 - \bar{K}_x^2\bar{K}_y\,\alpha_m^2 - \bar{K}_x\bar{K}_y^2\beta_n^2\\ \mathcal{G} &= \bar{K}_x\,\alpha_m^2 + \bar{K}_y\,\beta_n^2\end{aligned} \qquad (20)$$

If the frequency response functions $\bar{\Psi}_x = \bar{\Psi}_x(i\omega)$, $\bar{\Psi}_y = \bar{\Psi}_y(i\omega)$ $\bar{w} = \bar{w}(i\omega)$ are known, the frequency response function of any component of deformation or stress can be easily calculated.

Considering the stress σ_{xx} in the facing

$$\sigma_{xx} = E_{xx}\frac{h}{2}\frac{\partial \Psi_x}{\partial x} + E_{xy}\frac{h}{2}\frac{\partial \Psi_y}{\partial y} \qquad (21)$$

Consequently, the frequency response function for σ_{xx} is

$$\begin{aligned}\bar{\sigma}_{xx}(i\omega) &= -\bar{E}'_{xx}\frac{h}{2}\sum_m\sum_n \Psi_{xmn}\alpha_m \sin\alpha_m x\,\sin\beta_n y -\\ &- \bar{E}'_{xy}\frac{h}{2}\sum_m\sum_n \Psi_{ymn}\beta_n\sin\beta_n y\,\sin\alpha_m x =\\ &= -\sum_m\sum_n\left[E'_{xx}(1+i\,\eta'_{xx})\frac{h}{2}\,\Psi_{xmn}\alpha_m + \right.\\ &\left.\pm E'_{xy}(1+i\,\eta'_{xy})\frac{h}{2}\,\Psi_{ymn}\beta_n\right].\sin\alpha_m x\,\sin\beta_n y\end{aligned} \qquad (22)$$

and

$$\begin{aligned}max.\bar{\sigma}_{xx}(i\omega) = -\sum_m\sum_n\left[E'_{xx}(1+i\,\eta'_{xx})\frac{h}{2}\Psi_{xmn}\alpha_m + \right.\\ \left. + E'_{xy}(1+i\,\eta'_{xy})\frac{h}{2}\,\Psi_{ymn}\beta_n\right].\end{aligned} \qquad (23)$$

To determine the frequency response function of the shear stress in the core $\bar{\sigma}'_{xz}(i\omega)$, $\sigma_{xz} = \bar{\sigma}_{xz}(i\omega).e^{i\omega t}$, it is necessary to start from the relation [2] :

$$\sigma_{xz} = G_x \left(\Psi_x + \frac{\partial w}{\partial x} \right) \tag{24}$$

which implies

$$\sigma_{xz}(i\omega) = G_x(1 + i\mu_x) \sum_m \sum_n (\Psi_{xmn} + W_{mn}\alpha_m) \cos\alpha_m x . \sin\beta_n y \quad ..(25)$$

and

$$max. \bar{\sigma}_{xz}(i\omega) = G_x(1 + i\mu_x) \sum_m \sum_n (\Psi_{xmn} + W_{mn}\alpha_m) \tag{26}$$

Similarly

$$\bar{\sigma}_{yy}(i\omega) = -\sum_m \sum_n [E'_{yy}(1 + i\eta'_{yy}) \frac{h}{2} \Psi_{ymn} \beta_n + $$
$$+ E'_{xy}(1 + i\eta'_{xy}) \frac{h}{2} \Psi_{xmn} \alpha_m]. \sin\alpha_m x \sin\beta_n y \tag{27}$$

and

$$\bar{\sigma}_{yz}(i\omega) = G_y(1 + i\mu_y) \sum_m \sum_n (\Psi_{xmn} + W_{mn}\beta_n) \sin\alpha_m x \cos\beta_n y \quad ..(28)$$

Example

For the numerical evaluation of some of the above equations, a plate with the following dimensions has been taken

$$a = 20h \quad , \quad b = 16h \quad , \quad f = 0.05\,h$$

The elastic properties of the materials of the core and of the facings are described by the set of elastic constants

$$E_{xx} = E_{yy} = E$$
$$E_{xy} = E_{yx} = 0.3E$$
$$E'_{xx} = E'_{yy} = E' = 50\,E$$
$$E'_{xy} = E'_{yx} = 0.3E'$$

$$G_x = G_y = G_z = 0.3846\,E$$

The shear damping of the core only has been considered with the damping coefficients

$$\mu_x = \mu_y = \mu_z = 0.1$$

Fig. 3 shows the response of this plate to a loading uniformly distributed on the surface and harmonically varying in time with the angular frequency ω . The absolute value of the dimensionless deflection $\bar{w} D_x / q h^4$ is given as a function of the dimensionless frequency ω/ω_o , where $\omega_o^2 = D_x / h^2 I$. Some insight into the dynamical behavior of this plate may be obtained from Fig. 4, which represents the deflection $\bar{w}_{11}$ caused by the loading distributed sinusoidally over the surface and harmonic in time (in plotting the diagram, the effect of damping was neglected). As it should be expected, three "resonant" frequencies result from the theory employed for this calculation. The classical plate theory would give only one of these frequencies.

Concluding Remarks

The derived expressions for the frequency response functions of deformations and of stresses may be used to determine the behavior of a plate under normal pressure, uniformly distributed over the surface of the plate, and random in time. In particular, it is relatively easy to determine the power spectra of the components of deformations and of stresses for an arbitrary power spectrum of loading, even if it is not given in the form of a mathematical expression (it may be given, for instance, in the form of a diagram).

The material damping is considered in a manner which seems to be as exact as it is possible without employing non-linear equations. The analysis neglects the interaction with the surrounding medium (air, fluid); this effect will be discussed in one of future publications. Also another limitation of the presented analysis should be indicated: since the strains and stresses in the z-direction have been neglected, the results seem to be not applicable for the frequencies and modes for which the distance between nodal lines is smaller than about twice the thickness of the plate.

References

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2 R.D. MINDLIN, "Flexural Vibrations of Elastic Sandwich Plates", (ONR Report), Columbia University, 1959.

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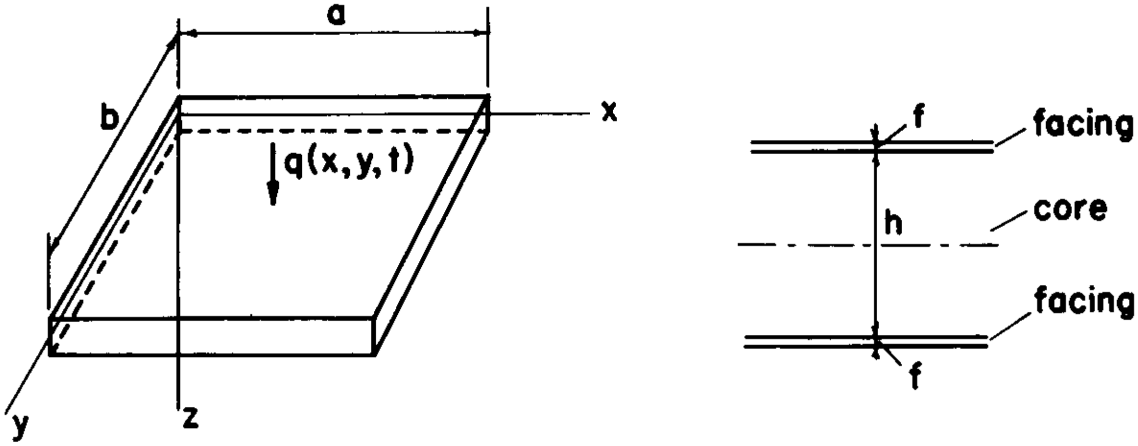

Fig.1. Coordinate system and dimensions of plate.

