STRUCTURAL OPTIMIZATION VIA STEEPEST DESCENT AND INTERACTIVE COMPUTATION

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In this paper a hybrid method of optimal structural design is developed and applied, using a steepest descent optimization technique in an interactive mode of computing. A method of computing design sensitivity is developed as part of a steepest descent optimization method. A technique is presented for utilization of this data by an experienced designer, interacting with the iterative computer algorithm. The designer is allowed to alter the structural configuration to seek a global, rather than just a local, optimum. Two examples are solved with a number of loading conditions and design constraints to illustrate the flexibility of the technique. Results for these problems are of interest in their own right.

Introduction

Structural optimization methods developed during the past decade generally fall into one of two categories. The first consists of a number of optimization techniques which seek to determine an optimum design, within a well defined mathematical structure, by purely mathematical techniques. The second approach consists of providing the designer with an interactive computing tool, such as computer graphics, with which he can try nominal designs, get rapid analysis feedback, and alter his initial design based on his knowledge of structural behavior. Both methods have been used with varying degress of success on a variety of design problems. In general, the first approach has been used on smaller scale structures with well defined optimality criteria, such as minimum weight or maximum stiffness. The second approach has been used to aid designers in large scale structural design problems, primarily airframe design, such as the Air Force C-5 transport aircraft.

The possibility of utilizing a combination of these two methods for structural design has been the subject of a recent paper [1]. The present paper presents the specifics of application of an optimization technique with, designer interaction. This hybrid approach is appealing from a number of points of view. First, the problem of topological design, i.e., determination of optimum structural configuration, has been addressed with very limited success from an analytical point of view [Kato]. Topological design, in practice, is done by experienced structural designers, occasionally with the aid of interactive computation. Combined analytical and interactive computing methods appear to be essential for this important class of problems. A second problem area arises due to the difficulty in formulating a single optimality condition and mathematically precise design constraints. Often, conflicting design constraints and objectives arise during design which require experienced judgment and defy

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apriori mathematical formulation. Such problems appear to require an interactive computing capability but should profit from analytical methods which are used in automated structural optimization.

In this paper, an iterative steepest descent optimal design method is used interactively to develop a hybrid method which allows the experienced designer to make key design decisions with the aid of analytical tools which have proved to be quite successful in automated structural optimization. Several structural optimization problems are solved to illustrate the applicability of the method. Prior to developing the hybrid method referred to above, a brief review of automated design optimization methods and interactive computing techniques will be presented to establish the rationale for the methods chosen.

Many numerical methods have been applied to solve structural optimization problems. The method of linear programming has been a very popular tool with many researchers in the area of optimal structural design. Moses [2], Romstad and Wang [3], Grierson and Cohn [4] and many others have used Kelley's [5] cutting plane method for nonlinear programming. In a recent paper, Johnson and Brotten [6] incorporated the force method of structural analysis with the linear programming technique to solve some redundant trusses. They have considered only stress constraints in their problems. Venkayya, et. al. [7] have used a scheme based on the study of distribution of strain energy in the structure to optimize trusses. Structures subject to stress and displacement constraints are considered. A simple recurrence relationship is used to resize the members when only stress constraints are present. However, in the presence of displacement constraints, the problem is handled in two stages.

In recent years, many authors have used SUMT, developed by Fiacco and McCormick [8], in optimal structural design. Kavlie and Moe [9, 10] have used an interior penalty technique to solve some frame and grillage foundation problems and have reported good success with the method. They have also extended the technique so that the initial designs may be infeasible. In order to compare linear programming and SUMT, LaPay and Goble [11] solved some truss problems. They have found the nonlinear formulation to be numerically superior and more efficient. A slightly different formulation of SUMT is suggested by Schmit and Fox [12]. The unconstrained function Ψ, that is constructed contains, among other quantities, an estimate of the optimum weight of the structure. The function Ψ is so constructed that, when $\Psi = 0$, all constraints and equations of equilibrium and compatability are satisfied.

The methods cited above are based primarily on mathematical optimization ideas which yield a minimizing sequence. These techniques, however, do not give the designer much useful design information at intermediate steps which would allow him to interact with the computer algorithm. There are techniques, however, which do give design trend information at intermediate steps of the computational algorithm. Most of these methods are based on gradient projection or steepest-descent ideas.

