AN INTRODUCTION TO ACOUSTIC FATIGUE

by

E. J. Richards
University of Southampton
England

I. INTRODUCTION

The science of aircraft damage due to random pressure fluctuations of fairly high frequency necessarily involves so many types of expert - aerodynamicist, acoustician, instrument man, vibration engineer, mathematician, and metallurgist - that I have been given the task of introducing as a whole, the matters to be discussed more learnedly later by the experts in these various fields. That it is not an academic field is shown quite clearly in Figure 1, which illustrates the early history of one of our British jet bombers in the early 1950's. Structural failures occurred regularly on the rear fuselage, mostly confined to rivet popping and to failures of cleats and stringers and generally in regions where stress concentrations occurred. It was noticed that the rate of failure increased when the engine thrust was increased and was noticeably decreased when a convergent divergent nozzle was fitted; thus relating the phenomenon to jet noise quite firmly. Strain gauge checks indicated clearly the reason for this improvement, Figure 2, showing the reduction in pressure fluctuations that occurred when the convergent divergent nozzle was fitted, not only throughout the frequency range, but at most places on the fuselage (Figure 3), in which the stress was such as to involve fatigue failure.

As time went on and as pressure ratios of engines increased, reports flowed in of more serious failures of skins themselves (Figure 4), even after everything had been done at the design stage to reduce local stress raising. On rocket engined aircraft and on missiles, experiences occurred of the disintegration of whole tail assemblies and missile structures, and it became the practice, indeed the expensive practice, of carrying out environmental tests on practical structures at as early a stage of development as possible.

Since it was found that the pressure fluctuations in flight were in general much smaller than those during static conditions, all these tests were on stationary aircraft. Since jet engines are so expensive to run, attempts to simulate the pressure field of the jet were made using loud-speakers, sirens, and choppers, and since aircraft are expensive to make, assemblies of panels were erected to simulate easily those parts of the aircraft subject to the greatest fluctuating air pressure.

Two queries immediately arose. First, did a siren give even an approximation to the loads actually occurring on a panel, bearing in mind that this load depends on the area over which the pressure is correlated as well as on the magnitude of the pressure itself? Secondly, did the panel array being tested respond as would an actual aircraft structure consisting of frames, stringers, and skin or would the modes be quite different and the stresses completely unrepresentative? A corollary to this last question arose from the fact that in general the excitation was sinusoidal and tuned as well as possible to the natural frequency of the panel: for a given excitation level, how did the fatigue life agree with that obtained from random pressure loading? Could Miner's Law (1) for cumulative damage be used or if not, what else?

At this stage, a further source of possible fatigue was observed, that due to pressure fluctuations in the turbulent boundary layer on a fuselage. Since, in a way, the air outside the boundary layer is a form of jet, placed very close to the surface with a strong mixing region in between, many fears were expressed that supersonic aircraft flying throughout their lives in this condition (and not just before take-off) might well give rise to really serious fatigue conditions. This last possibility has not materialized, as far as the author is aware, at least not yet. Even so, all the above question marks have led firmly to the need to establish a good theoretical background to explain the phenomenon of acoustic fatigue and has led to individual researches into the whole chain of investigations involved in the general problem.

II. THE STEPS IN THE PROBLEM

These links are indicated vividly in Figure 5, each one indicating in fact a different type of scientific investigation. Generalized methods of working out the fatigue life once the appropriate information is forthcoming have been presented in the United Kingdom by Powell (2) when at

Southampton, and by various investigators in the United States, notably Liepmann (3) of Cal. Tech., John Miles (4) of U.C.L.A., and others.

What are the basic studies in detail? They include studies of:

1. The fluctuating pressure or noise field from a jet, rocket, or along a boundary layer.

2. The exciting forces or regions of correlated pressure on structures in various parts of the noise field and with various orientations relative to the jet, etc.

3. An assessment of the stresses so caused. This involves a study of the likely modes of response of skin-stringer-frame structures.

4. The amount of energy dissipation available from both structural and acoustic damping.

5. The interpretation of these stresses in terms of the fatigue life of the structure.

III. PRESSURE FLUCTUATIONS NEAR A JET, ROCKET, OR ALONG A BOUNDARY LAYER

In order to understand the nature of the pressure excitations arising from moving streams generally, it is necessary to introduce two very elementary features of basic aerodynamics. First, longitudinal momentum is transferred laterally in a moving stream either by the interchange of the momentum of molecules colliding during their lateral journeys (laminar flow) or by the interchange arising from quite sizable masses of fluid eddying laterally and randomly (turbulent flow). The former type of flow occurs only near the leading edge of aerofoils or fuselages and for a very small length indeed of the mixing region of a jet. Apart from this, the flow is turbulent. These eddies are coherent for a small period of time, breaking up and coalescing with others to such a degree that after a short period of time they cannot be related to others in any way. If there is a general mean flow in any direction, they are convected downstream with the local velocity of the stream, so that in the mixing region of a jet, for example, there can be a whole series of convection velocities. They can also be large or small, large for example in atmospheric gusts, small for example in the very thin boundary layer of air along the nose of an aircraft fuselage. In fact, the turbulence is characterized by its magnitude (r.m.s. of velocity) and its scale.

The second idea is that no velocity change can occur without a pressure change. If the rates of change are slow, then Bernouilli's equation $p + 1/2\ U^2 = \text{constant}$ can be considered to hold, any change in velocity U being accompanied by a 180° out of phase pressure change, the density being constant. As the rate of change of velocity is increased, pressure and velocity changes cannot exactly balance, and density changes occur, the pressure and velocity not staying in exact antiphase. In the first case above, there is no energy interchange from point to point and the effect of the velocity change is very local, the induced pressure field near the fluctuating region of turbulence rapidly dying out with distance. On the other hand, the quick pressure changes give rise to non-balanced conditions with some part of the velocity in phase with the pressure, thereby radiating energy away in the form of sound. Thus, in Figure 6, representing the region around a jet engine, there are three regions of pressure fluctuation: first, the hydrodynamic field in which the pressure fluctuations obey Bernouilli's equation, the intensity $\overline{p^2}$ thus varying as the mean speed U raised to the fourth power (U^4) and with a close frequency relationship with the internal turbulence of the jet and its convection speed. Secondly, we have the distant or far field where the only pressure fluctuations are acoustically radiated and depend essentially on the rates of change of velocity, i.e., p varies as U^2 (strength) x frequency ^{2} (which again varies as U^2) or in terms of sound intensity $\overline{p^2}$ U^8. The greater index does not imply a higher intensity than in the near field since the radiated energy is only the marginal portion of the kinetic energy of the turbulence. For example, on a sonic jet, the turbulence energy in the noise producing region is 1% of the directed kinetic energy of the jet, the total radiated acoustic energy being in turn only 1% of this turbulent energy. On a rocket engine, it is thought that even with such high velocities and change rates, the radiated acoustic power cannot exceed that in the turbulence (e.g. 1 or 2%).

In between these two regions, the pressure fluctuations arise partly from (1) and partly from (2) and consist of a diffuse pattern of pressure fronts, capable of study only on an experimental basis. Thus, if a surface is very close to a jet or if we are concerned with the boundary layer, we can neglect the true radiated noise and can obtain a model of the pressure excitation field closely related to that of the flowing turbulence. If the structure is a long way away from the field of turbulence, such as upstream on a missile, we can get the pressure field only by using the radiated noise theory of Lighthill (5) while in the really important region in between, we must reconcile ourselves to empirical formulae based on the many measurements made in the near field of a jet.

A. Models of the Excitation Field

Since the loading on a structure depends on the correlated area of pressure fluctuation as much as on the pressure itself, it is necessary to study the excitation field far more deeply for structural response than for estimations of public reaction. For example, in Figure 7, which gives a very thorough analysis of the space correlation coefficients around a typical point in the fairly near field of a jet (6), a structure placed facing the obvious wave fronts would be subject to ten times the load on a structure placed at right angles to them.

In order to develop a model of this excitation, it is convenient to start with the boundary layer case, since it is easier to study physically. In Figure 8, the characteristics of the turbulent boundary layer are indicated for the purpose of discussion, in two dimensions, the mean velocity u at any point in the boundary has superimposed upon it fluctuating components u' and v', which are the instantaneous contributions at the point to the eddies being convected downstream. These eddies give rise to a fluctuating normal pressure p' at the surface, and, since the velocity profile is changing, a fluctuating shear stress τ_0' along the surface occurs also. Estimations and experiments regarding the magnitude of p' ($\overline{p^2}$) vary and we shall hear more about this during the conference, Willmarth (7) finding $\overline{p^2}$ equal to 3.5×10^{-3} times the free stream dynamic head q, Harrison (8) obtaining a larger value 9.5×10^{-3} approximately. The r.m.s. shear stress fluctuations $\sqrt{\overline{\tau_0'^2}}$ were found by Laufer (9) from experiments in pipes to be 0.3 times the mean skin friction, say about 0.8×10^{-4} times q for typical values of the mean skin friction τ_0. Kraichnan (10) has estimated from theoretical conditions that $\overline{p^2}$ equals 6 times the mean skin friction. Thus, the shear stress fluctuations can probably be neglected compared with the normal pressure fluctuations.

This gives us some idea of the pressures; now the question arises: over what area do they act? There is of course no single answer to this question, since the pressures arise from a near random field of turbulence and is generally described as a spectrum of the type shown in Figure 9 (11). H is the pressure component of frequency f, δ^* is the displacement thickness of the boundary layer, U is the free stream velocity, and $\overline{p^2}$ is the mean square fluctuating pressure as previously commented upon. The high frequencies relate to small scale turbulence with small areas of correlation, the low frequencies to larger scale eddies acting over larger areas.

Willmarth (12) and others found that the pressure fluctuations were best studied in a frame of reference travelling downstream with the main velocity of the eddies. Thus, by choosing the time delay to obtain the best correlation between points separated in a downstream direction, a much better idea of the physical nature of the excitation was found. It was found by both Willmarth and Harrison that the most important components of the turbulence were convected with a speed of about 0.8 times the free stream velocity and remained undissipated for some 10 boundary layer thicknesses in distance (correlation coefficient greater than 0.1). They remained really well correlated (coefficient greater than 0.5) for some twice the boundary layer thickness. Using this information and the power spectrum of Figure 9, we can think of the excitation as a random flow of pressure patches oscillating at various frequencies and being convected downstream at 0.8 U and acting over diameters, d (both along and transverse to the flow line) of an amount given by $f = \frac{0.8U}{d}$.

Thus, on the rear of a bomber, say, with a boundary layer thickness of six inches, and a speed of 600 knots, we can think of the maximum normal pressure excitation ($\frac{fd}{U} = 0.2$ in Figure 9) acting over patches of three inches diameter and convected as though "frozen" at a speed of 480 knots over lengths of some one or two feet, (e.g. over the length of typical panels, but not over the length of major assemblies). This, of course, is an oversimplification, but is very useful in tackling panel behavior due to boundary layer "noise".

B. Model of the Excitation in the Hydrodynamic or Very Near Field of a Jet

The same type of model may be formed of the pressure fluctuations on a structure very close to a jet. Here again we are not concerned with radiated noise but with the pattern of pressure fluctuations associated with the turbulence in the jet. At Southampton, Barrett(13) has studied the convection velocity of turbulence, in a jet, while Williams (14,15) has studied the turbulence spectrum both in respect to fixed and moving frames of reference. Figure 10 shows a typical picture obtained by space time correlation techniques of the variation across the jet of the convection speed of turbulence and a comparison with the mean speed variation. It may be seen that there is, in fact, a region (about the width of the transverse correlation length) in which the convection velocity is effectively constant and equal to 0.7 times the maximum jet velocity. As shown in Figure 11, the spectra relative to a fixed and moving frame are very different, the spectrum as observed effectively by a microphone moving with the speed of convection showing a much lower frequency characteristic and incidentally staying much more constant with distance downstream. These results again give rise to the concept of a flowing pattern of frozen turbulence, travelling downstream with a velocity of about 0.7 times the maximum speed and not altering its characteristics, in the length of two or three jet diameters.

Similar measurements of the pressure immediately adjacent to the jet (15,14) confirm this. Space-time correlations (Figure 12) with varying time delays give us an autocorrelation curve as shown dotted if the microphone were to move with the speed to give best correlation (the convection speed), this best speed being shown in Figure 12 to be about 0.72 times the jet nozzle speed. Thus, the spectrum of pressure fluctuations is very similar to that of the turbulence and that of the boundary layer and is convected in a similar manner (Figure 13). It must be pointed out that the above results all apply to subsonic flow and will be modified appreciably once the jet flow becomes supersonic.

C. Changes Due to High Mach Number

Once the pressure ratio of the jet has exceeded the critical value which gives M = 1 at the throat of the nozzle, a further source of sound or pressure fluctuation can occur. If the nozzle is not expanded accurately to give atmospheric pressure at the outlet, shocks (17) will occur and these will emit sound, if turbulence is convected through them (18). Indeed, under certain conditions a resonance or back reaction (19) can occur which can give rise to very strong pressure fluctuations (20) (Figure 14) in the vicinity of the jet. Since this is, in fact, the most realistic condition on practical jet engines at take-off power, moving frame correlation studies are currently in hand at Southampton and will be reported later by Dr. Clarkson.

D. Radiated Pressure Field From a Jet

If the structure is well away from the jet, the hydrodynamic field will have disappeared and the only pressure excitation will be that radiated acoustically. That this is a much more complicated item to analyze may be seen from the following simple argument. If the previously mentioned pattern of a convected frozen eddy is envisaged as put forward by G. I. Taylor many years ago, the pressure as observed by a person moving with the turbulence would be constant in the near field and atmospheric still further away. Thus, the pressure fluctuations observed by a stationary person far from the jet would only be that (Figure 15) which would arise if he were to run through the pressure field of a stationary vortex in the opposite direction to that of the jet and at the speed of convection. Since away from the immediate vicinity of the vortex, the pressure rapidly tends to fall to that of the atmosphere, it follows that no radiated sound field can occur from any field of turbulence which can be made to appear steady by suitable choice of moving axes. As Lighthill explains, far field sound arises from the rapid changes with time of the turbulence and is indeed a very strongly varying function of these quick temporal fluctuations. Thus, our previous simple models of frozen turbulence do not help us in predicting radiated sound, though they can be helpful if we move one step further and relate their rates of change to their frequencies.