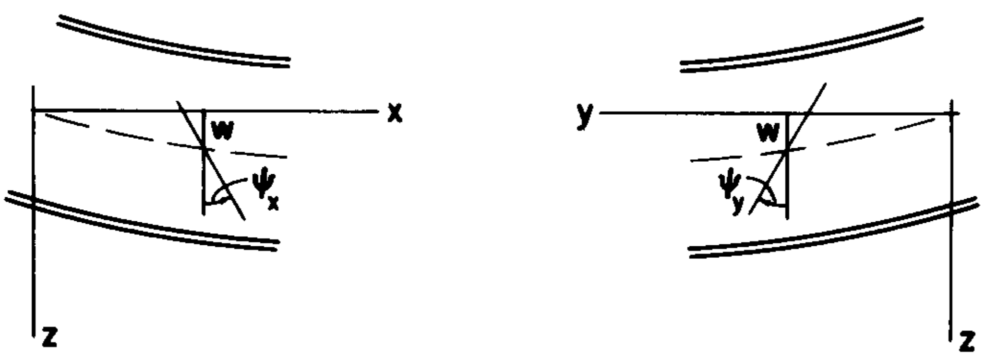

Fig.2. Components of deformation w, ψ_x, ψ_y

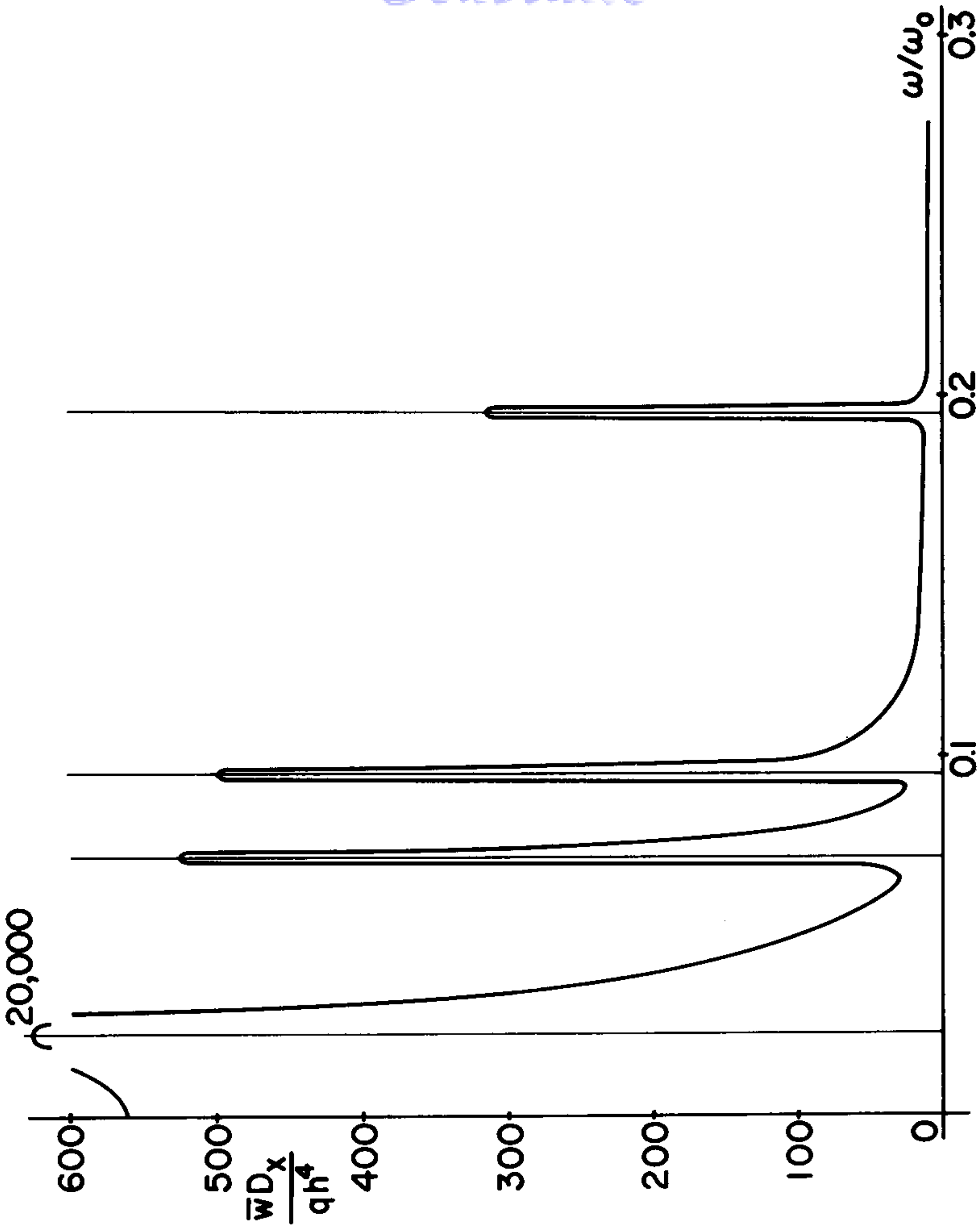

Fig. 3. Deflection, w, as a function of frequency, ω.

