ON AN ADAPTIVE AUTOPILOT USING A NONLINEAR FEEDBACK CONTROL SYSTEM

John D. McLean and Stanley F. Schmidt
National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California

As Mr. Triplett has mentioned, we at Ames Research Center have been studying adaptive autopilots to try to understand the fundamental principles by which they can be derived and determine methods for analysis of their behavior. In this paper I will present the results of our studies to date. I shall first show how conventional theories which are documented in many texts on servomechanisms can be used to design a system. These theories show that an on-off feedback control system can give the desired characteristics except that a chatter frequency must exist in the system. The theory will then be applied to a normal acceleration type autopilot and methods will be presented for predicting the chatter frequencies and amplitudes. Some results and problem areas with possible solutions will then be presented.

The theory by which one can make an adaptive autopilot is best understood with reference to the block diagram shown on the first slide. The aircraft is represented by G(s) which has variable characteristics, N(s) is a network or filter, H(s) is a feedback transfer function produced either by instruments or a network, and K is a gain constant. The ideal invariant transfer function which we would like the system to have, that is our model, is represented by M(s). The arrangement in the upper diagram is based on the concept of applying the same input to the aircraft system and the model and using the error between desired and actual outputs as a corrective feedback signal. We have not concerned ourselves with what the characteristics of the model should be but rather with how to make the system behave in the desired fashion. The principles involved can best be understood by transforming the system to the single loop equivalent in the lower diagram. Now we have a typical closed loop autopilot system preceded by a network. If H(s) is invariant with flight conditions, it is clear that in order to produce a transfer function from R to C which is independent of changes in G(s), the closed loop portion of the system must also be independent of G(s). Therefore, the problem of designing an adaptive autopilot is reduced to one of making the closed loop system response invariant over the flight envelope. Since this is the case, it is simpler to turn our attention strictly to the closed loop portion, and the next slide is more suitable for this purpose.

There are two possible approaches to keeping the transfer function of this closed loop system invariant with changes in G(s). One possibility is to devise means for measuring the characteristics of G(s). These measurements can then be used to adjust the network in a fashion that will compensate for changes in

G(s). The other approach depends upon the fact that closed loop systems inherently tend to be insensitive to changes in the forward loop parameters. The reason for this insensitivity can be shown by considering the closed loop transfer function below the diagram. As K becomes very large the transfer function approaches the reciprocal of H(s) and hence becomes invariant. The high gain alone is not sufficient to insure a desirable system because the system must also be stable. It will now be shown how linear methods of analysis can be used to insure stability.

We have chosen as our example system the normal acceleration autopilot shown in the next slide. The transfer function representing the aircraft has a natural frequency ω_a, a damping ratio ζ_a, and an aerodynamic gain K_a. This representation is reasonably valid for tail controlled aircraft. The transfer functions of the measuring instruments used to obtain the feedback have a second order denominator of natural frequency ω_i and damping ratio ζ_i and a second order numerator of natural frequency ω_o and damping ratio ζ_o. The limiter accounts for the rate limit of the servo which is assumed to be a pure integration. This integration is desirable to provide the system with a steady state gain of unity. The system is not practically realizable because the higher order dynamics of the servo have been neglected, but it is sufficiently complex for the development of theory which will apply to the realizable case.

Now let us consider the problem from the standpoint of stability as we try to make the gain high. We will use the root locus for this purpose. The pole at the origin represents the servo and the high frequency complex poles are those of the instruments. The zeros are formed by the numerator of the instrumentation transfer function. The aircraft poles are subject to motion dependent on flight conditions and might be anywhere within the shaded area. Irrespective of the location of the aircraft poles within the shaded area, the roots will move toward the zeros as the gain is increased. It can be seen that the problem associated with the high gain is the fact that loci from the instrumentation poles will cross the imaginary axis. Linear analysis indicates the system will become unstable if the gain is made high.

If the crossover gain is high enough so that the aircraft poles are near the zeros for all flight conditions the problem is reduced to one of keeping the gain of the system at the desired level. One scheme of insuring this is to measure the damping of the high order mode and adjust the gain K to keep this damping constant. A second alternative, which we have chosen to use, takes advantage of the limiting in the system. We simply make the gain K very high and due to the limiting action the system oscillates with a limit cycle or chatter of frequency ω_c at an amplitude determined by the frequency response of the linear elements following the limiter. You will recognize that this is the principle used in the pitch rate control system developed at Minneapolis-Honeywell. The methods presented in this paper for estimating chatter frequency and amplitude are just as applicable to such a pitch rate autopilot as to the normal acceleration example discussed here.