Lighthill (5) formalized the theory of acoustic radiation in terms of boundary layer turbulence by first showing that the Navier-Stokes equations of motion of a gas, and the continuity equation could be combined to give a very familiar equation:

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \Delta^2 = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

with the usual double suffix notation. T_{ij} is the internal stress tensor and can be reduced in certain circumstances to $\rho_o v_i v_j$, the i momentum flux in the j direction due to the turbulence of the stream. Outside the jet the turbulence is zero and so is the right hand side of the equation. Thus, we obtain the concept of the equation representing the acoustic field in the air at rest around the jet, this acoustic field being forced by the stress tensor T_{ij} and effectively related to its second space derivative. The turbulence in the high velocity shear layer of the jet is strongly amplified by the inclusion in the term $\frac{d^2 T_{ij}}{dx_i . dx_j}$ of such terms as $\frac{dU}{dy} \cdot \frac{dv'}{dx}$ for example, where for convenience, I have changed to the more elementary notation in which x is along the jet, y at right angles, U is the mean velocity at a point in the x direction, and v' is the turbulence component in the y direction. Thus, the contribution of the region of high shear towards the sound intensity has been integrated by Lilley (21) into the form

$$I \propto \frac{\overline{e_{ij}\ (0)}\ \ \overline{e_{kl}\ (0)}}{(1 - M_o \cos\ \theta)^6} \int \overline{\frac{\partial p}{\partial t}(o,t)}\ \ \overline{\frac{\partial p}{\partial t}(\bar{z},t)}\ d\bar{z}$$

where the integral covers the region of correlation, etc., and the axes are frame axes moving at the mean speed of the jet. Here the mean shear strains are represented by $\bar{e}$ with suitable suffices. Williams (14) has obtained similar relationships but in terms of the velocity fluctuations, not of the pressure fluctuations.

Since the hydrodynamic pressures can be shown (14) to take a form not dissimilar to the above, but without the time differentials, Williams has also put forward the argument that the radiated sound pattern can be obtained from the convected hydrodynamic pressure spectrum obtained as in Figure 11, by scaling up the amplitudes in proportional to the frequency squared (e.g. by 6 db. per octave).

E. Near Field Pressure Levels

If this is proved to be acceptable, no doubt a very much better semi-empirical method can be evolved of relating the near field noise levels to the jet hydrodynamic pressures. So far we can only go on measured pressure fields of the type given by Hubbard (22), Wolfe (23), and Franklin (6) (Figure 16) and using actual measured correlation patterns of the type shown in Figure 7.

F. Propeller Noise Pressure Fields

Although structures have not as yet fatigued from the fluctuating pressure fields of propellers, the gradual increase of forward speeds and the subsequent need for transonic and supersonic tip speeds to absorb the power, has raised serious problems of fuselage vibration and internal noise. Since internal noise is basically related to fuselage vibrations, a few words on this subject are not inappropriate. Ample data are available on the sound pressure levels arising from propeller thrust and torque, but it is impossible to apply these to structural response calculations without some knowledge of the phase relationships around the fuselage. These phase relations have been programmed on the Southampton Pegasus by Swift (24) and are given in Figure 17 for a typical high speed propellered airliner. While being complicated, even in the fundamental mode shown here, nevertheless, an approximation is feasible in the region of highest intensity which should allow the major structural response to be calculated. The overall significance of propeller phasing to reduce the peak excitations can also be calculated for any special cases, the vector addition of the pressures for two propellers on one side being shown, for example, in Figure 18. It is clear that specific work of this kind can be most valuable in reducing the major fuselage excitations though it is difficult to generalize usefully.

IV. STRUCTURAL RESPONSE

When a structure is subject to random pressure loads such as in the cases described above, the resultant stresses are also random in nature. In this case, instantaneous values of stress are meaningless and the problem must be treated statistically. The pressure loading has a wide frequency spectrum causing excitation of many natural modes of vibration of the structure. In these circumstances, the most convenient or apparently convenient method of analysis is to use Lagrange's equations to consider the excitation of each normal mode of the structure independently. This assumes that the coupling between the modes is negligible and is justified by the fact that the damping of the normal modes is small. The total response of the structure is then obtained by combining the effects in each mode. This method of generalized harmonic analysis was developed by Leipmann (3), etc., and applied by Powell (4) to this particular problem.

The equation in the Sth mode is examplified by the Lagrange equation for the mode

$$m_s \ddot{\xi}_s + c_s \dot{\xi}_s + k_s \xi_s = L_s(t) \qquad (s = 1 \text{ to } n)$$

where m_s, c_s, k_s, and $L_s(t)$ are the generalized mass, the generalized viscous damping, the generalized stiffness, and the generalized force, respectively, in the Sth mode, the displacement being ξ_s, it can be shown (2) that before any calculation can be made the following information is necessary:

1. The mode shapes.

2. The power spectral density of the fluctuating pressure and the narrow band spatial correlation of the noise pressures over the whole structure.

3. The mechanical impendence of the structure which in turn is dependent upon (a) the normal structural modes and natural frequencies of vibration and (b) the damping of each normal mode. This latter item includes both structural and acoustic damping.

Clearly, such a calculation is dogged with lack of information on a whole variety of items including the mode shapes, excitation fields, spatial correlations, and damping, not to mention the significance of the stress figures when finally they are obtained. Taking for instance the calculation of normal modes, Miller (26) has shown that on a cylindrical structure with closely spaced stiffeners in the form of frame rings and longerons, the number of modes is very large, even in limited frequency ranges. Figure 19 indicates that on one such structure of radius 60 in. skin thickness 0.048 in. and 15 feet long with a stringer pitch 8 in. and a frame pitch 20 in., the number of possible modes within quite a small natural frequency range (of say between 200 and 400 cycles per second) is about twenty, with some practically coinciding in frequency. Thus, even if the higher frequencies are neglected, there is no great likelihood of a general solution being obtained.

On the other hand, tests on large specimens consisting of a multi-panel assembly on the Comet rig at R.A.E., Farnborough, and analyzed by autocorrelation methods to give their narrow band response characteristics suggest (27), as in Figure 20, that the number of modes actually excited is strictly limited. Further work in this field is proceeding to relate more closely these panel response modes to the peak excitation. A very extensive and well planned program of a similar kind has been initiated on the Caravelle by the

Sud-Aviation Company. This program includes simultaneous recordings of strain over many panels of the rear fuselage, tail, and fin, in order to establish modal response and non-linear effects very clearly.

A. Special Cases

In view of the above difficulties, it is not surprising that many calculations have been made on idealized systems, the advantages of a calculable system being balanced in most instances by difficulties in its interpretation in terms of a practical structure.

In order to tackle the boundary layer problem, for example, Corcos and Leipmann (28) made a series of assumptions which allowed an answer to be obtained from the internal noise due to the "pseudo-noise" or pressure field of the boundary layer. The fuselage was treated as a flat plate, the correlation areas of the forcing function were assumed to be small compared with the size of the plate and the length scale associated with plate motion small compared to the flat plate dimensions. Thus, the response is that of a random load distribution on an infinite plate with no constraints at all at stiffeners or frames. Coincidence effect or panel "resonance" is the main contributor to the motion of the plate, thus any increase in damping has a marked effect on the amplitude of vibration and the internal noise in the cabin.

Ribner (29) makes practically the same assumptions in his analysis, which effectively treats the problem as that of flowing isotropic turbulence along an infinite plate without stiffeners and with negligible acoustic damping. The neglect of the stiffeners and frames is equivalent to treating the motion again as ripples rather than modes of standing waves. Thus, for vibrations involving the lower harmonics of a panel (in fact, those observed in practice), the difference in stresses, particularly in the region of the frames, must be grossly misleading. Ribner agrees with this view in subsonic flight, but considers the method reliable for supersonic flight speeds, and has made tentative estimates for the subsonic case, making due allowance for the standing waves.

In the United Kingdom, we have gone rather the other way, thinking in terms of drastic assumptions regarding the excitation field and the number of normal modes excited, but keeping the basic properties of the structure well represented. Thus, Powell (30) in a Douglas report has given an example of his general method, while Foxwell (31) has been building up experience of the structural response of stiffened cylinders to a series of progressively complicated sound fields. For example, Figure 21 indicates the modal deflections on a stiffened cylinder of the type treated by Miller (26) for the case of plane incident waves at various angles and for a frequency of 240 cycles per second.

At the moment, there is no sign of finality in this problem, the need for continued research in all its aspects being obvious. Nevertheless, the basic framework is available, so that in due course, sufficient information on excitation, modal shapes, etc., should be forthcoming. Dynamic considerations now enter into practically every aspect of structural design, so that the need for such methods, in addition to ad hoc testing, is paramount.

V. DAMPING

Little mention has yet been made of two other vital links in the chain, the energy absorption by damping and the fatigue life arising from the type of random loading distribution now occurring.

Since structural vibrations are essentially of a resonant character, the damping of the system is one of the most important properties which determines the amplitude of the vibration and the fluctuating stress level in the structure. Indeed, when the excitation is random, the mean square vibration amplitude is inversely proportional to the product of the damping and stiffness in the mode of vibration being excited. Thus, practical increases of damping by the addition of suitable damping compounds to the skin, can reduce the panel amplitudes and stresses to as little as 10% of the original values (32).

Two types of damping occur, structural and acoustic. The former derives principally from the riveted joints and to a much less extent to the internal hysteresis of the materials in the structure. Acoustic damping derives from the energy radiated acoustically by the structure by virtue of the phase lag between the exciting field and the motion of the structure.

A. Structural Damping

Structural damping is of a kind in which the cyclic dissipation of energy is independent of frequency, and arises from the hysteresis loop traced by the force-amplitude excursions during the vibration. That due to simple joints, for example, has been found to be sensibly linear over the low load range, but at a certain critical load, the equivalent damping coefficient rises rapidly with the load, this increase been due to rivet and skin slip. The energy dissipated per cycle may then increase as rapidly as with the cube instead of the square of the joint load.

The damping of a structure cannot, of course, be considered out of context with the modal response. For modes involving the whole fuselage (hour-glass modes) each joint in the structure will be subjected to oscillating loads, the magnitude of which depends on the inertia loading of the mode. It follows that some modes will cause certain joints to be highly loaded, while other modes cause them to be lightly loaded. The contribution to the damping from any one joint will, therefore, vary with the mode. Stringer skin joints appear to contribute very little to the damping of these modes whereas skin to skin joints contribute considerably. On the other hand, the stringer-skin joints provide nearly all the structural damping of a single vibrating panel. Some figures of typical damping ratios are worth quoting here for comparion with later figures. American tests on a large fuselage type cylinder indicated structural damping ratios of from .0007 to .003 for hour-glass modes, while tests done at Southampton on simple panels riveted around their edges show damping ratios of 0.003 (33).

B. Acoustic Damping

There seems to be some difference of opinion in various quarters regarding the importance of acoustic damping. A simple theory (32) may be derived for the damping of panel modes by splitting up the panel into a large number of elementary 'pistons', each vibrating with the local amplitude of the panel. If the panel is considered to be set in an infinite plane and rigid wall, then the acoustic pressure distribution from a simple small piston is readily obtainable from classical theory. The pressure has components in phase with and in quadrature with the piston velocity. The in-phase component opposes the piston velocity at the piston, but at a quarter wave length away, from the piston, its phase changes by 180°. The pressure due to one piston will, therefore, oppose the velocity of all the other in-phase pistons in the region within a quarter wavelength, and will act with the velocity of those pistons 180° out of phase with it. If the velocity and pressure are opposed, sound energy is radiated away from the panel. Where they act together, sound energy radiated from another part of the panel is being put back into the panel.

Now the damping of the whole panel is proportional to the product of velocity and total in-phase pressure, integrated all over the panel. Reference (34) gives the result of such integration for a rectangular aluminum panel of any length: breadth ratio, on the assumption that the wavelength of the radiated sound is greater than about two or three times the panel length as shown in Figure 22. A simply supported panel has a slightly higher acoustic damping ratio than a fully fixed panel, at low length:breadth ratios, but at the higher values, the acoustic damping from each is equal.

If two adjacent (side by side) panels are vibrating in anti-phase, the acoustic damping pressure from one will annul the damping pressure from the other, whereas if they vibrate in-phase, the damping is doubled.

For aluminum panels of length:breadth ratio (n) of about 3, acoustic damping ratio is approximately 0.004. If n = 1, the acoustic damping ratio has a minimum value of 0.002 for a fully-fixed panel.

The acoustic radiation from an infinite cylinder vibrating in hour-glass modes has been investigated by Junger (35). The damping coefficient of these modes varies according to the number of waves around the circumference, the ratio of radiated sound wavelength to cylinder diameter, and the ratio of sound wavelength to wavelength of the mode along the cylinder. The latter is effectively the variation of damping with frequency. At a certain value of the frequency, the acoustic damping drops to zero, and below this frequency it remains zero. This "cut-off" frequency corresponds to a sound wavelength equal to the longitudinal mode wavelength. As the frequency increases above cut-off, the damping increases rapidly, but then reaches a maximum after which it falls off to an asymptotic value.

Below the cut-off frequency, the air has only a "reactive" effect upon the vibrating cylinder, i.e., the effect can be regarded solely as a change of the mass or stiffness of the system.

Typical values for the acoustic damping ratio are quoted in Reference (34), e.g. for a 10 ft. diameter stiffened fuselage structure vibrating at frequency f with n full-waves around the circumference, having a longitudinal wavelength λ, the acoustic damping ratio, δ_{ac} is as follows:

f(c.p.s.)	n	ft.	δ_{ac}
400	2	6	0.021
400	3	3.75	0.031
400	7	5	0.030
200	4	10	0.031

From the above it is clear that acoustic damping may be of the same order as the basic structural damping before the addition of damping adhesives, and in the above case was ten times as high.

C. Methods of Increasing Damping

Since it seems unlikely that any increase is forthcoming in friction damping of structural joints (such as that obtained from putting friction elements between skin and stringers) without the risk of fretting fatigue, the only promising method available to us is that of adding to the structure some anti-vibration compound. We, ourselves, have investigated two of these in Southampton, which are essentially different in their way of behavior and are, therefore, worth enlarging upon. They are both "plastic" in the sense of having qualities sensitive to temperature, the one being a Permacell damping tape, consisting of a metal foil with a thin soft layer of damping material attached, the other being a low density light polymer "Aquaplas" of very high natural damping properties.