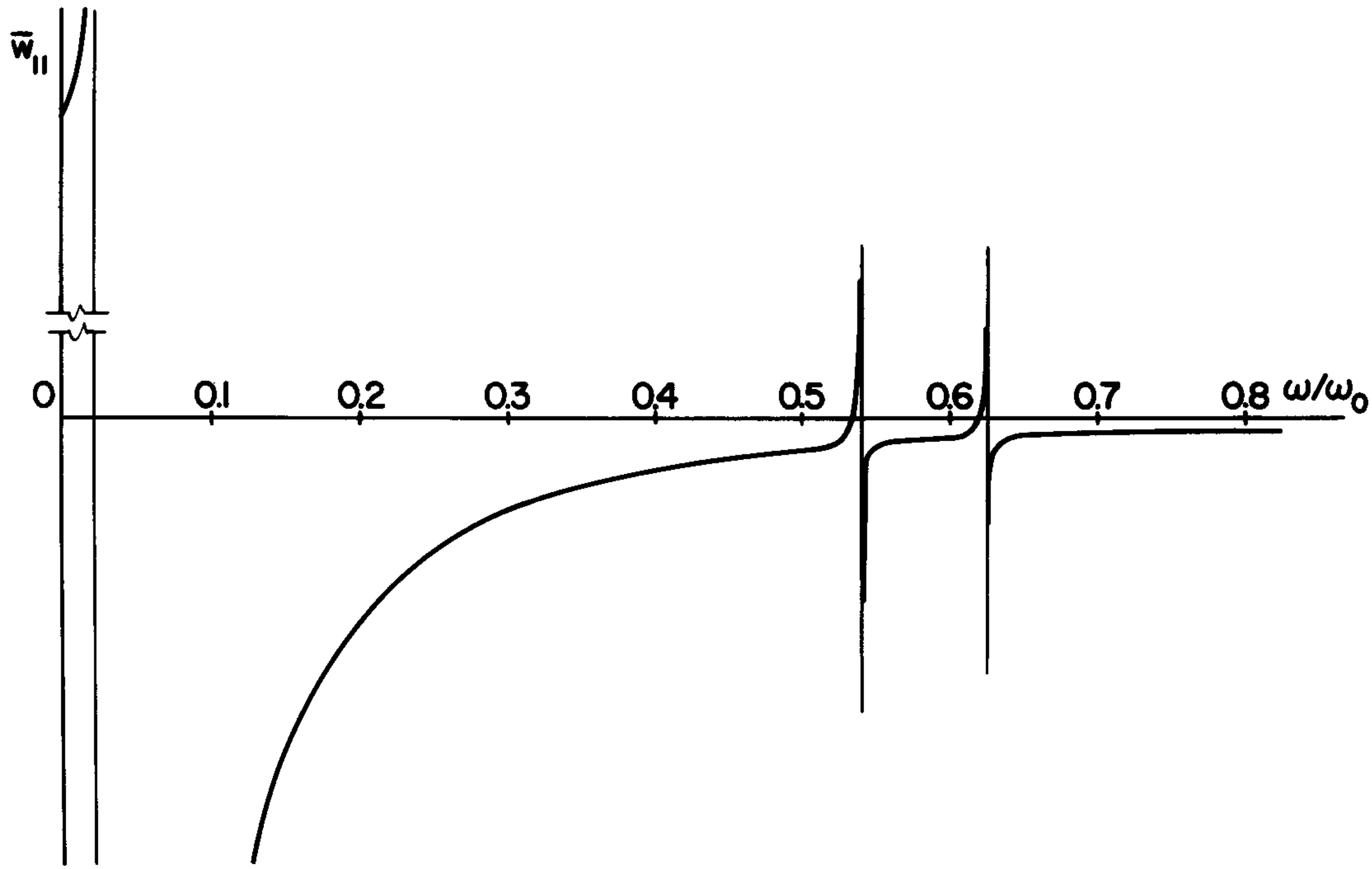

Fig. 4. Deflection w_{11} $(m = 1, n = 1)$ as a function of frequency ω/ω_0
(plate without damping)

PART II: VIBRATIONS OF CYLINDRICAL SANDWICH SHELLS

Introduction

The dimensions of the shell and the coordinate system are shown in Fig. 5. It is assumed that the components of the displacement of any point of the shell may be represented in the form

$$
\begin{aligned}
u_x &= u + \varphi z \\
u_y &= v + \psi z \qquad (1) \\
u_z &= w
\end{aligned}
$$

where $u = u(x, y; t)$, $v = v(x, y; t)$, and $w = w(x, y; t)$ are the components of the displacement of the middle surface in the direction of x, y, and z, respectively; $\varphi = \varphi(x, y; t)$ and $\psi = \psi(x, y; t)$ are the angles of rotation of the fibers which in the initial state are perpendicular to the middle surface.

The materials of the core and of the facings are assumed to be orthotropic and dissipative. If the components of stress and the components of strain vary in time harmonically, $\sigma_{ij} e^{i\omega t}$, and $\varepsilon_{ij} e^{i\omega t}$, the elastic and damping properties of these materials may be described by the use of complex moduli [4]. The stress-strain relations for the material of the core are

$$
\begin{aligned}
\sigma_{11} &= \bar{E}_{11}\varepsilon_{11} + \bar{E}_{12}\varepsilon_{22} = E_{11}(1 + i\eta_{11})\varepsilon_{11} + E_{12}(1 + i\eta_{12})\varepsilon_{22} \\
\sigma_{22} &= \bar{E}_{22}\varepsilon_{22} + \bar{E}_{12}\varepsilon_{11} = E_{22}(1 + i\eta_{22})\varepsilon_{22} + E_{12}(1 + i\eta_{12})\varepsilon_{11} \\
\sigma_{12} &= 2\bar{G}_3\varepsilon_{12} = 2G_3(1 + i\mu_3)\varepsilon_{12} \qquad (2) \\
\sigma_{13} &= 2\bar{G}_2\varepsilon_{13} = 2G_2(1 + i\mu_2)\varepsilon_{13} \\
\sigma_{23} &= 2\bar{G}_1\varepsilon_{23} = 2G_1(1 + i\mu_1)\varepsilon_{23}
\end{aligned}
$$

Similarly, the stress-strain relations for the material of the facings are

$$
\begin{aligned}
\sigma_{11} &= \bar{E}'_{11}\,\varepsilon_{11} + \bar{E}'_{12}\,\varepsilon_{22} = E'_{11}(1+i\eta'_{11})\varepsilon_{11} + E'_{12}(1+i\eta'_{12})\varepsilon_{22} \\
\sigma_{22} &= \bar{E}'_{22}\,\varepsilon_{22} + \bar{E}'_{12}\,\varepsilon_{11} = E'_{22}(1+i\eta'_{22})\varepsilon_{22} + E'_{12}(1+i\eta'_{12})\varepsilon_{11} \qquad (3) \\
\sigma_{12} &= 2\bar{G}_3\,\varepsilon_{12} = 2G'_3(1+i\mu'_3)\varepsilon_{12}
\end{aligned}
$$

In the relations (2) and (3), the quantities $E_{11}, \eta_{11}, \ldots G_1, \mu_1$, and $E'_{11}, \eta'_{11}, \ldots G'_3, \mu'_3$ are material constants of the core and of the facings, respectively.

In some considerations, the resultant forces and moments are used rather than the components of stress themselves. The positive directions of these resultants are shown in Fig. 6.

In the derivation of the equations of the problem, the quantities containing h/R in the power higher than one will be neglected. This leads, therefore, to a similar degree of accuracy as accepted by Donnell [1] and Vlasov [2] (in an alternate form of his theory), but here the transverse shear deformations are taken into account. The equations used in this paper can also be obtained from the equations derived by Grigoliuk [3] after the latter are extended for the materials with damping.

Equations of the Problem

Using the relations between the components of strain and the components of displacement in cylindrical coordinates, and taking into account the relations (1), the components of strain may be expressed in terms of u, v, w, φ, and ψ

$$
\begin{aligned}
\varepsilon_{11} &= \frac{1}{R}\left(\frac{\partial u}{\partial \alpha} + \frac{\partial \varphi}{\partial \alpha} z\right) \\
\varepsilon_{22} &= \frac{1}{R}\left(\frac{\partial v}{\partial \beta} + \frac{\partial \psi}{\partial \beta} z + w\right) \\
2\varepsilon_{12} &= \frac{1}{R}\left(\frac{\partial u}{\partial \beta} + \frac{\partial w}{\partial \alpha} + \frac{\partial \varphi}{\partial \beta} z + \frac{\partial \psi}{\partial \alpha} z\right) \\
2\varepsilon_{13} &= \varphi + \frac{1}{R}\,\frac{\partial w}{\partial \alpha} \\
2\varepsilon_{23} &= \psi + \frac{1}{R}\,\frac{\partial w}{\partial \alpha}
\end{aligned}
\tag{4}
$$

where $\alpha = x/R$.