The fact that limiting will restrict the oscillation to a limit cycle can be predicted by treating the limiter or on-off controller as a gain which is dependent on input signal level. This concept is useful in determining qualitative performance since it allows one to use linear methods of analysis. For this analysis we need not regard these roots as being fixed but rather they move back along the root loci dependent upon the amount of limiting. For example, if a large step were applied to the system, it can be seen that saturation would immediately occur and the response of the system would be governed by the open loop poles.

For this system, then, the output will have an approximately linear relationship to low frequency inputs and there will be a chatter superimposed on this low frequency response. If the chatter can be made small enough, this approach would seemingly produce an adaptive autopilot.

Next, I will show a means of predicting the chatter frequency by an approximate method which is less time consuming than conventional linear techniques. In most practical systems ω_c will be much larger than either the natural frequency of the aircraft or that of the zeros. A pole zero plot of such a system is shown in the next slide. Note that the phase shifts produced at ω_c by the zeros and aircraft poles are nearly equal and we may assume for purposes of determining ω_c that these poles and zeros cancel each other. Thus, in the practical system the chatter frequency is determined primarily by the dynamic characteristics of the servo and instruments. Also we can see that the higher the chatter frequency, hence the more desirable the system, the more valid this approximation becomes.

If the zeros and aircraft poles are eliminated there remain a pole at the origin and several higher order poles. The pole at the origin produces a phase shift of 90° at every point on the imaginary axis so that at ω_c the combined phase shifts of the higher order poles is also 90°. We can use geometric relations to find the tangent of this combined phase angle and then ω_c can be found by equating the denominator to zero. For the sixth order example shown, this means that if vectors from the two complex instrument poles and the one real pole are drawn to ω_c, then the sum of the angles of these vectors with the real axis must equal 90°. The double angle tangent formula can be used to find the tangent of the combined phase angle. The results of several such calculations are shown in the table. The ω's and the ζ's, and P_1 represent dynamics of the instruments or higher frequency characteristics of the servo. The seventh order case results in a quadratic equation in ω_c^2, but we know that 90° phase shift occurs at the lower frequency (270° being the higher one). Realistic values of ζ_1 and ζ_2 in this case give a maximum ω_c of about $0.6\sqrt{\omega_1\omega_2}$. Considering only an even number of higher order poles it can be shown that ω_c will always be a multiple of the geometric mean of their frequencies and, of course, it must be lower than the lowest frequency present. Once the chatter frequency is known the amplitude can be quite readily predicted by describing function techniques which will be shown later.

These methods of prediction were applied to the fifth order normal acceleration system taken as an example earlier and the results were checked by analog computer simulation. The aerodynamics used were those of a hypothetical missile chosen to be representative of high performance air-to-air missiles. The variations in parameters over the flight envelope were about 7 to 1 in natural frequency, 80 to 1 in aerodynamic gain and the damping ratio varied from 0 to 0.3. Three flight conditions were chosen, the two extremes of the flight envelope and an intermediate case. The gain constant was incorporated into the limiter and a very high open loop gain was obtained by removing the feedback from the limited computer amplifier. Different chatter frequencies were produced by changing the instrumentation denominator.

The frequency predicted by the approximate method is plotted as a function of the exact frequency computed by the root locus method on the next slide. Note that with these approximate formulas we can get a reasonably close estimate of the exact chatter frequency. For the example chosen the predicted frequency was always lower than the exact one.

The chatter amplitude was predicted using the frequencies measured on the computer. The method of prediction and the comparison of predicted amplitudes with the measured amplitudes are shown on the next slide. The prediction method is to consider the output of the limiter as a square wave of amplitude equal to the limit level. The input to the aircraft and servo can be approximated by the fundamental component of this square wave which is a sine wave of amplitude $4/\pi$ times the limit level. The chatter amplitude C_c is simply this amplitude times the magnitude of $G(j\omega_c)$.