Aquaplas, having some strength and stiffness in its own right, is usually applied as an unconstrained damping layer (Figure 24) and is under direct strain when the panel or stringer is deflected. Permacell, on the other hand, is applied with a metal foil constraining the soft damping material (Figure 24) and relies on the high shear in the plastic arising from differential movement of the foil and skin.

Many applications are now being investigated, in the light of the optimization arising from careful consideration of the actual movements of the various parts of the structures. For example, it is sensible to locate the Aquaplas as far from the neutral axis by the interleaving of a foam plastic; it may be worthwhile to cut the foil of the Permacell in order to increase the local shearing action; and it is worth thinking of adding a damping compound into sandwich types of structures. Already we know that Aquaplas, the most promising of the two, when added in the form of two skin thicknesses to an aluminum panel, gives an increase of damping ratio of as much as 0.18, while a quarter of an inch on a stringer flange can increase the damping ratio by about 0.03 (of the stringer mode). If we assume an initial total damping of about 0.01, such an addition of Aquaplas will reduce panel amplitudes to about 10%, surface stresses by slightly less, and inertia forces (and, therefore, the loads on attachment rivets) to 30%. Care must, however, be taken, since further additions, in fact, increase inertia forces.

Summing up, it is likely that further research in damping methods will prove to be very advantageous both to reduce stresses, increase temperature ranges, and also to indicate how and how not to apply the material effectively. A further point may also be made here: Acousticians are in the habit of talking in terms of decibels; the reductions in stresses obtainable, while appearing very large, are surprisingly small compared with the reductions (usually hidden by the logarithmic decibel scale) arising from sensible repositioning of the engines. There is a lesson in instrumentation to be made here also. There is little satisfaction in fighting to reduce stress levels by a few percent by damping when acoustic instrumentation and acoustic theory cannot measure such small differences.

VI. FATIGUE LIFE ESTIMATION

In due course, it is to be hoped that given the relevant data, and with acceptable simplifications to the theory, the root mean square stress levels in the structure may be calculated. It then remains to provide an estimate of the fatigue life of the structure under the type of spectrum of stress which has been calculated. At the moment, fatigue failures occur at stress-raisers and would not be predicted theoretically in any case; as these obvious defects in design are eliminated, the need for fatigue prediction at the calculation stage becomes vital.

Of the laws of cumulative damage, now available, the most widely in use is Miner's Law (1) or some modification of it, such as that of Shanley (39). In Miner's hypothesis, if N cycles can cause failure at any load amplitude I, a single application of this load contributes Ith/N of failure of the structure. Thus, in Figure 25, if n cycles occur at a stress requiring N cycles to fail, then

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots\dots\dots \frac{n_m}{N_m} = 1$$

Shanley's approach is to calculate a reduced stress on the same basis, such that under sinusoidal loading, it has the same fatigue life as that for the real more complicated loading. It is claimed that this reduced stress is confirmed experimentally to within 4%; since, however, the number of cycles to failure is inversely proportional roughly to the stress raised to a high exponent between 10 and 25, such an error as 4% is equivalent to a life factor of between 1.5 and 2.65. Certainly, it is known that Miner's hypothesis can be grossly in error in predicting fatigue life as it fails to take cognizance of recovery induced by rest periods and understressing, and variations in residual stress patterns caused by changes in sequence of loading or the application of static stresses. When high loads are applied in the initial stages of the loading, program values of $\frac{n}{N}$ of approximately 0.6 are reported, whereas in the reverse condition the ratio is up to 1.5.

Schijve and Jacobs (36) have investigated cumulative damage in riveted lap joints and show that the interruption of program load patterns to apply static tensile or compressive loads can alter fatigue life by a factor of seven.

Torbe (37) has compared programmed block loading with random loading and suggests that the fundamental difference between the two types of loading is in the effect of residual stress patterns on fatigue life. Whereas in block loading the residual stress patterns persist and can result in a wide scatter in fatigue results, the rapidly fluctuating stress levels of the random load sequences tend to neutralize this effect and so make fatigue predictions less difficult. He claims that the important parameters of fatigue life under random loading conditions are the r.m.s. and the maximum stress amplitude attained, which must be accurately assessed.

Information and theory are being developed rapidly (38) and it is hoped that a really satisfactory framework will soon be developed so that the significant test data can be abstracted from the materials available to the engineers.

VII. IN CONCLUSION

A description has been given of how acoustic or aerodynamic excitation of structures occurs as a result of turbulent movements, either in a jet efflux or in a boundary layer along a fuselage, wing, or engine intake. The need to know not only the pressure field, but sufficient of its correlation characteristics to calculate the loadings on the structure has been stressed. The parameters involved in the response of the structure have been emphasized, the lesson again being the need to know the modes of oscillation, the damping, both structural and acoustic, in these modes, and the need to relate more closely than at present the relationship between the r.m.s. stress levels so calculated to the fatigue life of the structure.

Even after many years of research, the impression now still remains of the problem being insoluble in its general form until much urgently required research is carried out in all the above fields. It also raises the question of whether or not ad hoc methods of panel testing will not give the results sooner and satisfactorily. No specific section in this paper has been devoted to panel testing, though the difficulties of interpretation are implied in every section. The difficulties of obtaining a representative random loading distribution, the correct mode shapes, and the right end constraints imply that such tests are best thought of as comparative ones only, the relative fatigue lives of honeycomb, and other structures, being well established for equal. though not necessarily representative, excitations.

In this context, it is worth mentioning that some inspections of vibrating structures on a practical aircraft during acoustic excitation tests on the ground reveals still further the complexity of the modes of oscillation. Not only did the panels, frames, and stringer modes exist as already envisaged in this paper, but cross sectional distortion modes occurred which involved the bending of stringer flanges and of frame walls, which undoubtedly caused high bending stresses in the bends of the cross sections. Thus, brackets joining frames and stringers may be twisted and pulled by the combined loading, while rivets may have to carry repeated loading in tension while also transmitting shear loads. Thus, the need for realistic representation of structures, even to the extent of fatigue testing of the actual craft, may yet be needed as a routine measure.

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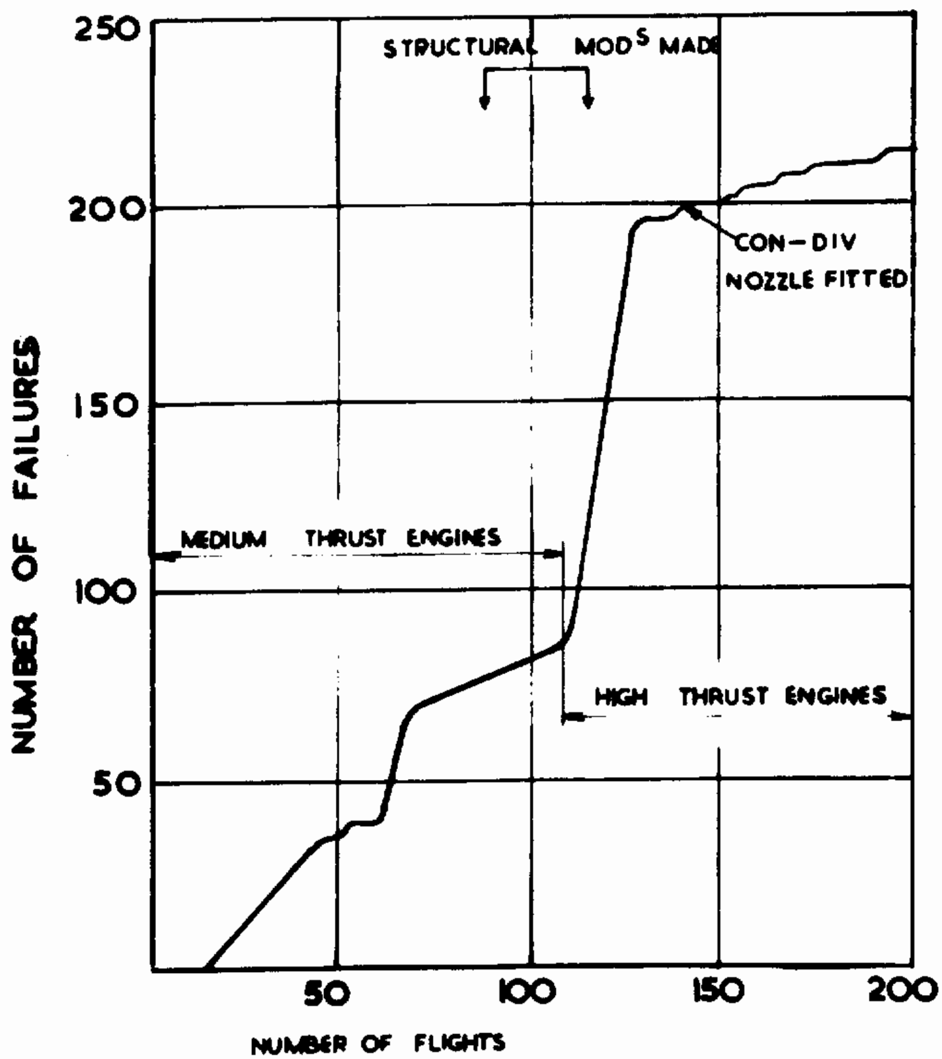