Combining the relations (4) and Hooke's law (Eqs. (2) and (3)), the following expressions for the components of stress are obtained:

For the core:

$$
\begin{aligned}
\sigma_{11} &= \bar{E}_{11}\frac{1}{R}\left(\frac{\partial u}{\partial \alpha} + \frac{\partial \varphi}{\partial \alpha} z\right) + \bar{E}_{12}\frac{1}{R}\left(\frac{\partial v}{\partial \beta} + \frac{\partial \psi}{\partial \alpha} z\right) \\
\sigma_{22} &= \bar{E}_{22}\frac{1}{R}\left(\frac{\partial v}{\partial \beta} + \frac{\partial \psi}{\partial \beta} + w\right) + \bar{E}_{12}\frac{1}{R}\left(\frac{\partial v}{\partial \alpha} + \frac{\partial \varphi}{\partial \alpha} z\right) \\
\sigma_{12} &= \bar{G}_{3}\frac{1}{R}\left(\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + \frac{\partial \varphi}{\partial \beta} z + \frac{\partial \psi}{\partial \alpha} z\right) \\
\sigma_{13} &= \bar{G}_{2}\left(\varphi + \frac{1}{R}\,\frac{\partial w}{\partial \alpha}\right) \\
\sigma_{23} &= \bar{G}_{1}\left(\psi + \frac{1}{R}\,\frac{\partial w}{\partial \beta}\right)
\end{aligned}
\tag{5}
$$

For the facings:

$$
\begin{aligned}
\sigma_{11} &= \frac{1}{R}\left(\frac{\partial u}{\partial \alpha} \pm \frac{\partial \varphi}{\partial \alpha}\,\frac{h}{2}\right) + \bar{E}'_{12}\frac{1}{R}\left(\frac{\partial v}{\partial \beta} \pm \frac{\partial \psi}{\partial \beta}\cdot\frac{h}{2} + w\right) \\
\sigma_{22} &= \bar{E}'_{22}\frac{1}{R}\left(\frac{\partial v}{\partial \beta} \pm \frac{\partial \psi}{\partial \beta}\,\frac{h}{2} + w\right) + \bar{E}'_{12}\frac{1}{R}\left(\frac{\partial u}{\partial \alpha} \pm \frac{\partial \varphi}{\partial \alpha}\cdot\frac{h}{2}\right) \\
\sigma_{12} &= \bar{G}'_{3}\frac{1}{R}\left(\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} \pm \frac{\partial \varphi}{\partial \beta}\,\frac{h}{2} \pm \frac{\partial \psi}{\partial \alpha}\cdot\frac{h}{2}\right)
\end{aligned}
\quad ..(6)
$$

where the plus sign applies to the upper facing, and the minus sign to the lower facing.

Finally, the relations between the resultant forces and moments and the displacements u, v, w, φ, and ψ are

$$N_1 = (\bar{E}_{11} h + 2\bar{E}'_{11} f)\frac{1}{R}\cdot\frac{\partial u}{\partial \alpha} + (\bar{E}_{12} h + 2\bar{E}'_{12} f)\frac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right) =$$

$$= \bar{B}_{11}\frac{1}{R}\frac{\partial u}{\partial \alpha} + \bar{B}_{12}\frac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right).$$

$$N_2 = (\bar{E}_{22} h + 2\bar{E}'_{22} f)\frac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right) + (\bar{E}_{12} h + 2\bar{E}'_{12} f)\frac{1}{R}\cdot\frac{\partial u}{\partial \alpha} =$$

$$= \bar{B}_{22}\frac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right) + \bar{B}_{12}\frac{1}{R}\cdot\frac{\partial u}{\partial \alpha}$$

$$N_{12} = (\bar{G}_3 h + 2\bar{G}'_3 f)\frac{1}{R}\left(\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha}\right) = \bar{C}\frac{1}{R}\left(\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha}\right) \qquad (7)$$

$$M_1 = (\bar{E}_{11} h^3/12 + \bar{E}'_{11} f h^2/2)\frac{1}{R}\cdot\frac{\partial \varphi}{\partial \alpha} + \bar{E}_{12} h^3/12 + \bar{E}'_{12} f h^2/2)\frac{1}{R}\cdot\frac{\partial \psi}{\partial \beta} =$$

$$= \bar{D}_{11}\frac{1}{R}\cdot\frac{\partial \varphi}{\partial \alpha} + \bar{D}_{12}\frac{1}{R}\cdot\frac{\partial \psi}{\partial \beta}$$

$$M_2 = (\bar{E}_{22} h^3/12 + \bar{E}'_{22} f h^2/2)\frac{1}{R}\cdot\frac{\partial \psi}{\partial \beta} + (\bar{E}_{12} h^3/12 + \bar{E}'_{12} f h^2/2)\frac{1}{R}\cdot\frac{\partial \varphi}{\partial \alpha} =$$

$$= \bar{D}_{22}\frac{1}{R}\cdot\frac{\partial \psi}{\partial \beta} + \bar{D}_{12}\frac{1}{R}\cdot\frac{\partial \varphi}{\partial \alpha}$$

$$M_{12} = (\bar{G}_3 h^3/12 + \bar{G}'_3 f h^2/2)\frac{1}{R}\left(\frac{\partial \varphi}{\partial \beta} + \frac{\partial \psi}{\partial \alpha}\right) = H\frac{1}{R}\left(\frac{\partial \varphi}{\partial \beta} + \frac{\partial \psi}{\partial \alpha}\right)$$

$$Q_1 = \bar{G}_2 h\left(\varphi + \frac{1}{R}\cdot\frac{\partial w}{\partial \alpha}\right) = \bar{K}_1\left(\varphi + \frac{1}{R}\frac{\partial w}{\partial \alpha}\right)$$

$$Q_2 = \bar{G}_1 h\left(\psi + \frac{1}{R}\cdot\frac{\partial w}{\partial \beta}\right) = \bar{K}_2\left(\psi + \frac{1}{R}\cdot\frac{\partial w}{\partial \beta}\right)$$

The system of equations for the functions u, v, w, φ, ψ may be obtained either by substitution of the relations (7) into the equations of equilibrium, or by the use of the principle of virtual work. For the considered problem, these equations are given in Table 1. The quantities p_x, p_y, p_z are the components of external loading; M and I are the mass and the moment of inertia, respectively, per unit area of the middle surface.

The uniqueness of the solution requires that, at the boundaries $x = const.$ one quantity of each of the pairs

$$N_1 u \quad , \quad N_{12} v \quad , \quad M_1 \varphi \quad , \quad M_{12} \psi \quad , \quad Q_1 w$$

be prescribed, and at the boundaries $\beta = const.$ one quantity of each of the pairs

$$N_2 v \quad , \quad N_{12} u \quad , \quad M_2 \psi \quad M_{12} \varphi \, , \quad Q_2 w$$

be prescribed. For this problem, the following boundary conditions are assumed:

For $x = 0$ and $x = L$

$$N_1 = 0 \, , \quad v = 0 \, , \quad M_1 = 0 \, , \quad \psi = 0 \, , \quad w = 0 \tag{8}$$

For $\beta = 0$ and $\beta = b/R$

$$N_2 = 0 \, , \quad u = 0 \, , \quad M_2 = 0 \, , \quad \varphi = 0 \, , \quad w = 0 \tag{9}$$

Solution

It is assumed that only the normal pressure p_z acts on the shell, while $p_x \equiv 0$; $p_y \equiv 0$. The loading $p_z = p_z(\alpha, \beta ; t)$