Since the amplitude prediction is more accurate than the frequency prediction the best design procedure would appear to be first to determine the chatter frequency which would give a tolerable amplitude and then design for that frequency. It should be noted here that actual hardware will have phase shifts due to nonlinearities that cannot be readily accounted for in this analysis. Therefore, it would be advisable to make the design chatter frequency somewhat higher than one which would give an acceptable amplitude.

We have a means for predicting chatter frequency and amplitude rapidly and the question now arises as to whether the low frequency characteristics of the practical system will be as desired. For the answer to this question we again turn to the analog computer and the results are presented in the next slide. The system used is the one taken as an example earlier except that instrument damping is greater than critical, and the instrument frequency was chosen to give a fairly low chatter frequency. The root loci are shown below the corresponding step responses and these responses are for the two extremes of the flight envelope. The one on the left is for an aircraft natural frequency of 3.6 radians/sec, a gain of 0.11 and zero damping. The instrument frequency is about 16 times that of the aircraft and results in the chatter frequency being high compared to the aircraft natural frequency. Thus, the chatter amplitude is well filtered and the step response corresponds quite closely to the zero positions, that is, to the desired response. The response on the right is for an aircraft natural frequency of 25 radians/sec, a gain of 8.8 and 0.3 damping ratio. The instrument frequency here is only a little more than twice that of the aircraft and the chatter frequency is less than three times the aircraft frequency. This low ratio of chatter frequency to aircraft frequency coupled with the high gain of the aircraft results in a very high chatter amplitude. In addition, one can see that instead of following the desired response, shown by the dashed line, the step response is very heavily damped. This means that the roots are quite far back on the loci with the dominant one probably near the origin. There are then two important questions to be answered. These are: First, for those examples where the chatter frequency cannot be made high compared to the aircraft frequency, what can be done to reduce the amplitude to an acceptable value? Second, what technique could be used to predict the actual pole locations for small signal inputs in the presence of chatter?

Considering the question of how to reduce the chatter amplitude, two methods are shown on the next slide. The upper system measures the chatter amplitude by means of a high pass network, rectifier, and filter. The output of the filter adjusts the limit level through a dead zone. If the chatter amplitude exceeds the threshold of the dead zone, the limit level is reduced. This results in a proportional decrease in chatter amplitude. The lower system consists of a lead network before the limiter and a network with the reciprocal transfer function following the limiter. The chatter frequency is not altered since the phase shifts of these networks cancel. Since the amplitude at the output of the limiter is constant, attenuation is provided by the lag network. The step responses for the high chatter amplitude case from the previous slide are shown to the right of each system and the desired response is again shown in dashed lines. Both systems are effective in reducing the amplitude and no attempt has been made to determine which is better.

The second question, that is, why the step response is so heavily damped, is best answered with reference to the next slide. As was mentioned a high gain limiter, or on-off controller, acts for sinusoidal signals as a variable gain which is dependent on the ratio of the input magnitude to the limit level. The graph on the left is a normalized curve illustrating how the gain varies as a function of the ratio of input amplitude to limit level. The solid curve is for a single low frequency input to the limiter. Note that since the gain is essentially infinite for very low inputs the curve goes to infinity. It is shown in Tsien's " Engineering Cybernetics" that the addition of a high frequency sinusoidal dither signal causes the low frequency average characteristic to behave as indicated by the dotted line. The characteristics of this dotted line are determined by the dither amplitude which for this graph was 0.1 the limit level. Thus, the addition of dither to a limiter gives, for low frequency inputs, an equivalent gain reduction which can be computed if the dither amplitude is known. For our example, the dither was assumed to be the chatter frequency where the amplitude at the input to the limiter could be computed using linear techniques. By using the equivalent low gain obtained to find the pole positions on the root loci the low frequency characteristic of the step response could be predicted. The results of this prediction are shown in the step response on the right and very good correspondence can be noted.