FIG. 1 - DAMAGE RATE FOR A TYPICAL AIRCRAFT

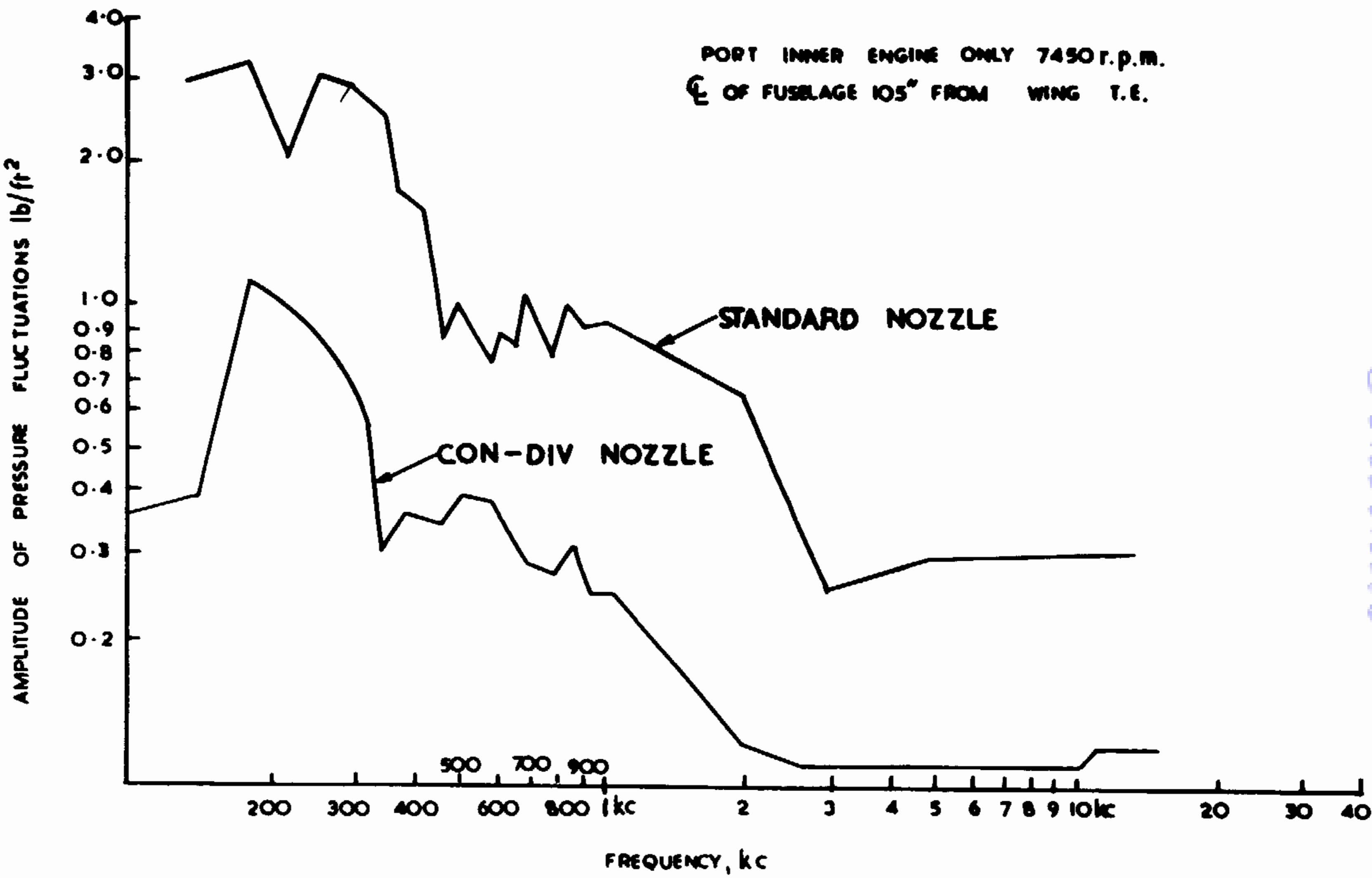


FIG. 2 - PRESSURE FLUCTUATIONS ON FUSELAGE OF A TYPICAL BOMBER

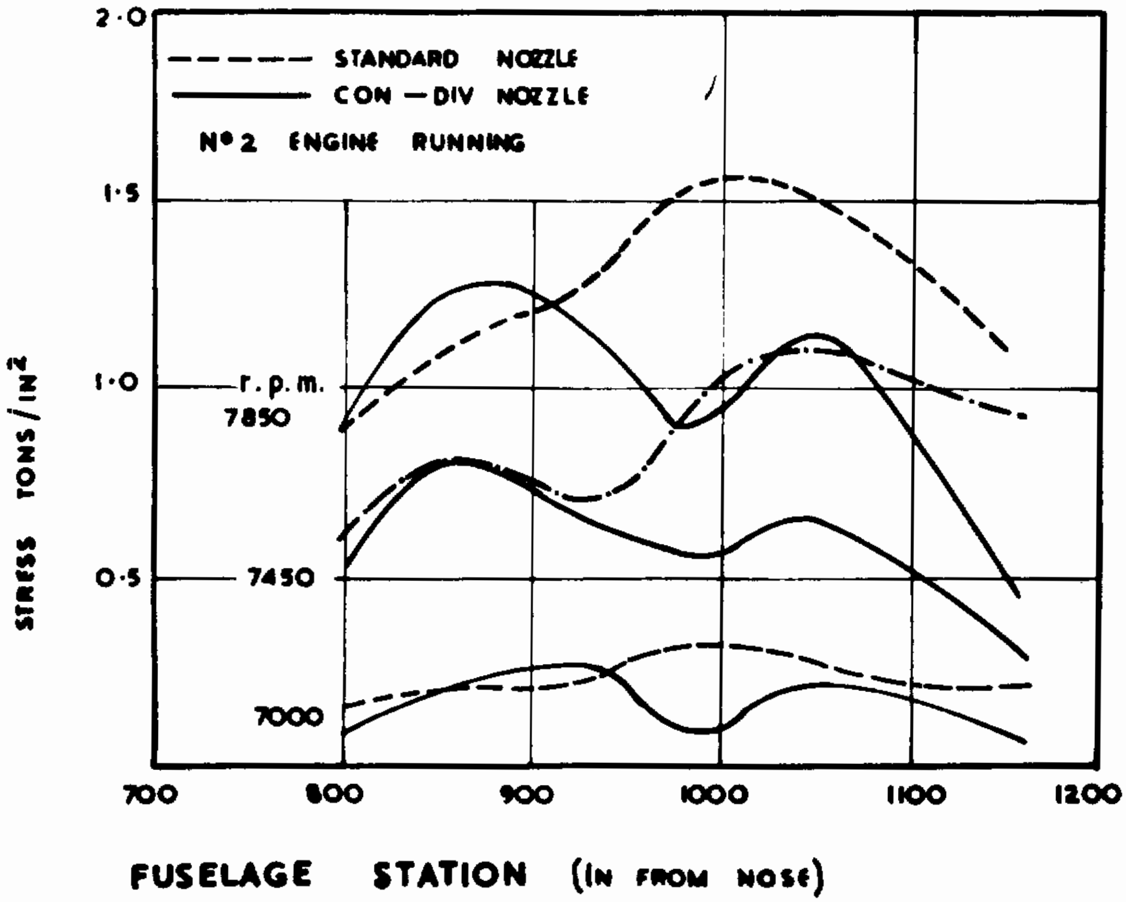


FIG. 3 - FLUCTUATING STRESS LEVELS IN SKIN ALONG FUSELAGE

FIG. 4a - SPECIMEN DAMAGE - STAR CRACKS IN REAR FUSELAGE FAIRING

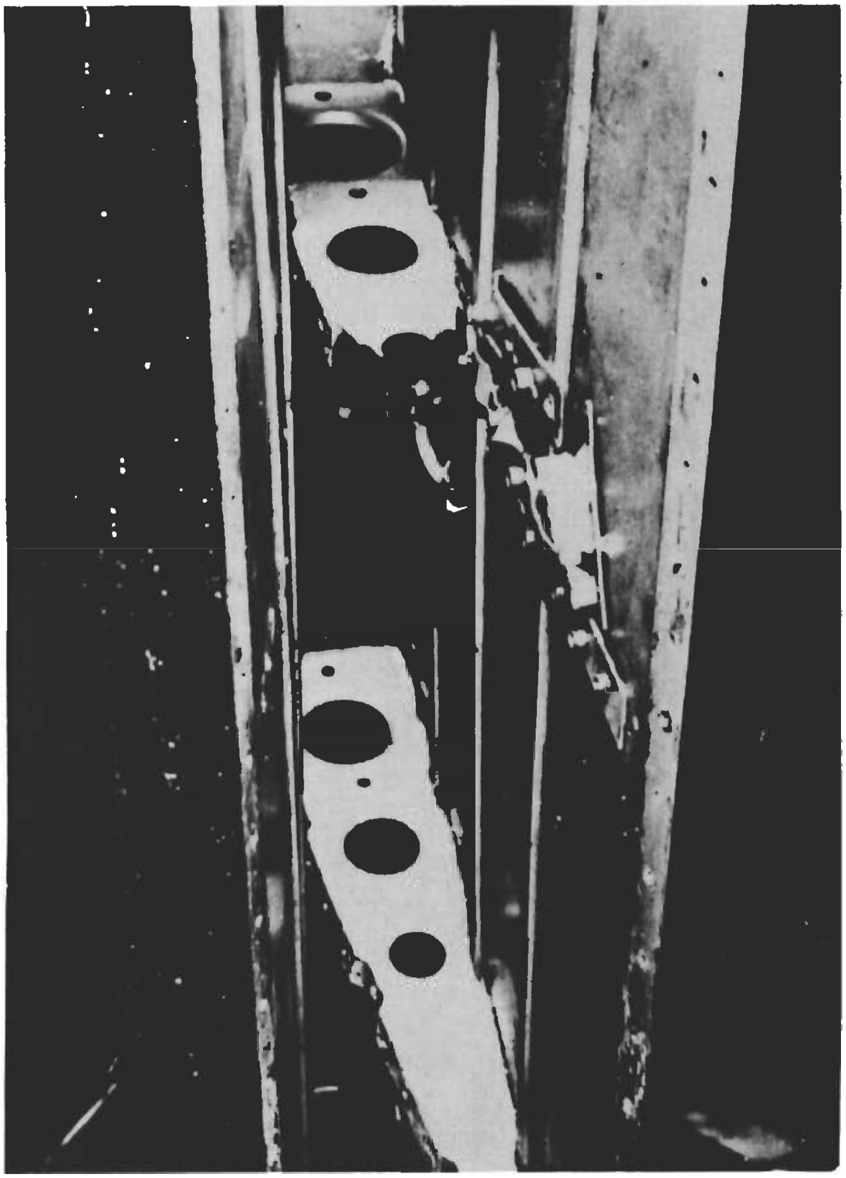


FIG. 4b - SPECIMEN DAMAGE - T. P. TRAILING RIB

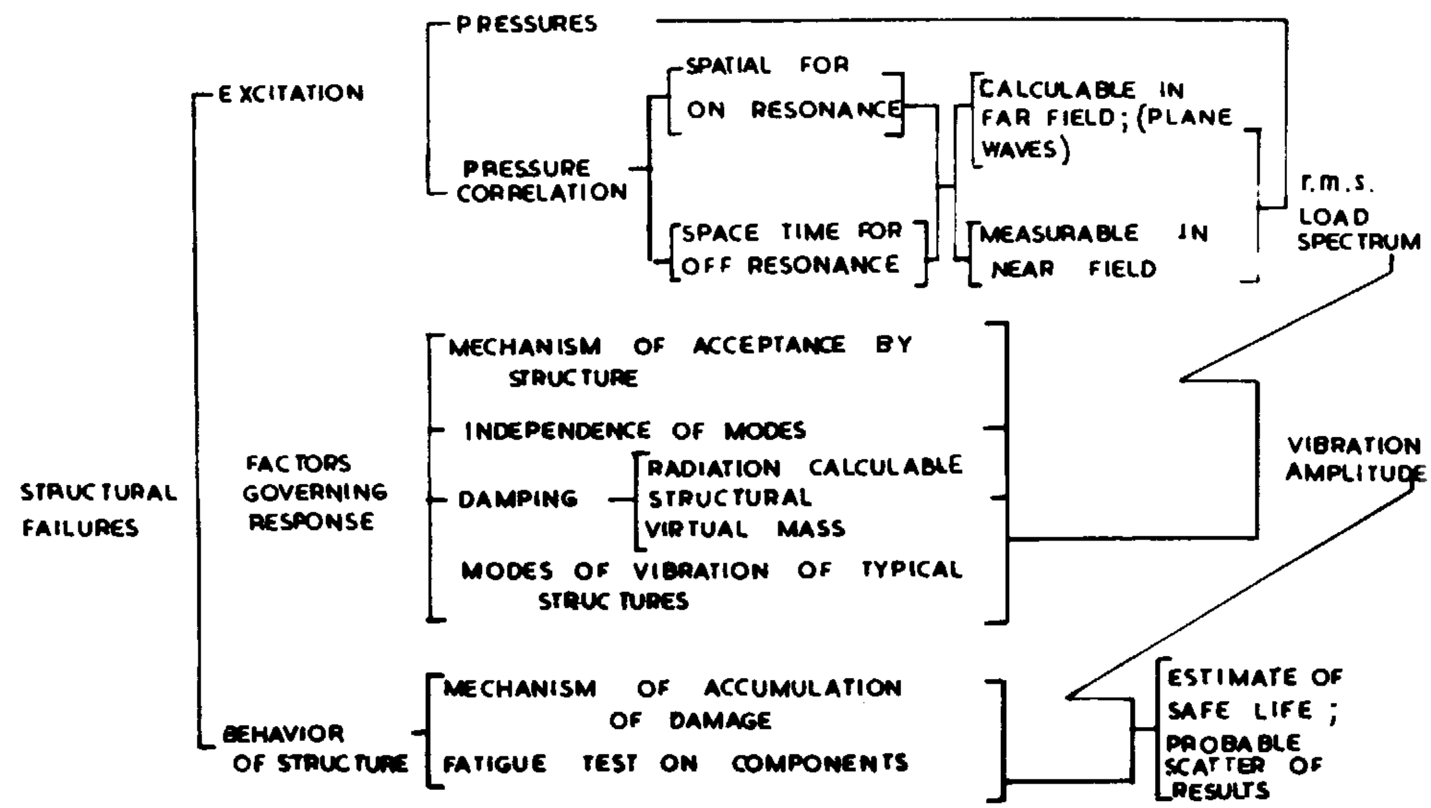


FIG. 5 - RESEARCH NEEDED TO OVERCOME THE PROBLEM OF THE EFFECT OF NOISE ON STRUCTURAL FATIGUE

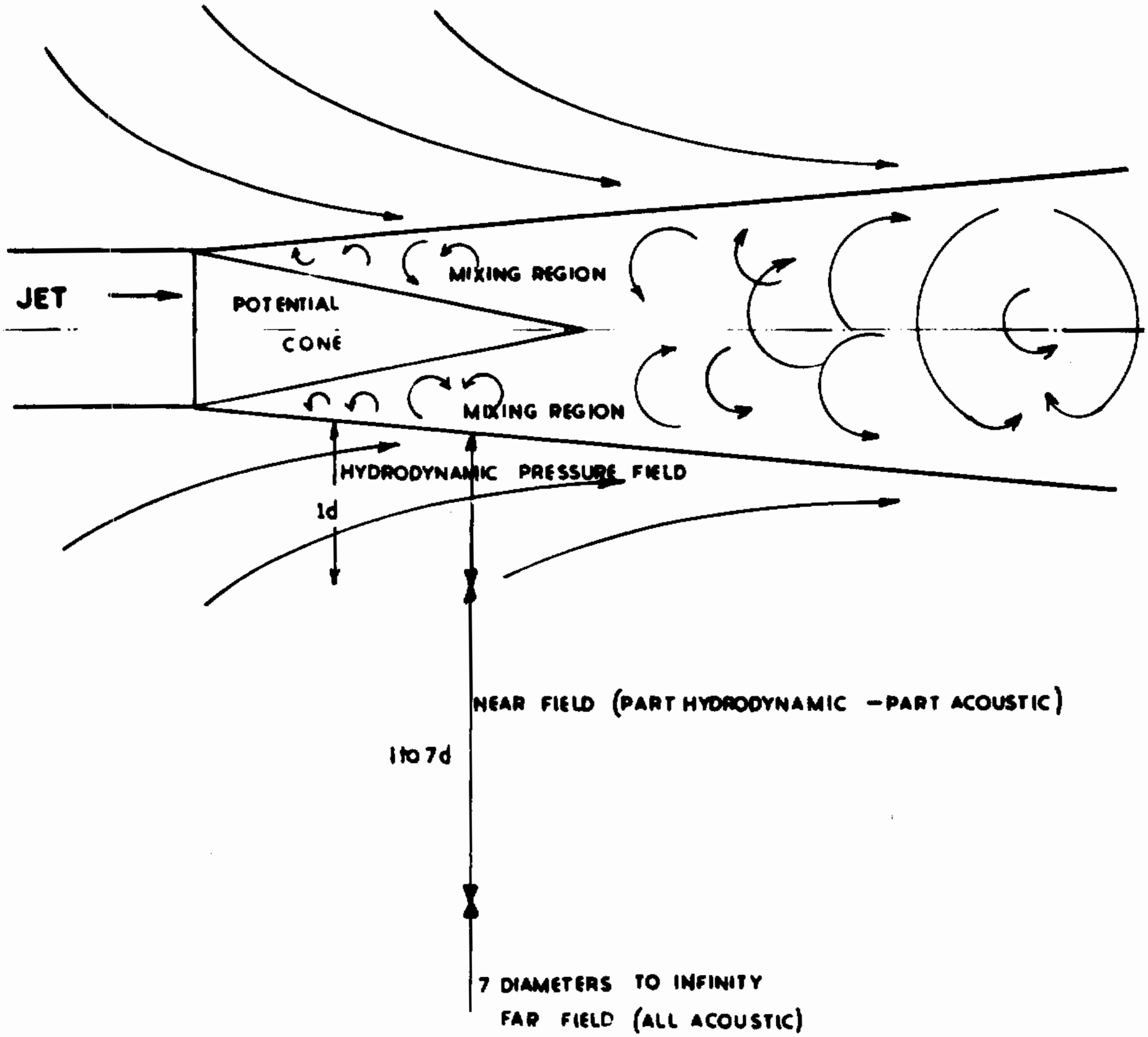