TABLE I

u	v	w	ϕ	ψ	right-hand side
$\frac{B_{11}}{R}\frac{\partial^2}{\partial\alpha^2}+\frac{C}{R}\frac{\partial^2}{\partial\beta^2}-RM\frac{\partial^2}{\partial t^2}$	$\frac{B_{12}}{R}\frac{\partial^2}{\partial\alpha\partial\beta}+\frac{C}{R}\frac{\partial^2}{\partial\alpha\partial\beta}$	$\frac{B_{12}}{R}\frac{\partial}{\partial\alpha}$	0	0	$-Rp_x$
$\frac{B_{12}}{R}\frac{\partial^2}{\partial\alpha\partial\beta}+\frac{C}{R}\frac{\partial^2}{\partial\alpha\partial\beta}$	$\frac{B_{22}}{R}\frac{\partial^2}{\partial\beta^2}+\frac{C}{R}\frac{\partial^2}{\partial\alpha^2}-RM\frac{\partial^2}{\partial t^2}$	$\frac{B_{22}}{R}\frac{\partial}{\partial\beta}$	0	0	$-Rp_y$
$-\frac{B_{12}}{R}\frac{\partial}{\partial\alpha}$	$-\frac{B_{22}}{R}\frac{\partial}{\partial\beta}$	$\frac{K_1}{R}\frac{\partial^2}{\partial\alpha^2}+\frac{K_2}{R}\frac{\partial^2}{\partial\beta^2}-\frac{B_{22}}{R}-RM\frac{\partial^2}{\partial t^2}$	$K_1\frac{\partial}{\partial\alpha}$	$K_2\frac{\partial}{\partial\beta}$	$-Rp_z$
0	0	$-K_1\frac{\partial}{\partial\alpha}$	$\frac{D_{11}}{R}\frac{\partial^2}{\partial\alpha^2}+\frac{H}{R}\frac{\partial^2}{\partial\beta^2}-K_1R-RI\frac{\partial^2}{\partial t^2}$	$\frac{H}{R}\frac{\partial^2}{\partial\alpha\partial\beta}+\frac{D_{12}}{R}\frac{\partial^2}{\partial\alpha\partial\beta}$	0
0	0	$-K_2\frac{\partial}{\partial\beta}$	$\frac{H}{R}\frac{\partial^2}{\partial\alpha\partial\beta}+\frac{D_{12}}{R}\frac{\partial^2}{\partial\alpha\partial\beta}$	$\frac{D_{22}}{R}\frac{\partial^2}{\partial\beta^2}+\frac{H}{R}\frac{\partial^2}{\partial\alpha^2}-K_2R-RI\frac{\partial^2}{\partial t^2}$	0

will be represented in the form

$$p_z = \bar{p}_z\, e^{i\omega t} = \sum_m \sum_n P_{zmn} \sin \lambda_m \alpha \sin \chi_n \beta\, e^{i\omega t} \tag{10}$$

where

$$\lambda_m = m\pi R/l \quad , \qquad \chi_n = n\pi R/b \tag{11}$$

If the pressure p_z is constant over the surface of the shell

$$P_{zmn} = p_z\, 16/\pi^2 mn \tag{12}$$

Solutions for u, v, w, φ, ψ will be sought in the forms

$$\begin{aligned}
u &= \bar{u}\, e^{i\omega t} = \sum_m \sum_n U_{mn} \cos\lambda_m \alpha . \sin \chi_n \beta . e^{i\omega t} \\
v &= \bar{v} . e^{i\omega t} = \sum_m \sum_n V_{mn} \sin \lambda_m \alpha . \cos \chi_n \beta . e^{i\omega t} \\
w &= \bar{w} . e^{i\omega t} = \sum_m \sum_n W_{mn} \sin\lambda_m \alpha . \sin \chi_n \beta . e^{i\omega t} \\
\varphi &= \bar{\varphi} . e^{i\omega t} = \sum_m \sum_n \phi_{mn} \cos\lambda_n \alpha . \sin \chi_n \beta . e^{i\omega t} \\
\psi &= \bar{\psi}\, e^{i\omega t} = \sum_m \sum_n \Psi_{mn} \sin\lambda_m \alpha . \cos \chi_n \beta . e^{i\omega t}
\end{aligned} \tag{13}$$

which satisfy the boundary conditions (8) and (9).

The substitution of the expressions (13) into the equations shown in Table 1, results in the system of algebraic linear equations for the coefficients $U_{mn}, V_{mn}, W_{mn}, \phi_{mn}$, and Ψ_{mn} as shown in Table 2.

The solution of this system is relatively simple. The quantities $U_{mn}, V_{mn}, \phi_{mn}, \Psi_{mn}$ may be expressed in terms of W_{mn} in the following way:

TABLE 2

U_{mn}	V_{mn}	W_{mn}	Φ_{mn}	Ψ_{mn}	right-hand side
$\frac{B_{11}}{R}\lambda_m^2 + \frac{C}{R}\chi_n^2 - RM\omega^2$	$\left(\frac{B_{12}}{R} + \frac{C}{R}\right)\lambda_m \chi_n$	$-\frac{B_{12}}{R}\lambda_m$	0	0	0
$\left(\frac{B_{12}}{R} + \frac{C}{R}\right)\lambda_m \chi_n$	$\frac{B_{22}}{R}\chi_n^2 + \frac{C}{R}\lambda_m^2 - RM\omega^2$	$-\frac{B_{22}}{R}\chi_n$	0	0	0
$-\frac{B_{12}}{R}\lambda_m$	$-\frac{B_{22}}{R}\chi_n$	$\frac{K_1}{R}\lambda_m^2 + \frac{K_2}{R}\chi_n^2 + \frac{B_{22}}{R} - RM\omega^2$	$K_1\lambda_m$	$K_2\chi_n$	RP_{zmn}
0	0	$K_1\lambda_m$	$\frac{D_{11}}{R}\lambda_m^2 + \frac{H}{R}\chi_n^2 + K_1R - RI\omega^2$	$\left(\frac{H}{R} + \frac{D_{12}}{R}\right)\lambda_m \chi_n$	0
0	0	$K_2\chi_n$	$\left(\frac{H}{R} + \frac{D_{12}}{R}\right)\lambda_m \chi_n$	$\frac{D_{22}}{R}\chi_n^2 + \frac{H}{R}\lambda_m^2 + K_2R - RI\omega^2$	0

$$
\begin{aligned}
U_{mn} &= \frac{\mathcal{K} RM\omega^2 + \mathcal{F}}{R^2M^2\omega^4 - RM\omega^2\mathcal{A} + \mathcal{B}} W_{mn} \\
V_{mn} &= \frac{\mathcal{L} RM\omega^2 + \mathcal{G}}{R^2M^2\omega^4 - RM\omega^2\mathcal{A} + \mathcal{B}} W_{mn} \\
\phi_{mn} &= \frac{\mathcal{M} RI\omega^2 + \mathcal{H}}{R^2I^2\omega^4 - RI\omega^2\mathcal{C} + \mathcal{D}} W_{mn} \\
\psi_{mn} &= \frac{\mathcal{N} RI\omega^2 + \mathcal{J}}{R^2I^2\omega^4 - RI\omega^2\mathcal{C} + \mathcal{D}} W_{mn}
\end{aligned} \tag{14}
$$

and the coefficients W_{mn} are determined by the expression

$$
W_{mn} = RP_{zmn} \Big/ \left\{ \frac{\mathcal{O} RM\omega^2 + \mathcal{P}}{R^2M^2\omega^4 - RM\omega^2\mathcal{A} + \mathcal{B}} + \mathcal{E} - RM\omega^2 + \frac{\mathcal{R} RI\omega^2 + \mathcal{S}}{R^2I^2\omega^4 - RI\omega^2\mathcal{C} + \mathcal{D}} \right\} \tag{15}
$$