Schmit and his co-workers [13] have used a steepest-descent alternate-step procedure for many years. Gellatly and Gallagher [14] have applied this technique and have given it a theoretical basis. Felton and Hofmeister [15] and Ridha and Wright [16] have also used it to solve certain structural design problems. A variation of the method of steepest-descent is Zoutendijk's method of usable-feasible directions. In this method, the steepest-descent mode is also followed until some constraints are encountered, at which point the direction of travel lies between the gradients of the objective and the constraint functions. Fox and Kapoor [17], Pope [18], and Moses and Onoda [19] used this technique in conjunction with linear programming to solve some structural design problems. They have found the method to be quite efficient.

Best [20] was probably the first to suggest the use of Rosen's gradient projection method [21] to solve optimal structural design problems. Brown and Aug [22], and Seaburg and Salmon [23] have used this method quite successfully. Recently, a method of constrained steepest-descent with state equations has been used by Haug [24, 25]. This method is a variation of Rosen's gradient projection method and uses Kuhn-Tucker necessary conditions along with Lagrange multipliers. Bartel and Rim [26] have compared this method with SUMT for two and three member space frames and have found the method to be superior to SUMT in these cases.

The second element of the interactive optimal design technique is the mode of designer-computer interaction. There are several types of interactive equipment available to the designer today, the most promising of which appears to be interactive graphics. For discussions of graphics systems previously considered in structural design the reader is referred to [1, 27, 28]. The interactive system intended for use in this study consisted of a PDP-8/I computer and a Tektronic 4001A graphics terminal located remotely to an IBM 360-65 computer which operates in an interactive mode. Due to a computer center decision to delay implementation of large scale, interactive system, for this study had to be simulated. Instructions were prepared and computations were run in the batch mode. Output data was then displayed and analyzed just as it would be in the interactive mode and instructions for recomputation were given by the designer and the process repeated. The delay in designer interaction is felt to degrade performance somewhat over true interactive computing since the designer tends to forget pertinent detailed data during the time delay. For this reason, the results of this study should be a conservative estimate of the designers performance in a truly interactive mode.

Design Sensitivity Analysis and Optimization

A first step in providing information for structural design and optimization is mathematical formulation of the design problem and determination of the effect of design changes on structural performance. In this paper, a finite element formulation is utilized which allows design variables to be member size, material parameters, or structure dimensions. A distinction is made between design variables $b = [b_1, \ldots, b_m]^T$ and state variables $q = q_1, \ldots, q_n]^T$ which are generalized displacements. To allow treatment of buckling and natural frequency considerations, an additional state variable $v = [v_1, \ldots, v_p]^T$ is defined along with the associated eigenvalue ζ.

As a simple example to illustrate the notation and problem formulation, consider the three bar truss of Figure 1. The design variables b_1, b_2, and b_3 are cross-sectional areas of the members. The displacements q_1 and q_2 of the free joint constitute the state variables. If a frequency constraint is involved then v_1 and v_2 would be the same two displacements which specify the mode of vibration. This simple problem will be used later as an illustrative example in design for minimum weight with constraints on stress, displacement, buckling, and natural frequency.

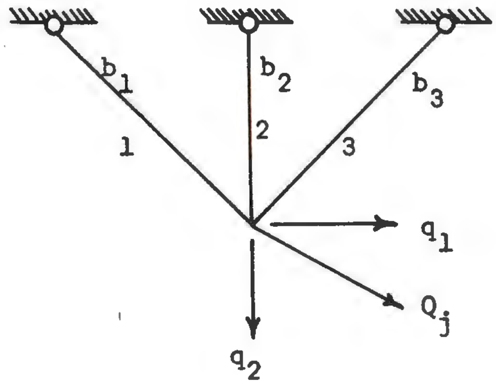

Figure 1. Three Bar Truss

In this section a steepest-descent technique is outlined for a broad class of optimal structural design problems. This technique has been used successfully in the batch, or noninteractive, mode and provides useful data for the experienced designer when it is employed in the interactive mode.