FIG. 6 - REGIONS OF INVESTIGATION AROUND A HIGH SPEED JET

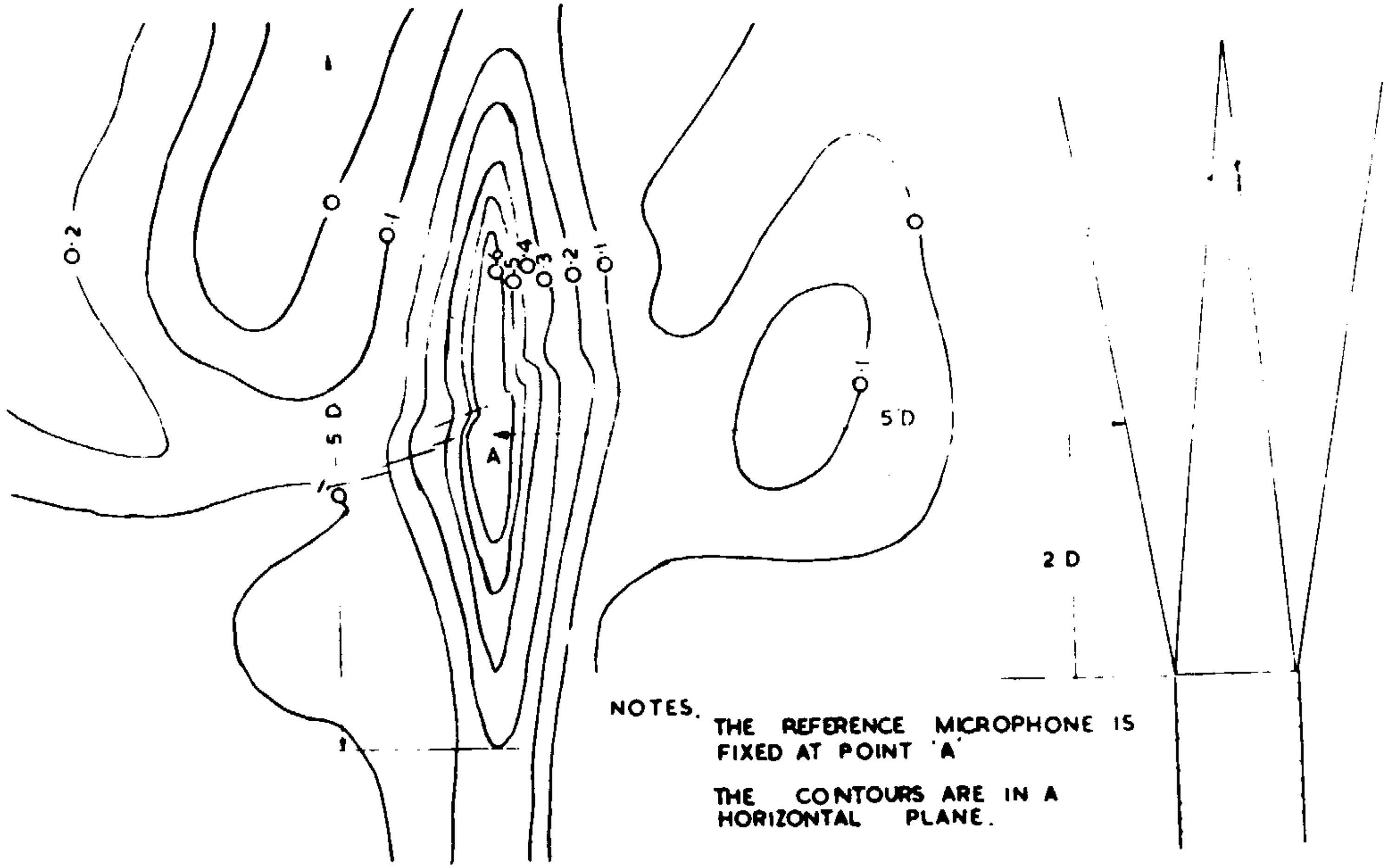


FIG. 7 - SOUND PRESSURE CORRELATION IN THE NOISE FIELD OF A JET ENGINE

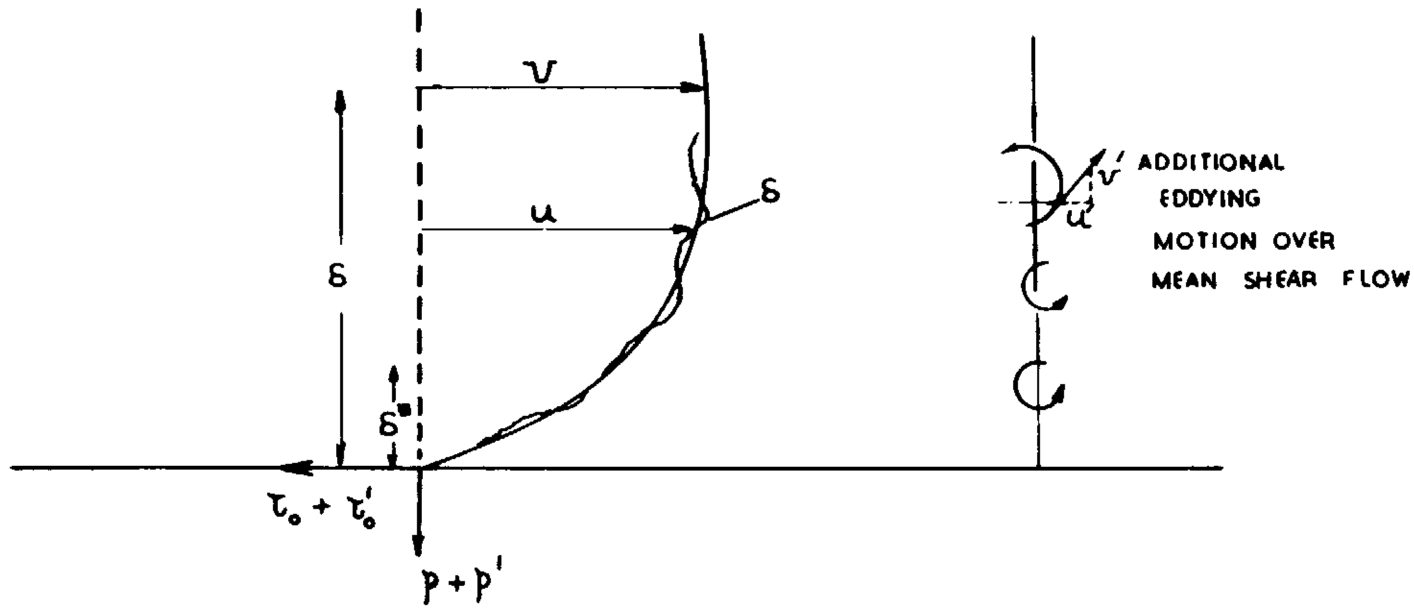


FIG. 8 - TYPES OF EDDYING FLOW IN A TURBULENT BOUNDARY LAYER

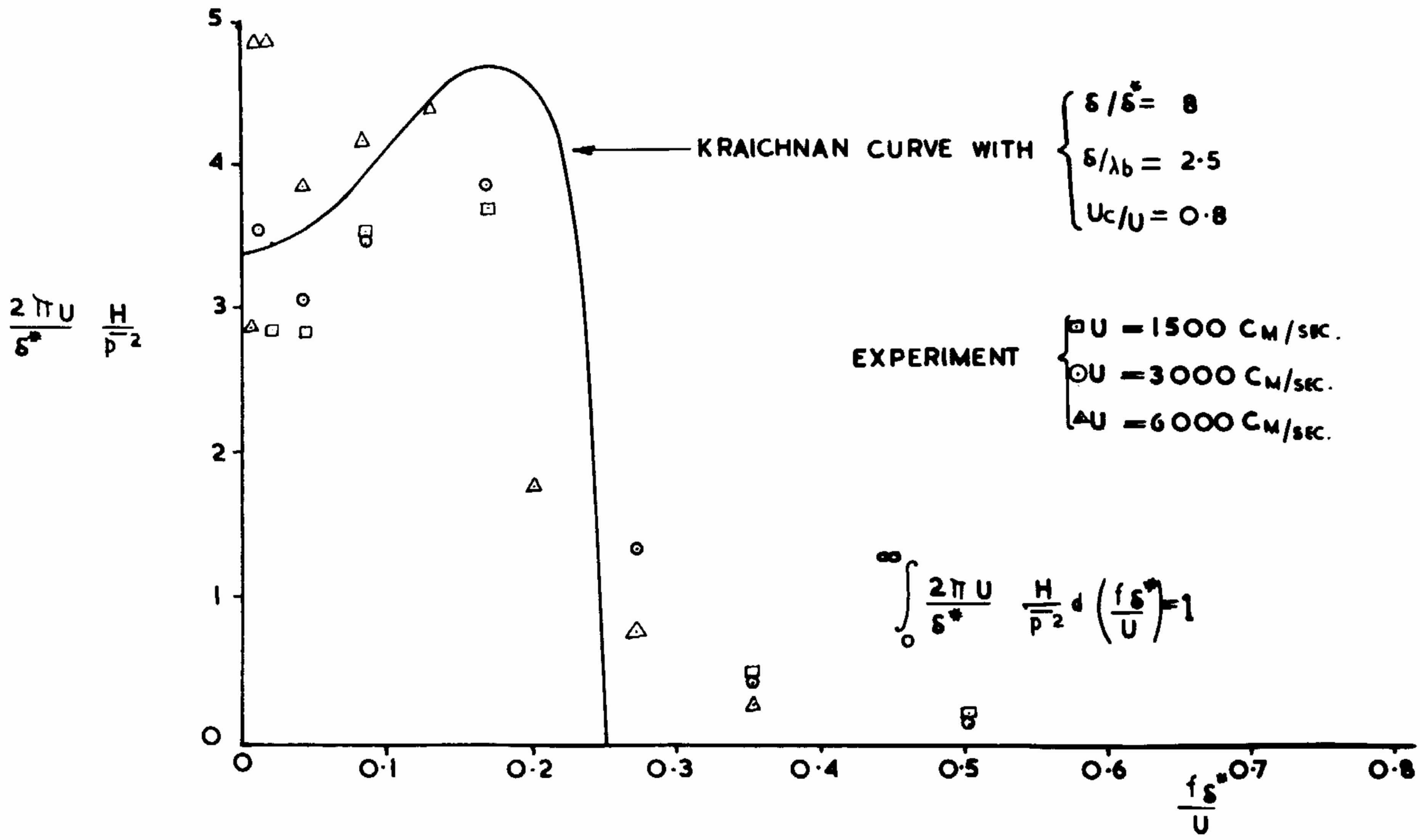


FIG. 9 - FREQUENCY SPECTRUM(EXPERIMENTAL) OF PRESSURE FLUCTUATIONS(H) IN A BOUNDARY LAYER

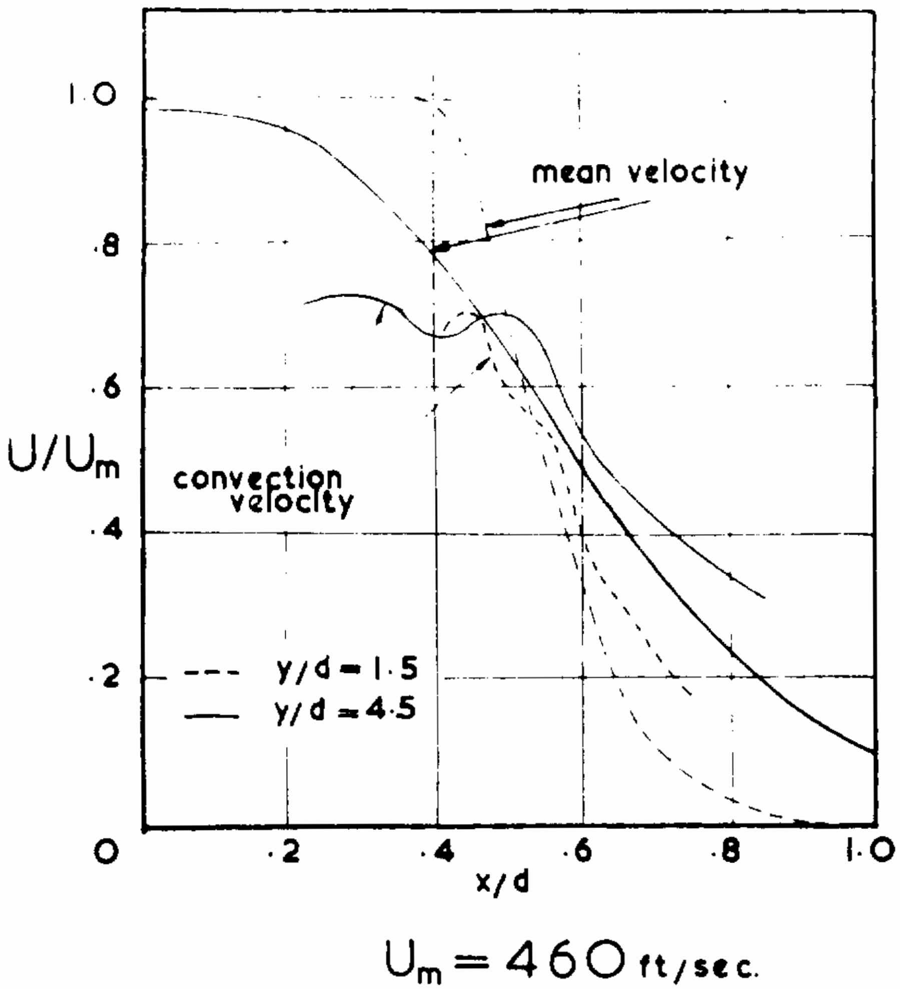


FIG. 10 - VELOCITY PROFILES ACROSS THE MIXING REGION OF A ONE-INCH JET

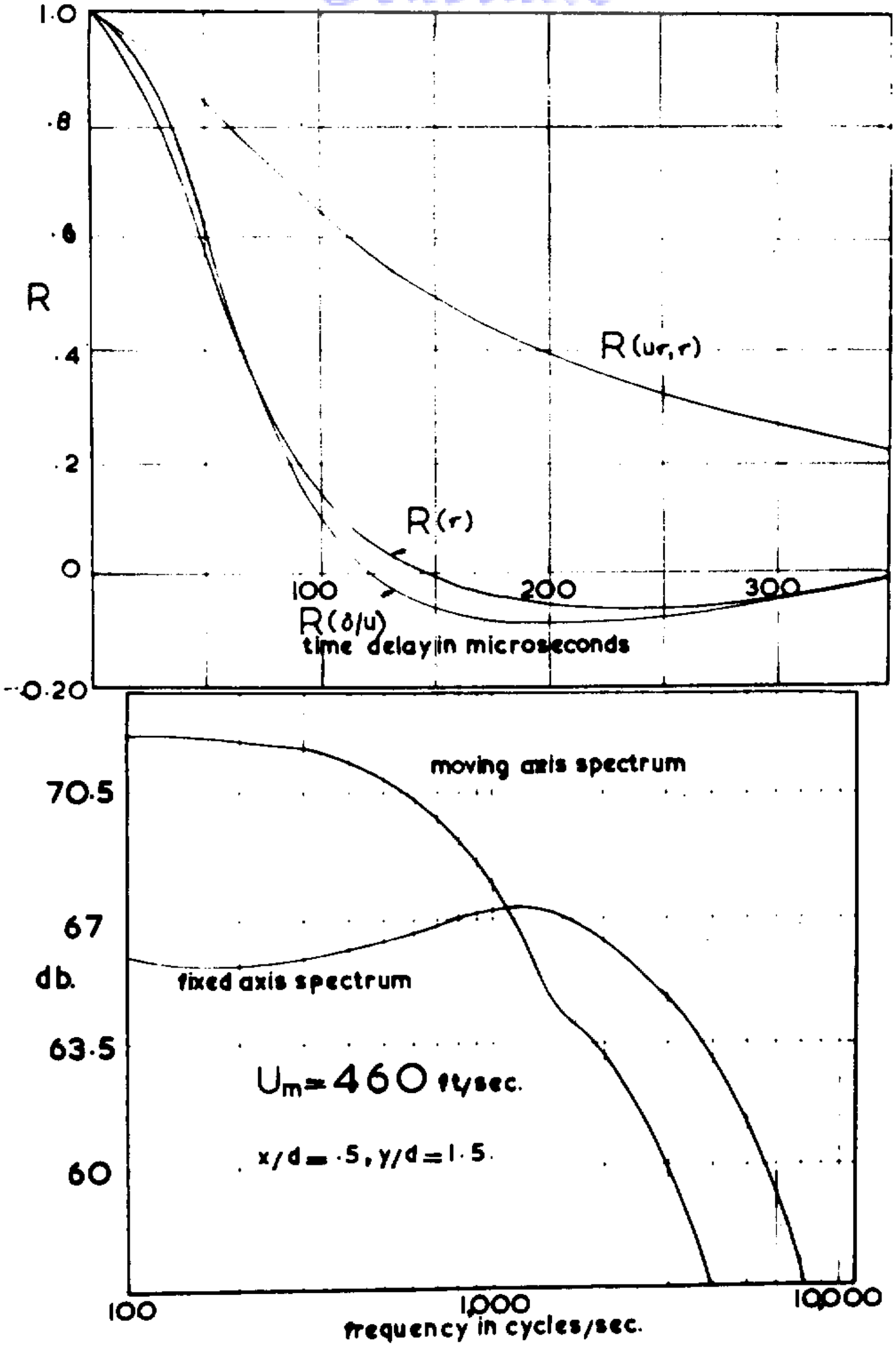