In summary, I would like to review the following points. First, it has been shown that one way of designing an adaptive autopilot is by the principle of using high gain. There are many ways of insuring the gain be high, and we exploited only one of these ways, the use of a high gain limiter. Second, we have shown that for a high gain saturated control system suitable for adaptive autopilot application, a limit cycle or chatter must exist. Techniques have been shown for analyzing such a system by which one can predict the chatter frequency, chatter amplitude, and the low frequency response in the presence of chatter. Third, the application chosen was for a normal acceleration type autopilot; however, there is no reason why the same type system, that is a high gain saturated control system, could not be used in any of the other modes. And last, the chatter frequency has been shown to be dependent primarily on instrument and servo characteristics. In order for this system to function satisfactorily, the chatter frequency must be quite high compared to the aircraft natural frequency. The example shows it is desirable to have the ratio of instrument to aircraft frequency in excess of 10 in order that the chatter frequency be high. This suggests that if conventional servos and instruments are to be used, a reduction in static margin of the aircraft which reduces its natural frequency would be helpful.

REFERENCES

1. Chestnut, Harold and Mayer, Robert W., " Servomechanisms and Regulating System Design" , Vol I, General Electric Co., 1951. John Wiley and Sons, Inc., and Chapman and Hall, Ltd., London.

2. Truxal, John G., " Automatic Feedback Control System Synthesis, McGraw-Hill Book Co., Inc., 1955.

3. " A Study to Determine an Automatic Flight Control Configuration to Provide a Stability Augmentation Capability for a High-Performance Supersonic Aircraft" , WADC Technical Report 57-349 (a series of four progress reports covering work done from March 1, 1957 through February 28, 1958).

4. Tsien, H. S., " Engineering Cybernetics" , McGraw-Hill Book Co., Inc., 1954.

ADAPTIVE SYSTEM WITH MODEL

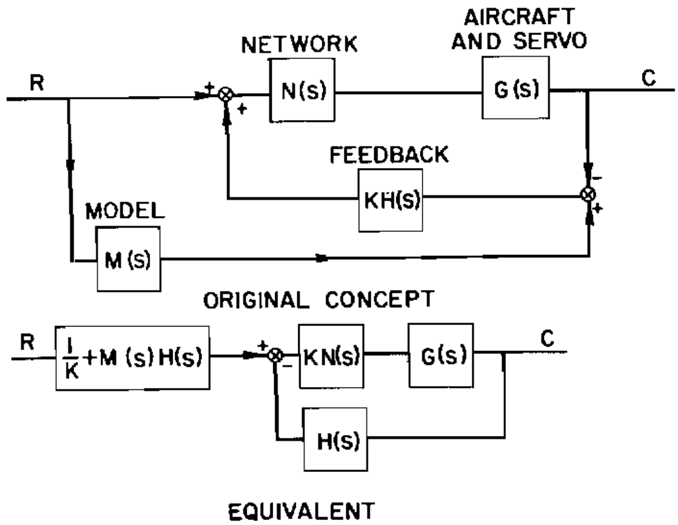
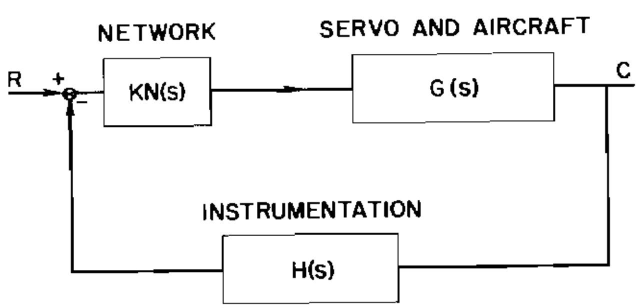


Fig 1

BLOCK DIAGRAM OF TYPICAL AUTOPILOT



$$\frac{C}{R} = \frac{KN(s)\,G(s)}{1 + KN(s)\,G(s)\,H(s)} \approx \frac{1}{H(s)}$$