The notations used in the expressions (14) and (15) are

$$
\begin{aligned}
\mathcal{A} &= (B_{11}\lambda_m^2 + C\chi_n^2 + B_{22}\chi_n^2 + C\lambda_m^2)/R \\
\mathcal{B} &= [(B_{11}\lambda_m^2 + C\chi_n^2)(B_{22}\chi_n^2 + C\lambda_m^2) - (B_{12} + C)^2\lambda_m^2\chi_n^2]/R^2 \\
\mathcal{C} &= (D_{11}\lambda_m^2 + H\chi_n^2 + K_1R^2 + D_{22}\chi_n^2 + H\lambda_m^2 + K_2R^2)/R \\
\mathcal{D} &= (D_{11}\lambda_m^2 + H\chi_n^2 + K_1R^2)(D_{22}\chi_n^2 + H\lambda_m^2 + K_2R^2)/R^2 - \\
&\quad - (H + D_{12})^2\lambda_m^2\chi_n^2/R^2 \\
\mathcal{E} &= (K_1\lambda_m^2 + K_2\chi_n^2 + B_{22})/R \\
\mathcal{F} &= [B_{12}\lambda_m(B_{22}\chi_n^2 + C\lambda_m^2) - B_{22}\chi_n(B_{12} + C)\lambda_m\chi_n]/R^2 \\
\mathcal{G} &= [B_{22}\chi_n(B_{11}\lambda_m^2 + C\chi^2) - B_{12}\lambda_m(B_{12} + C)\lambda_m\chi_n]/R^2
\end{aligned} \tag{16}
$$

$$\mathcal{H} = \left[-K_1 \lambda_m \left(D_{22} \chi_n^2 + H \lambda_m^2 + K_2 R^2 \right) + K_2 \lambda_n \left(H + D_{12} \right) \lambda_m \chi_n \right] / R$$

$$\mathcal{J} = \left[-K_2 \chi_n \left(D_{11} \lambda_m^2 + H \chi_n^2 + K_1 R^2 \right) + K_1 \lambda_m \left(H + D_{12} \right) \lambda_m \chi_n \right] / R$$

$$\mathcal{K} = -B_{12} \lambda_m / R \ ; \quad \mathcal{L} = -B_{22} \chi_n / R \ ; \quad \mathcal{M} = K_1 \lambda_m \ ; \quad \mathcal{N} = K_2 \chi_n$$

$$\mathcal{O} = \left[\left(B_{12} \lambda_m \right)^2 + \left(B_{22} \chi_n \right)^2 \right] / R^2 \quad ; \quad \mathcal{R} = \left(K_1 \lambda_m \right)^2 + \left(K_2 \chi_n \right)^2$$

$$\mathcal{P} = \left[-\left(B_{12} \lambda_m \right)^2 \left(B_{22} \chi_n^2 + C \lambda_m^2 \right) - \left(B_{22} \chi_n \right)^2 \left(B_{11} \lambda_m^2 + C \chi_n^2 \right) + \right.$$
$$\left. + 2 B_{12} B_{22} \left(B_{12} + C \right) \lambda_m^2 \chi_n^2 \right] / R^3$$

$$\mathcal{S} = \left[-\left(K_1 \lambda_m \right)^2 \left(D_{22} \chi_n^2 + H \lambda_m^2 + K_2 R^2 \right) - \left(K_2 \chi_n \right)^2 \left(D_{11} \lambda_m^2 + H \chi_n^2 + K_1 R^2 \right) + \right.$$
$$\left. + 2 K_1 K_2 \left(H + D_{12} \right) \lambda_m^2 \chi_n^2 \right] / R$$

The use of the above equations is quite simple if the external loading p_2 has only one harmonic component of frequency ω. It is necessary to calculate $U_{mn}, \ldots, \Psi_{mn}$ for this frequency, and then all the components of deformation and of stress.

The above equations may also be used if the external loading has a continuous spectrum over a range of frequencies, for instance from ω_1 to ω_2. In this case, it is necessary to determine $U_{mn}, \ldots, \Psi_{mn}$ as functions of ω in the interval (ω_1, ω_2) assuming that the load intensity over the surface of the shell is equal to 1. After this is accomplished, the dynamical response of the shell may be determined by the known methods of generalized harmonic analysis for practically arbitrary spectra of external loading.

It is advisable to transform the expression (15) into a dimensionless form for more convenient numerical calculations.

Example

Numerical calculations were carried on for a panel with the following dimensions:

$$l = b = 30h \quad , \quad R = 30h \quad , \quad f = 0.05h$$

The materials of the facings and of the core were assumed isotropic with the core being fifty times "softer" than the facings. The Poisson's ratios of the core and of the facings were assumed equal to 0.3. Only shear damping of the core was considered with the dissipation coefficient equal to 0.1. The above assumptions lead to the following relations between the elastic constants:

$$E'_{11} = E'_{22} = E'$$

$$E_{11} = E_{22} = E = 0.02E'$$

$$E'_{12} = 0.3E'$$

$$E_{12} = 0.3E$$

$$G_1 = G_2 = G_3 = E/2(1+\nu) = 0.3846E$$

$$G'_3 = E'/2(1+\nu) = 0.3846E'$$

$$\mu_1 = \mu_2 = \mu_3 = 0.1$$

The ratio of densities of the core and of the facings was $\rho'/\rho = 10$

The external loading p_z was assumed uniform over the surface of the shell and harmonically varying in time $p_z = \bar{p}_z \exp(i\omega t)$.

Fig. 7 shows the dimensionless deflection, $\bar{w}E'/\bar{p}_z R$, as a function of the dimensionless frequency ω/ω_0 (where $\omega_0^2 = B_{22}/R^2M$) in the interval $0.5\omega_0 < \omega < 10\omega_0$. Fig. 8 shows a similar diagram for the same shell without damping. The comparison of Figs. 7 and 8 indicates that the effect of damping reduces considerably the amplitudes of shear modes, while it is less significant in the case of extensional modes.