FIG. 11 - CORRELATION AND SPECTRUM CURVES FOR TURBULENCE IN ONE-INCH JET

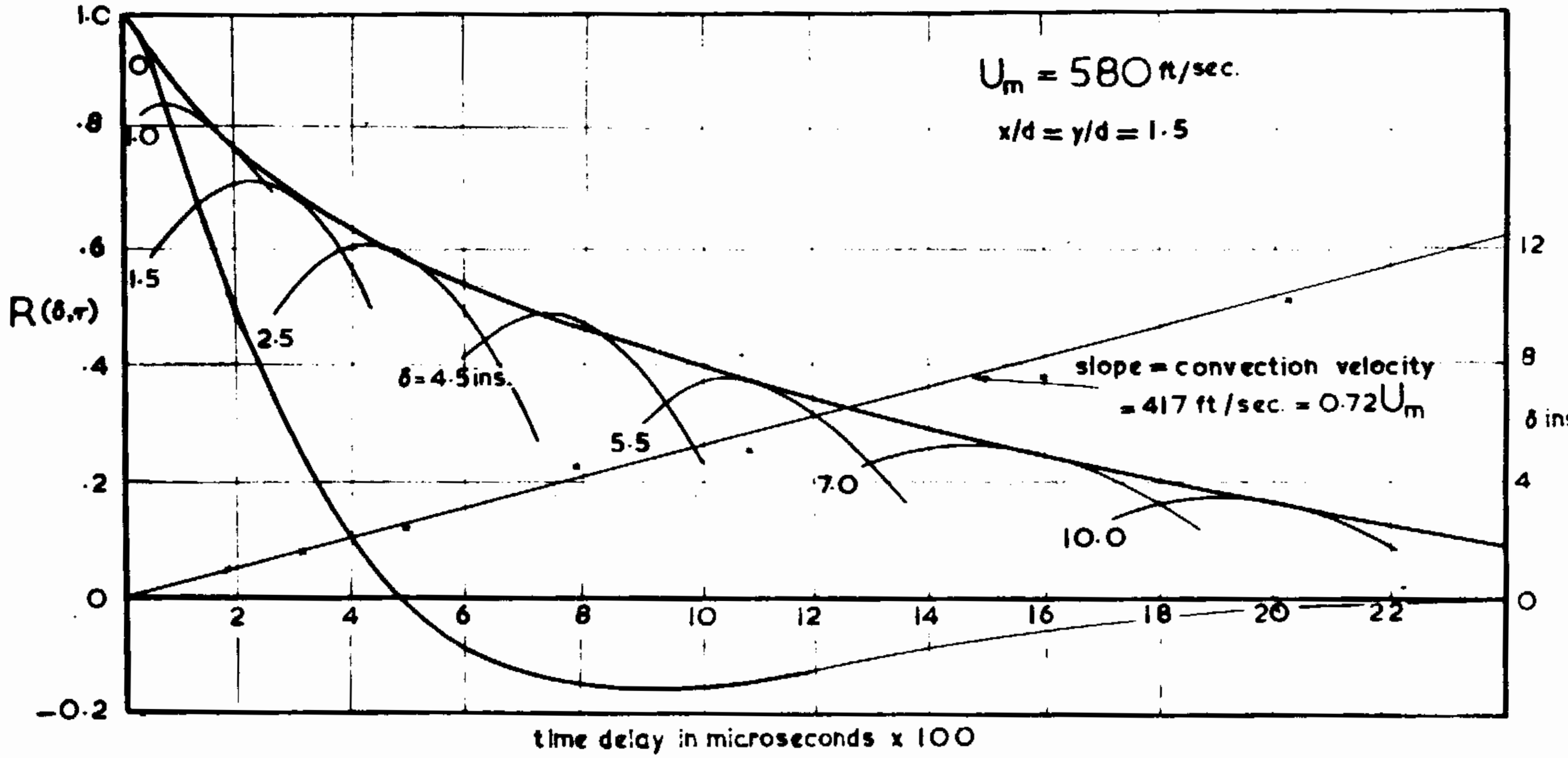


FIG. 12 - CROSS CORRELATIONS IN THE NEAR FIELD OF A TWO-INCH DIAMETER CIRCULAR JET

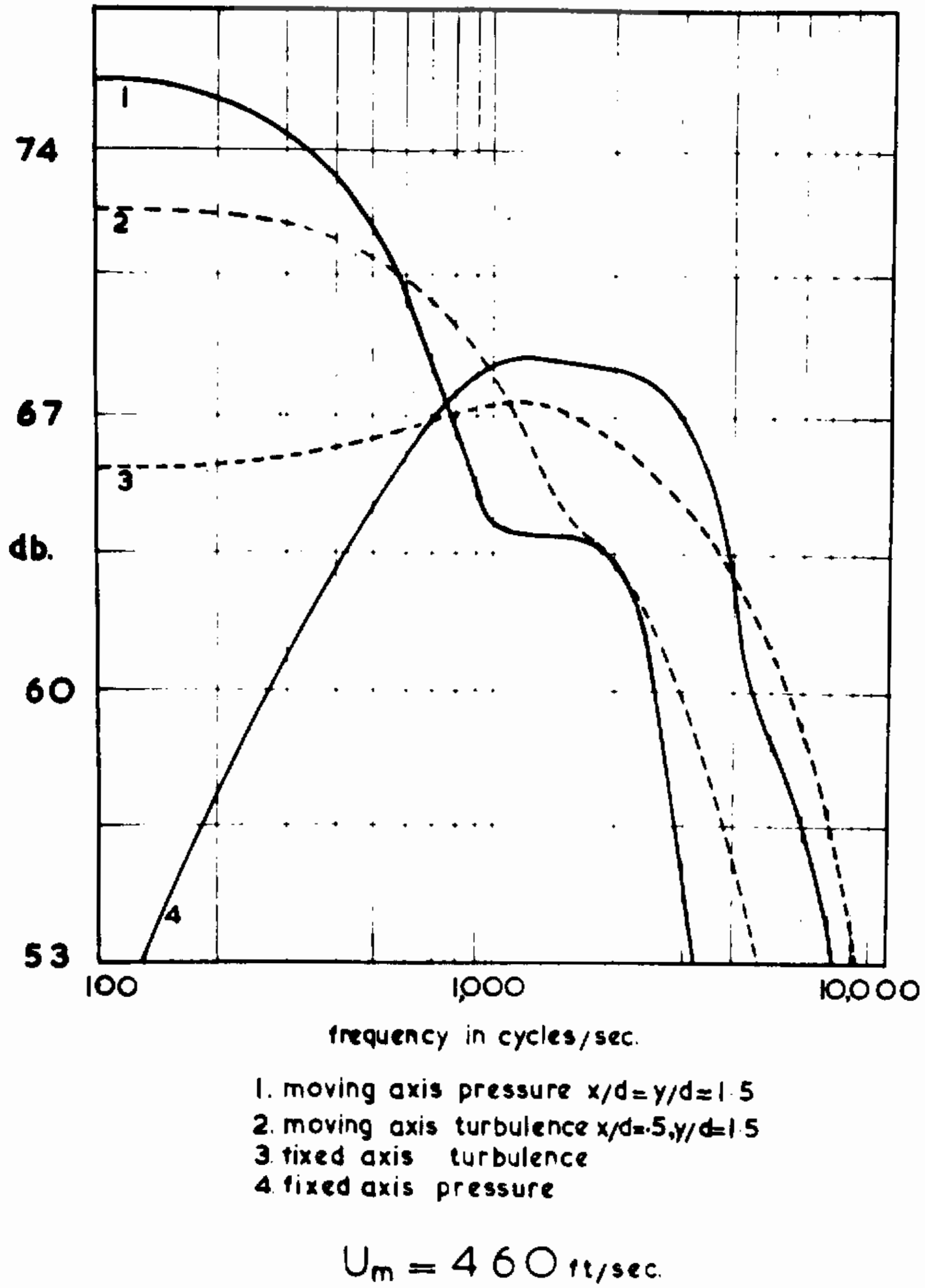


FIG. 13 - SPECTRAL DENSITY OF PRESSURE AND TURBULENCE IN ONE-INCH JET

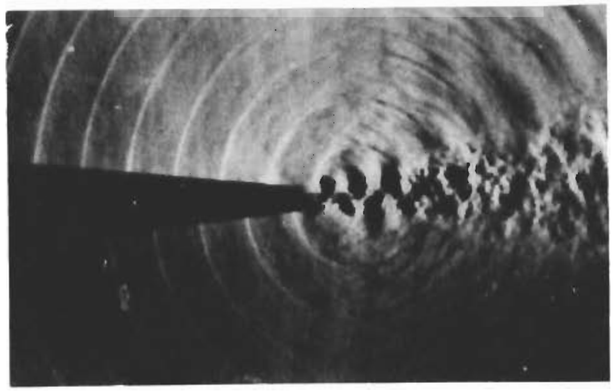


FIG. 14 - EXAMPLE OF INTENSE SOUND FIELD GENERATED BY BACK REACTION MECHANISM

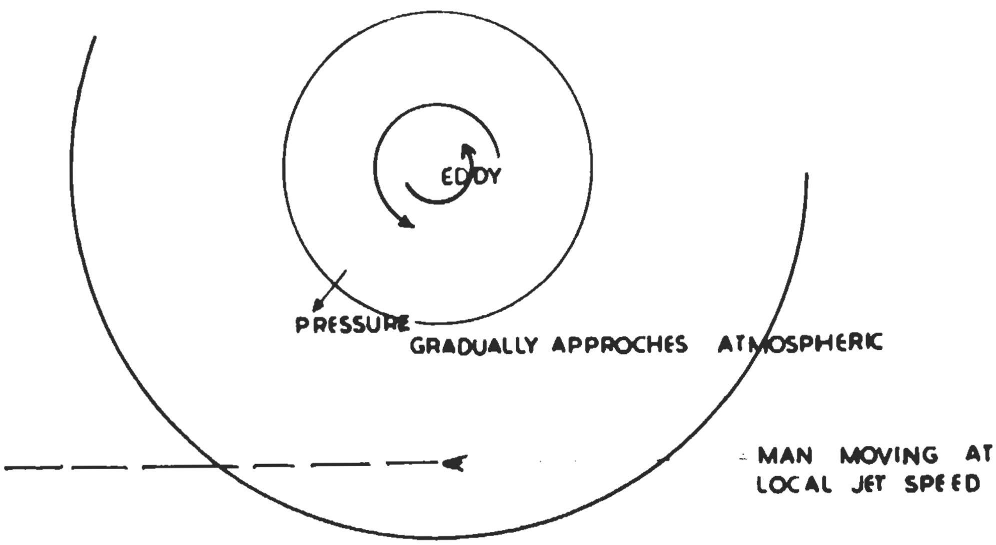


FIG. 15 - INDICATION THAT FROZEN TURBULENCE CANNOT GENERATE RADIATED SOUND