Fig 2

FIFTH ORDER NORMAL ACCELERATION AUTOPILOT

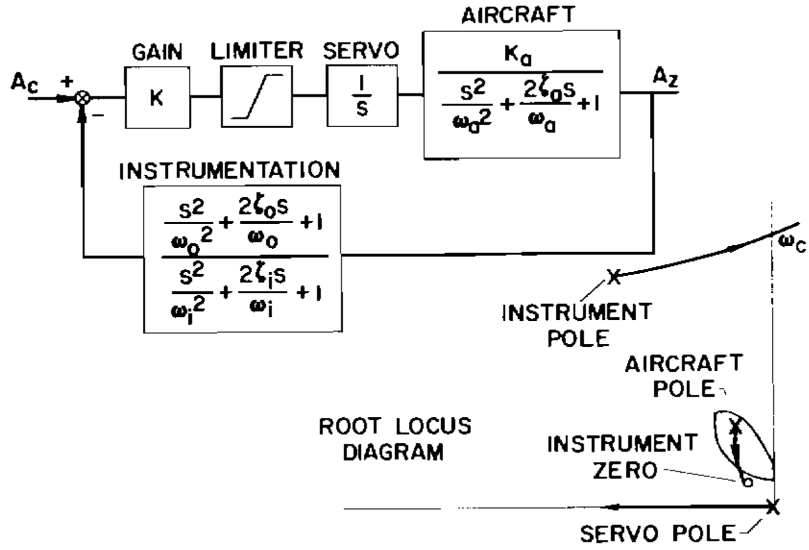
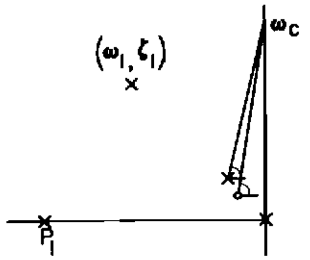


Fig 3

APPROXIMATE METHOD FOR DETERMINING CHATTER FREQUENCY



POLE-ZERO PLOT FOR SIXTH ORDER EXAMPLE

SYSTEM ORDER	CHATTER FREQUENCY FOR TWO ZEROS
4	$\omega_c = \infty$
5	$\omega_c = \omega_1$
6	$\omega_c = \sqrt{\dfrac{P_1 \omega_1^2}{P_1 + 2\zeta_1 \omega_1}}$
7	$\omega_c = \sqrt{\dfrac{\omega_1 \omega_2 (b - \sqrt{b^2 - 4})}{2}}$

$$b = \left(\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} + 4\zeta_1\zeta_2\right)$$