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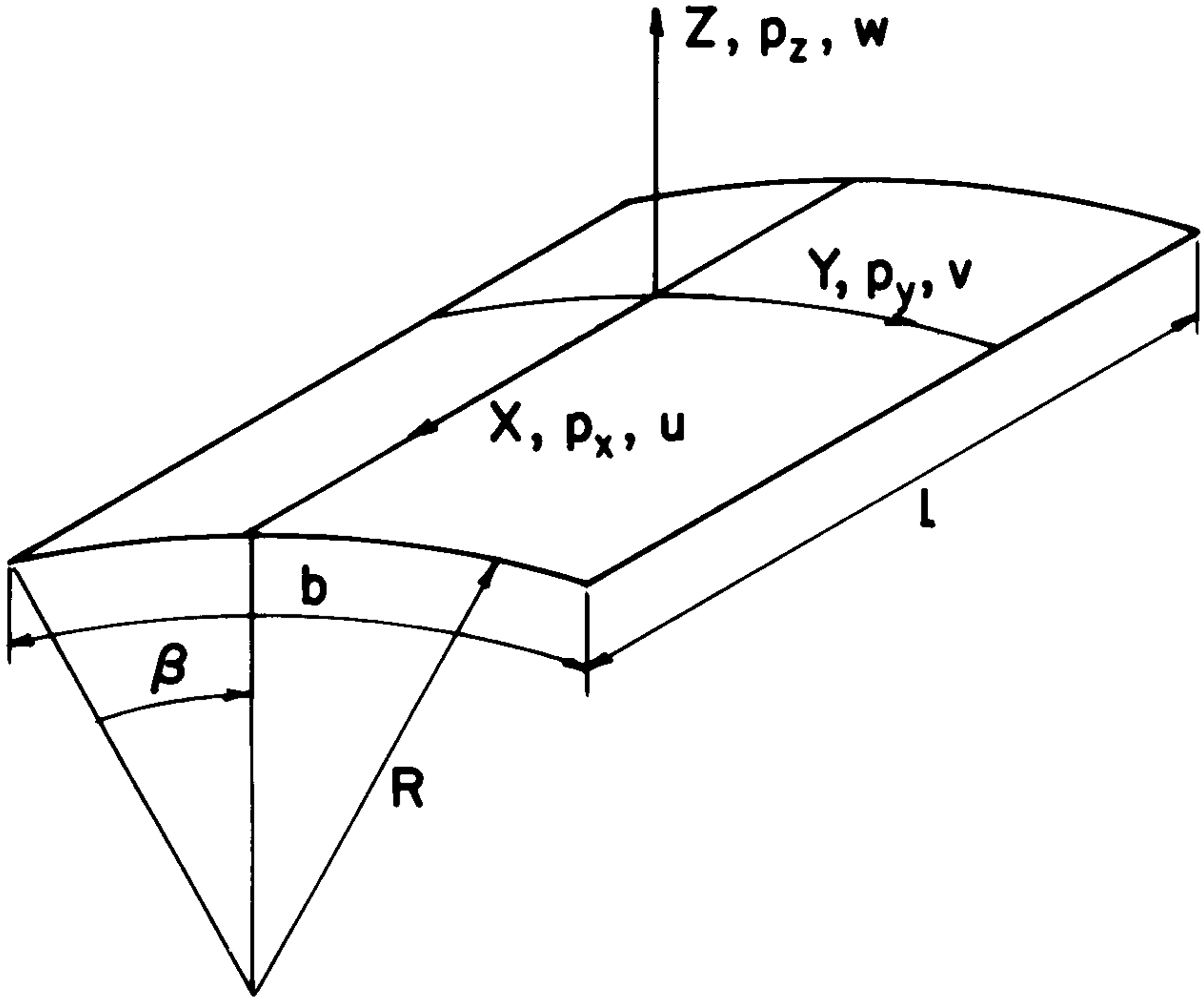

Fig. 5 Coordinate system, components of external loading, and components of displacement.

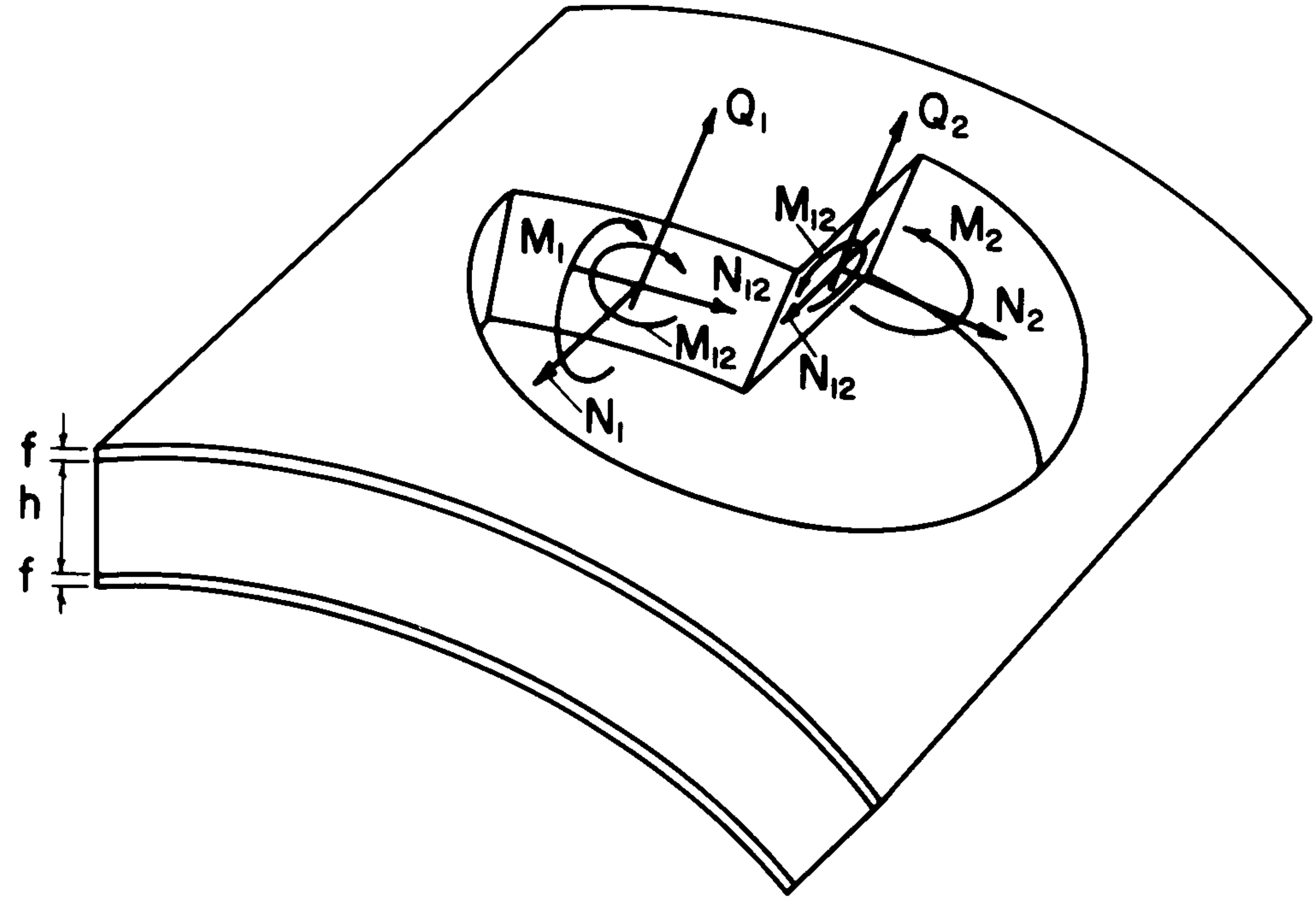

Fig. 6. Internal forces in the shell.