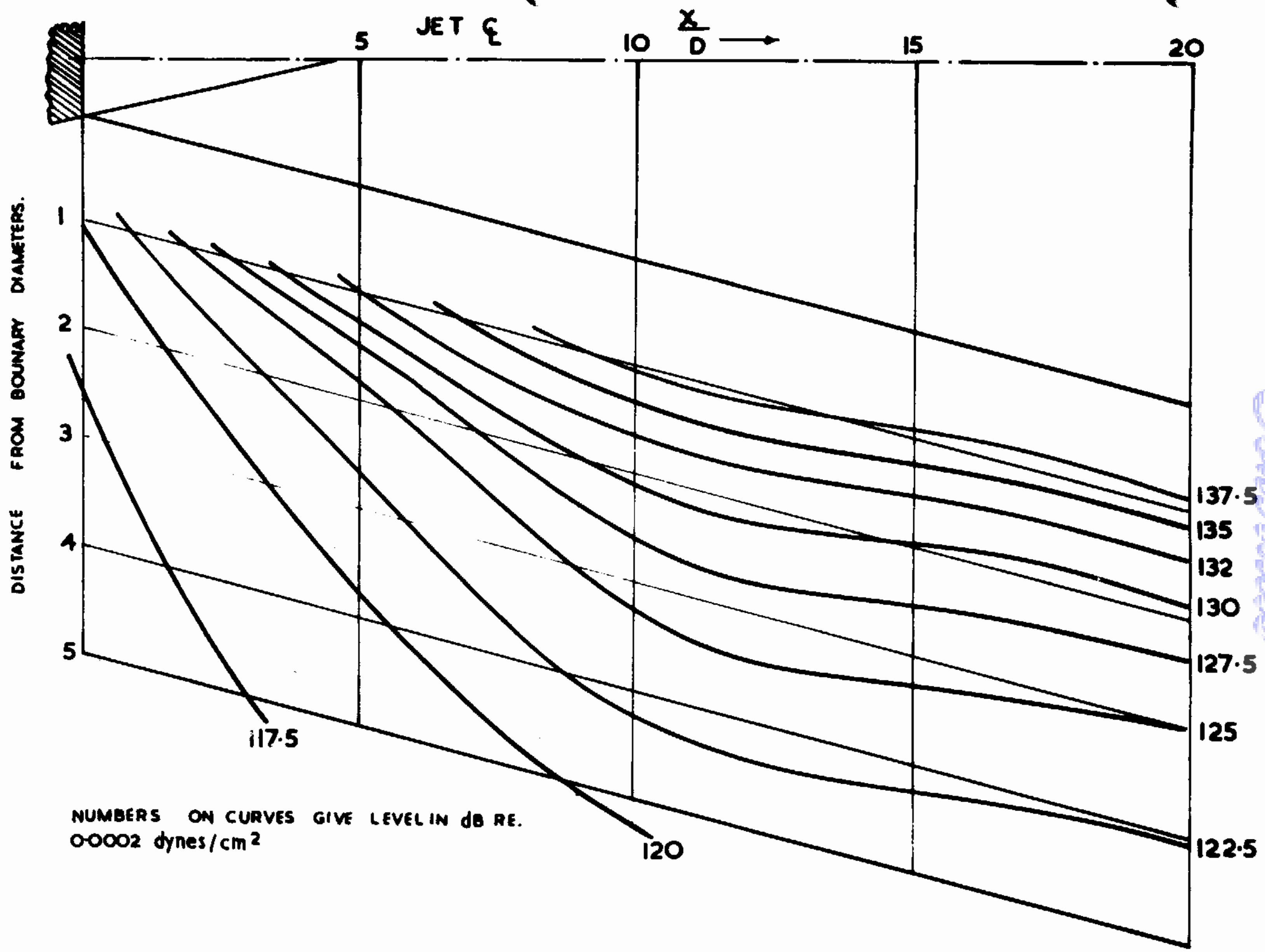


FIG. 16 - CONTOURS OF EQUAL R.P.M. PRESSURE NEAR A 4-INCH. DIAMETER JET.
EXIT VELOCITY 1000 FT/SEC

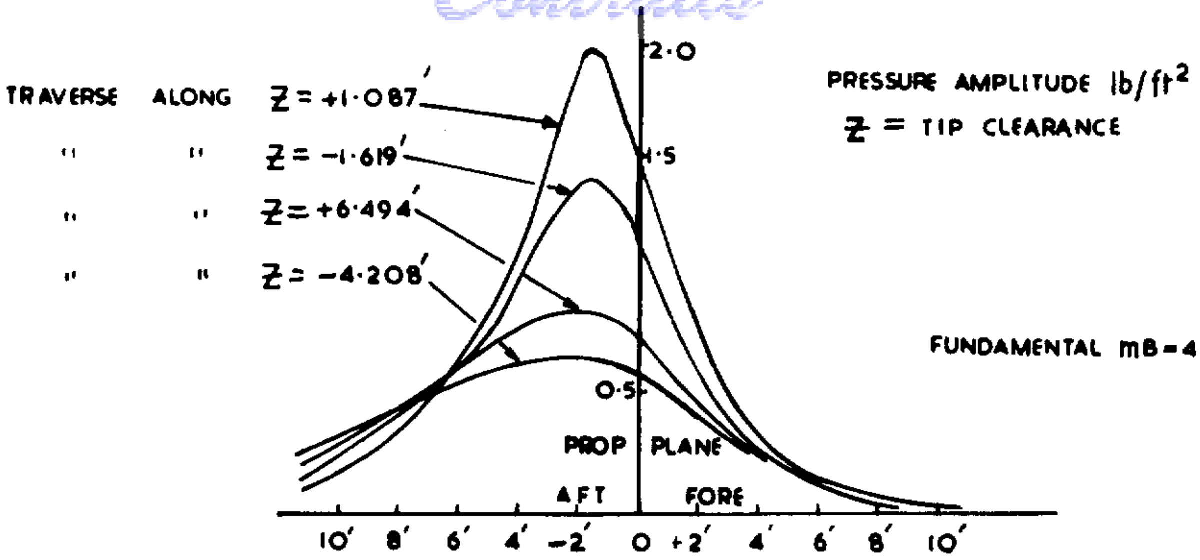


FIG. 17a - VARIATION OF FREE SPACE SOUND PRESSURE AMPLITUDES ALONG SOME GENERATORS OF THE FUSELAGE AT CRUISING SPEED AND ALTITUDE

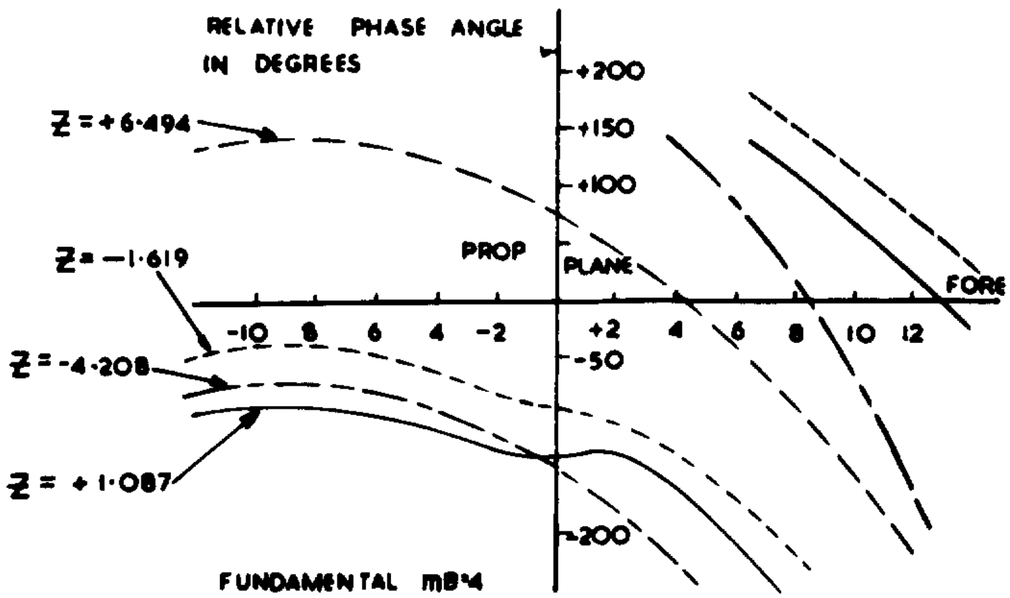


FIG. 17b - VARIATION OF RELATIVE PHASE ANGLE ALONG GENERATORS OF THE FUSELAGE

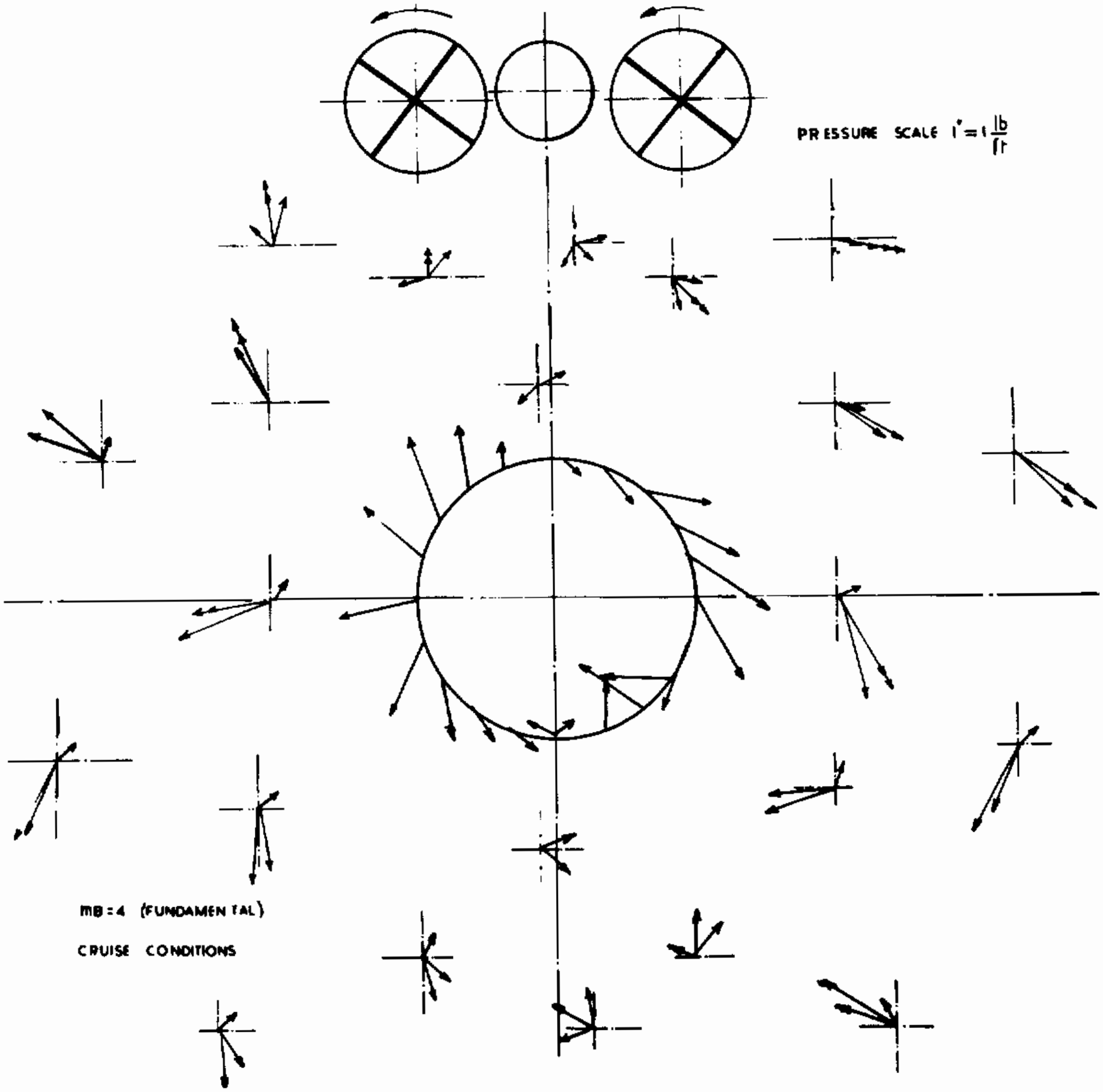


Fig. 18 - Diagram to Illustrate the Summation of Pressures Due to the Above Configuration at the Fuselage Surface