Fig 4

FREQUENCY PREDICTION FOR FIFTH ORDER SYSTEM

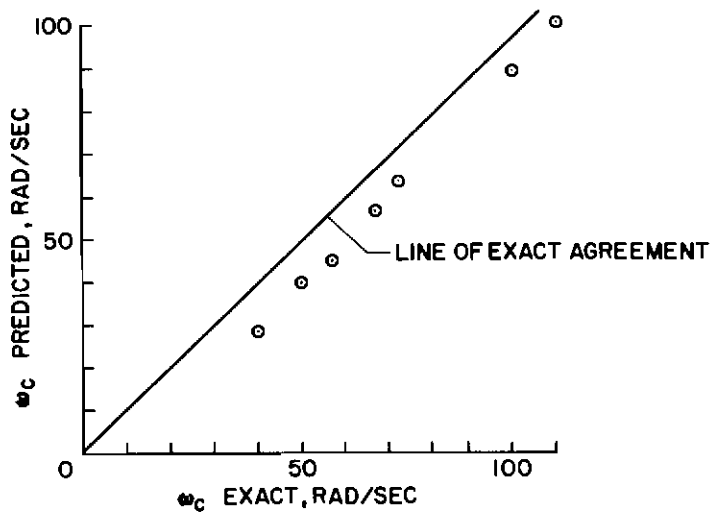


Fig 5

AMPLITUDE PREDICTION FOR FIFTH ORDER SYSTEM

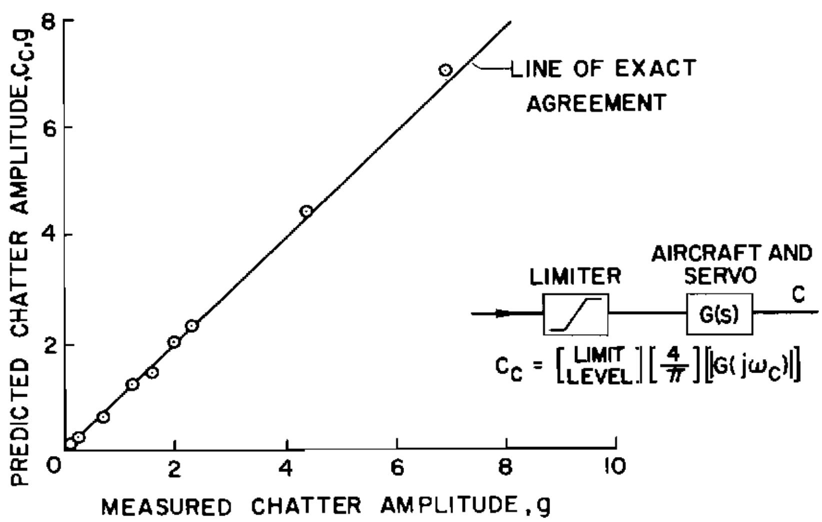


Fig 6

FIFTH ORDER SYSTEM FOR TWO FLIGHT CONDITIONS

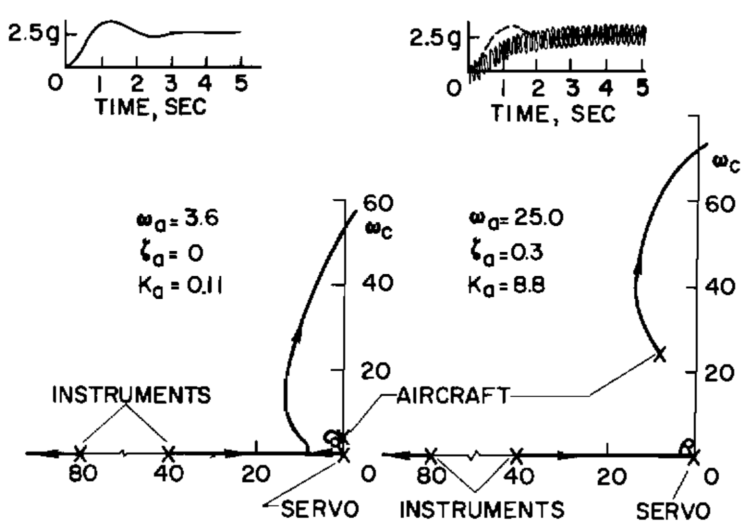


Fig 7

TWO METHODS FOR REDUCING CHATTER AMPLITUDE

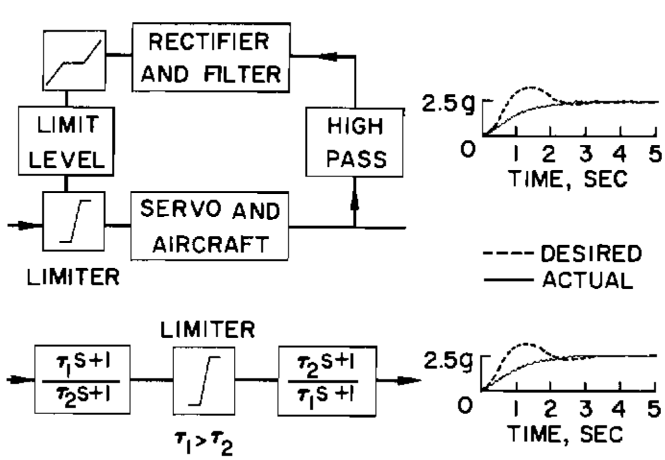


Fig 8

LOW FREQUENCY CHARACTERISTICS OF A HIGH GAIN LIMITER

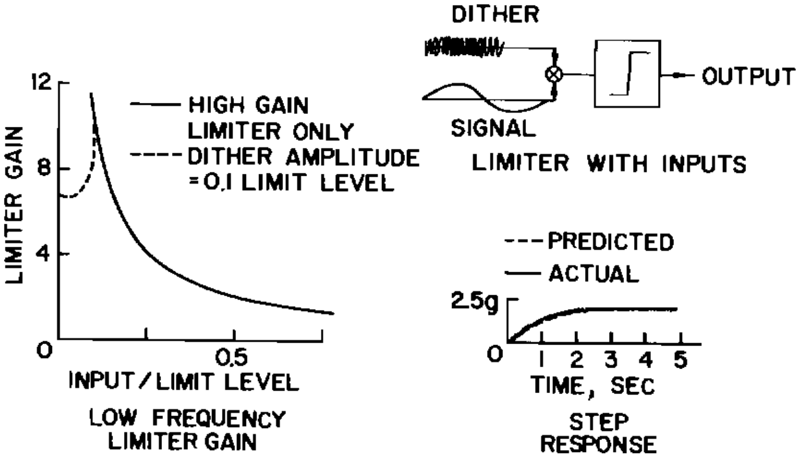


Fig 9