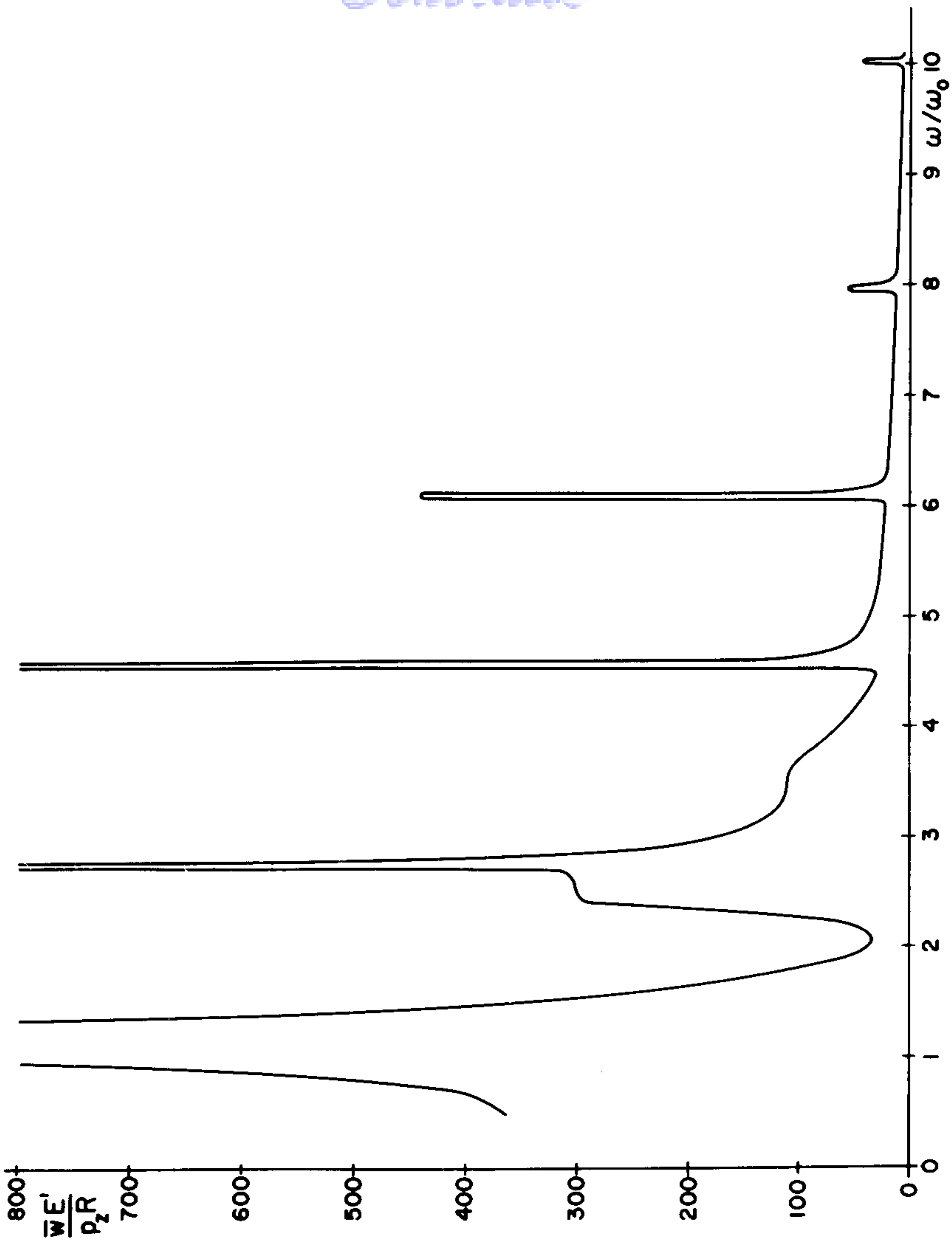

Fig. 7. Displacement $\overline{w}$ as a function of frequency ω.

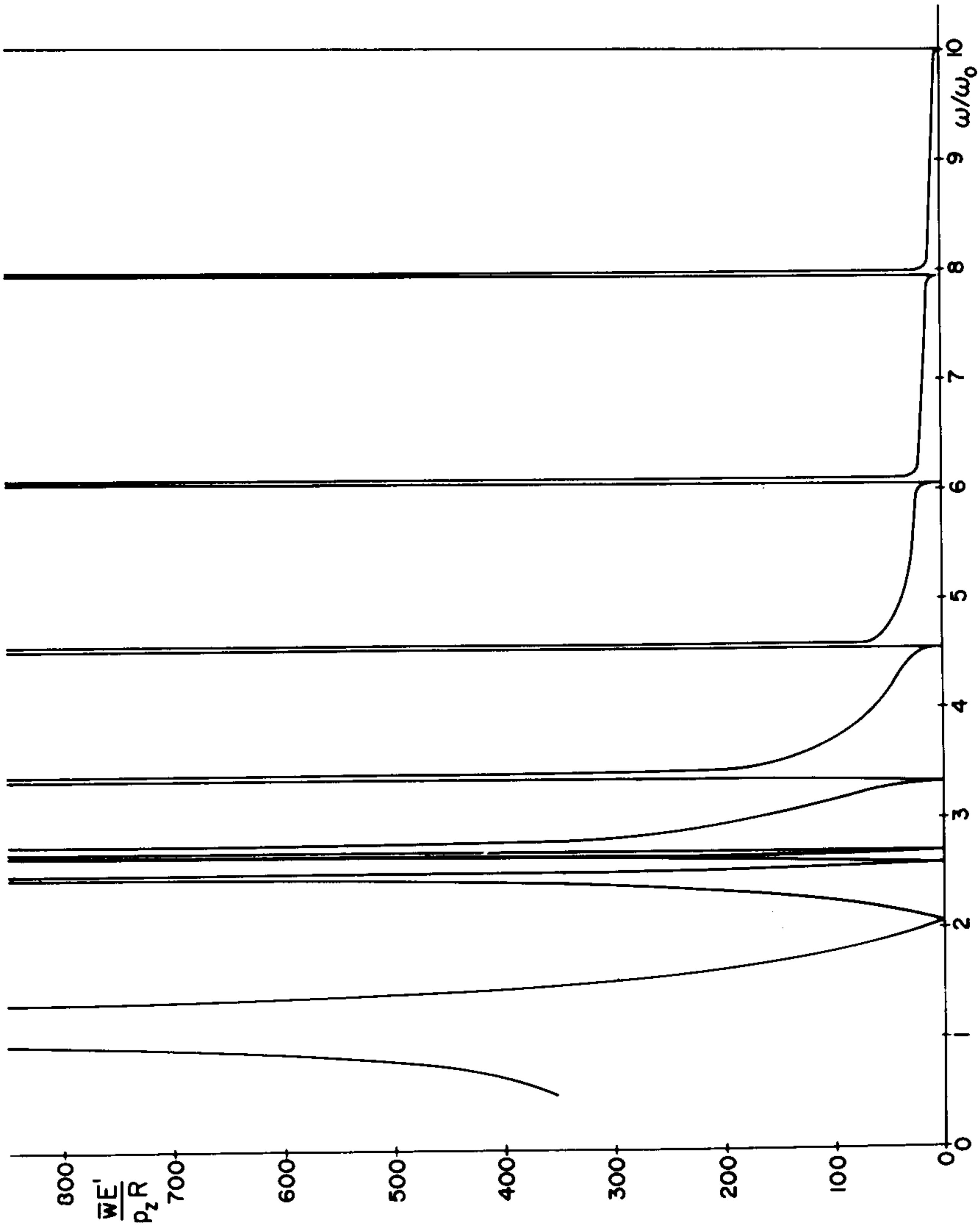

Fig. 8. Displacement $\overline{w}$ as a function of frequency ω (shell without damping).