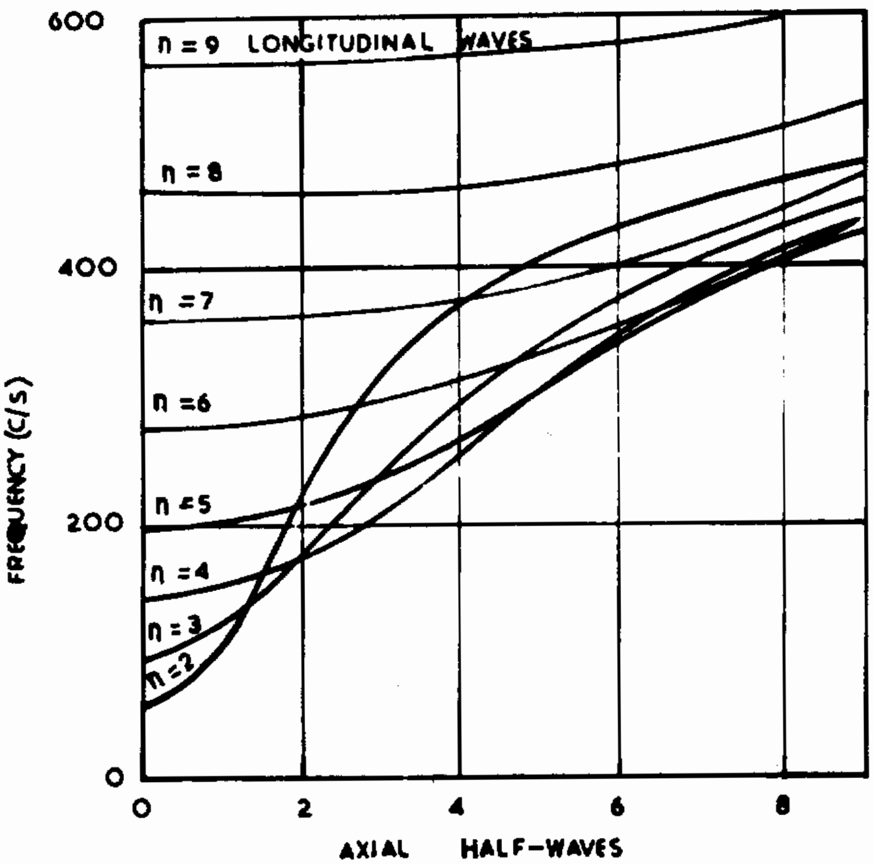


FIG. 19 - VIBRATION MODES OF A STIFFENED CYLINDER

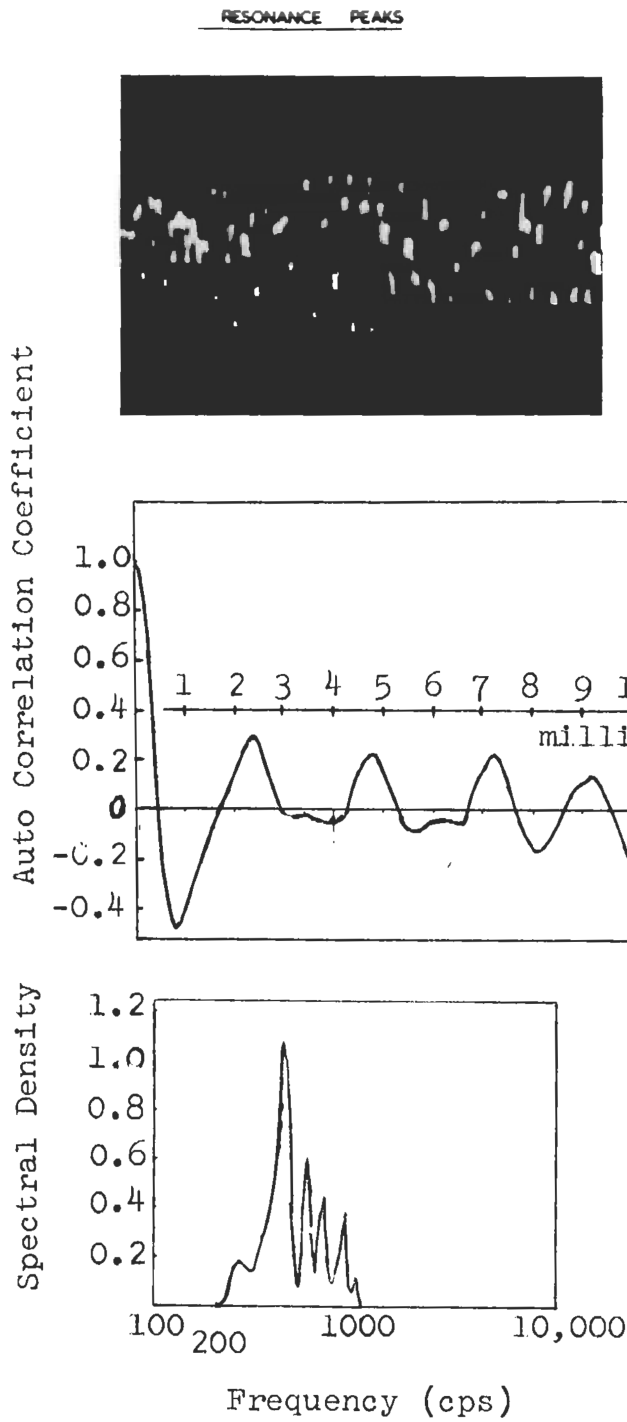


FIG. 20 - STRAIN ON A T/P PANEL SUBJECTED TO JET NOISE

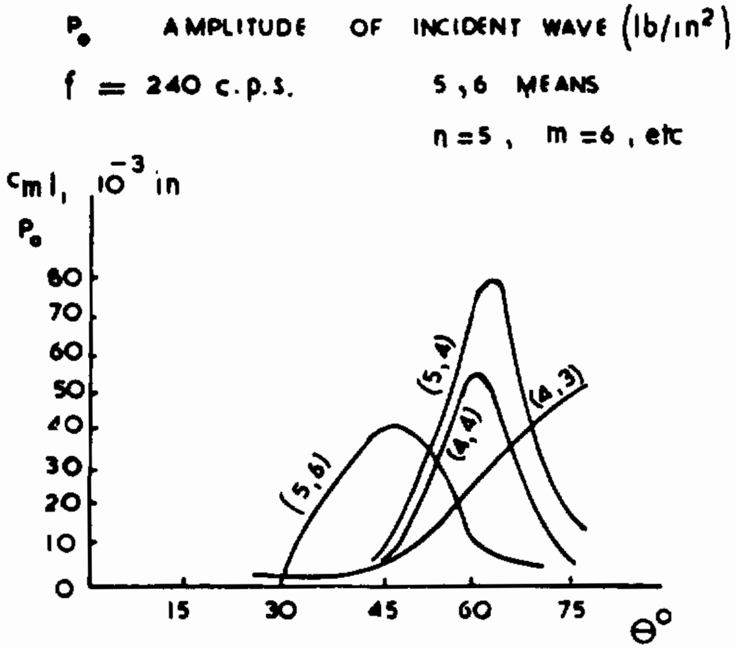


FIG. 21 - MODAL DEFLECTIONS FOR PLANE INCIDENT WAVE AT ANGLE θ°

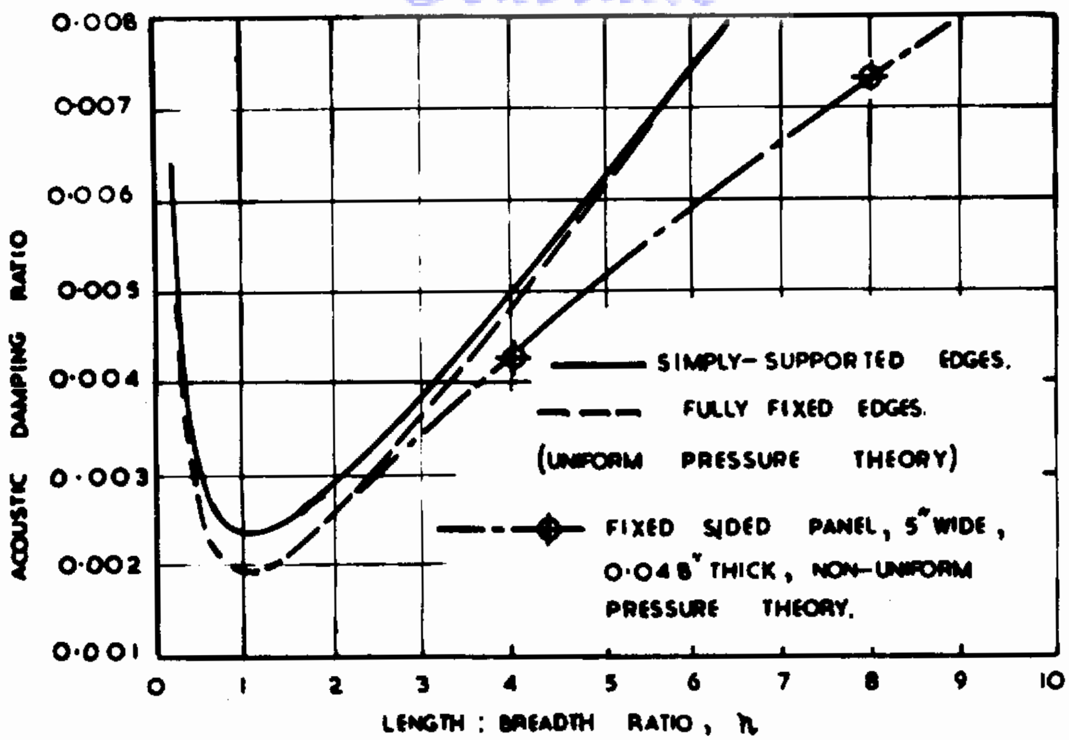


FIG. 22 - VARIATION OF ACOUSTIC DAMPING RATIO OF FLAT ALUMINUM PANELS WITH LENGTH TO BREADTH RATIO

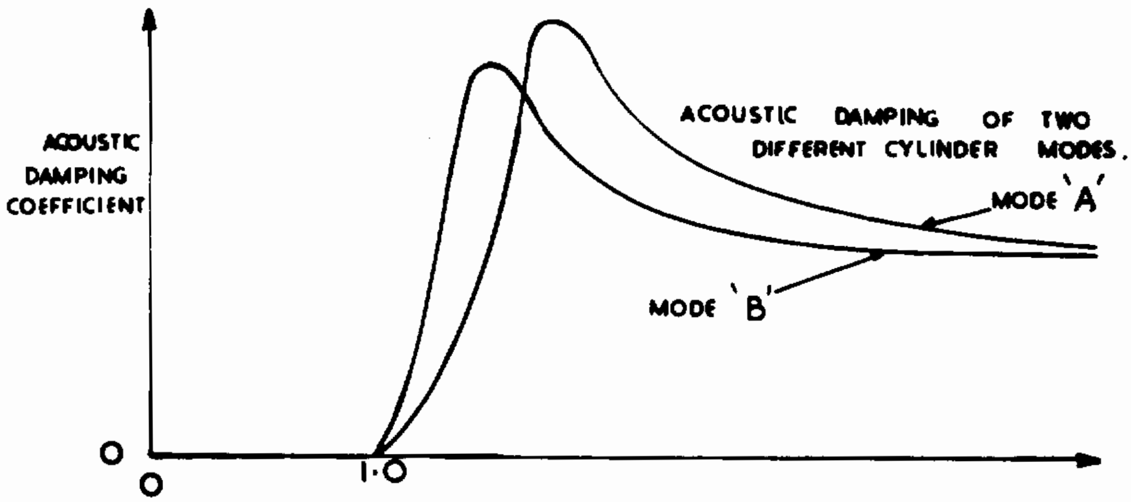
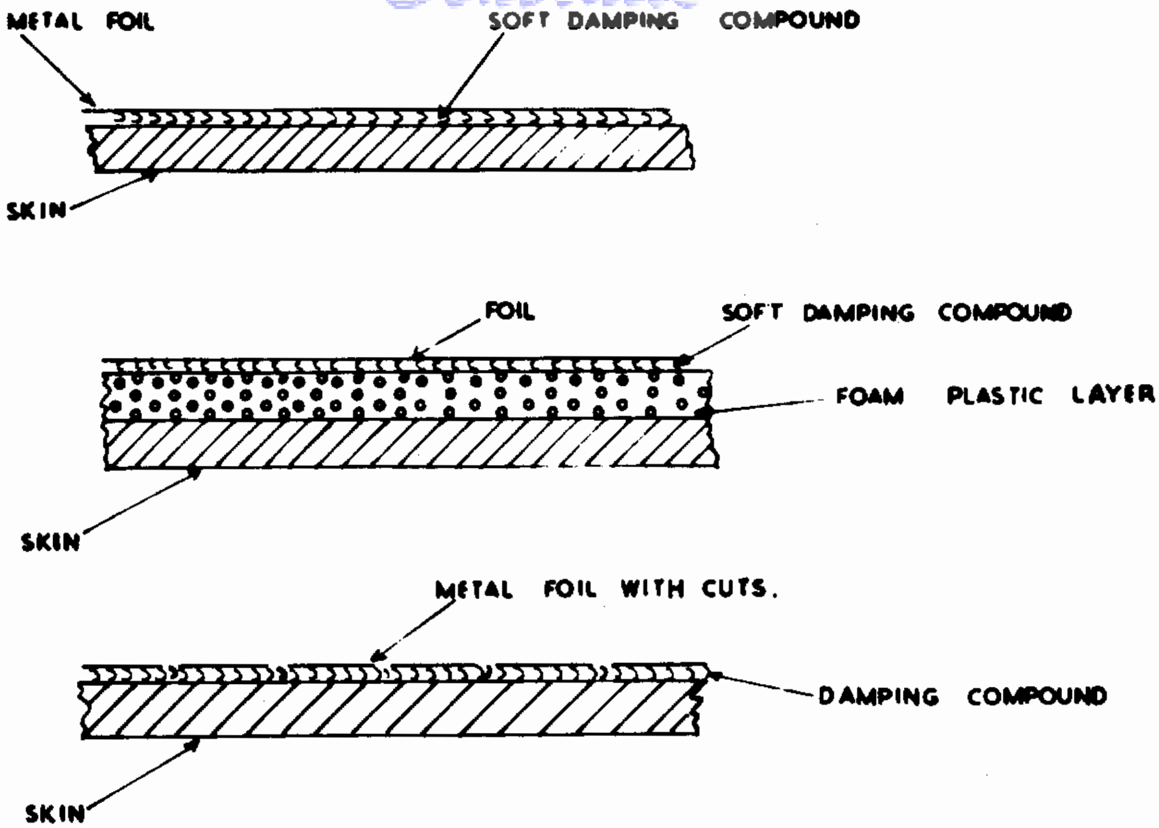


FIG. 23 - ACOUSTIC DAMPING COEFFICIENT VERSUS RATIO OF LONGITUDINAL WAVELENGTH OF CYLINDER MODE TO WAVELENGTH OF RADIATED SOUND ($\propto$ FREQUENCY)



EXAMPLES OF CONSTRAINED TYPE OF DAMPING ADHESIVE SYSTEM.

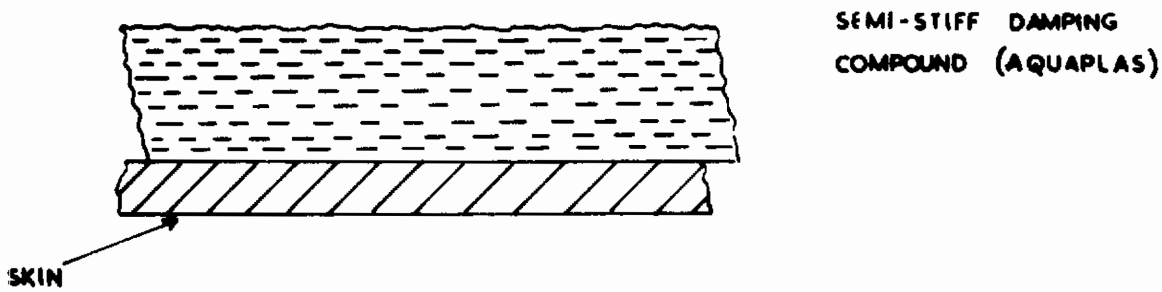


FIG. 24 - EXAMPLES OF UNCONSTRAINED TYPE OF DAMPING ADHESIVE SYSTEM

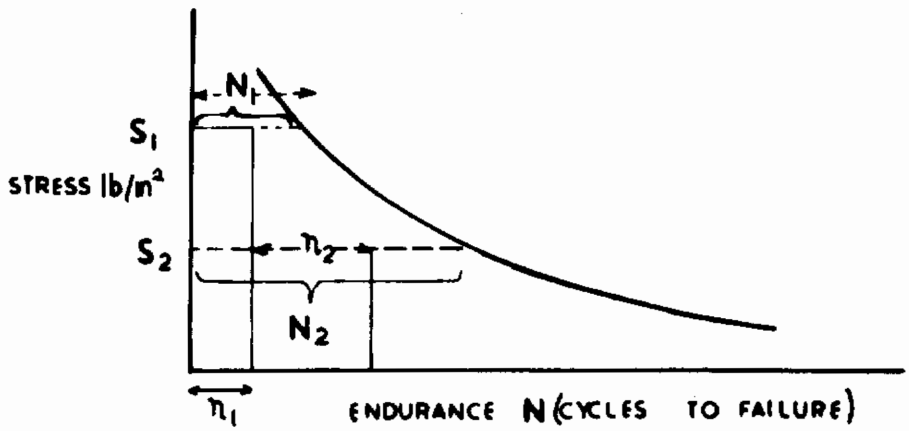


FIG. 25 - STRESS-CYCLE DIAGRAM