The mathematical model of the problem to be solved is to find b, q, v, and ζ which minimize a cost functional $J(b,q,\zeta)$, satisfy the equations

$$A(b)q = Q(b) \tag{1}$$

$$K(b)v = \zeta M(b)v, \tag{2}$$

and satisfy the constraints

$$\phi(b,q,\zeta) \leq 0. \tag{3}$$

In this formulation, (1) represents the structural response to a load Q. The matrix A(b) plays the role of stiffness matrix whose elements depend on member size b. Likewise, (2) is the eigenvalue problem for natural frequency, where K(b) is a stiffness matrix and M(b) is a mass matrix. The inequality constraint (3) represents stress, displacement, eigenvalue, and member size constraints, where $\phi(b,q,\zeta)$ is a vector of functions and the inequality applies to each component of the vector.

Since analytic methods of finding a solution to the above problem are unwieldy, a numerical procedure, developed in [25] will be summarized here. In the derivation of this procedure, an initial estimate of the design variable vector $b = b^{(0)}$ is made and a small change, δb, in the design variable vector, b is sought such that $b^{(1)} = b^{(0)} + \delta b$ is an improved design in some sense. The procedure is repeated with $b^{(1)}$ as the estimated design point until a convergence criterion is satisfied. In seeking the small change, δb the equations of the above problem are approximated and the theory of nonlinear programming is used to obtain an explicit expression for δb. For example, the change in $J(b,q,\zeta)$ due to changes δb, δq, and $\delta\zeta$ is approximated by

$$\delta J = \frac{\partial J}{\partial b}\,\delta b + \frac{\partial J}{\partial q}\,\delta q + \frac{\partial J}{\partial \zeta}\,\delta\zeta$$

where matrix calculus notation is used freely. Note that in the linearized form of the objective function, $J(b,q,\zeta)$ and the constraint function $\phi(b,q,\zeta)$, small changes δq, δv and $\delta\zeta$ in q, v, and ζ respectively, will be appearing. These variables are eliminated from δJ and $\delta\phi$ by the use of linearized versions of the state equations. From this procedure one obtains [24, 25]:

$$\delta J = \ell^{J^T}\delta b \tag{4}$$

$$\delta\tilde{\phi} = \Lambda^T \delta b, \tag{5}$$

where

$$\ell^J = \frac{\partial J}{\partial b}^T(b,q,\zeta) + \left[\frac{\partial Q(b)}{\partial b} - \frac{\partial}{\partial b}\{A(b)q\}\right]^T \lambda^J$$

$$+ \frac{\frac{\partial J(b,q,\zeta)}{\partial \zeta}}{v^T M(b) v}\left[\frac{\partial}{\partial b}\{K(b)v\} - \zeta\,\frac{\partial}{\partial b}\{M(b)v\}\right]^T v,$$

$$\Lambda = \frac{\partial \tilde{\phi}^T}{\partial b}(b,q,\zeta) + [\frac{\partial Q(b)}{\partial b} - \frac{\partial}{\partial b}\{A(b)q\}]^T \lambda^{\tilde{\phi}}$$

$$+ \frac{1}{v^T M(b) v} [\frac{\partial}{\partial b}\{K(b)v\} - \zeta \frac{\partial}{\partial b}\{M(b)v\}]^T v \frac{\partial \tilde{\phi}^T}{\partial \zeta},$$

$$\tilde{\phi} = [\phi_i(b,q,\zeta); \text{ for each } i \text{ such that } \phi_i(b,q,\zeta) \geq 0], \tag{6}$$

$$\Delta\tilde{\phi}_i = - \tilde{\phi}_i(b,q,\zeta), \tag{7}$$

and λ^J and $\lambda^{\tilde{\phi}}$ are solution of

$$A^T(b)\, \lambda^J = \frac{\partial J^T}{\partial q}(b,q,\zeta) \tag{8}$$

$$A^T(b)\, \lambda^{\tilde{\phi}} = \frac{\partial \tilde{\phi}}{\partial q}(b,q,\zeta). \tag{9}$$

In the above, $\tilde{\phi}$ consists simply of those elements of ϕ which are zero or positive at the nominal design. The vector $\Delta\tilde{\phi}$ defined by (7) is the desired change in the violated constraints $\tilde{\phi}$. To verify that one obtains (4) and (5) by the above calculations, the equations (1) and (2) may be linearized about the nominal design and the resulting equations along with (8) and (9) used to eliminate explicit dependence on δq, δv, and $\delta\zeta$. These calculations are carried out in detail in [25].

To assure that δb is small, as required by the linear approximations, introduce a quadratic constraint on δb as follows:

$$\delta b^T W \delta b = \xi^2 \tag{10}$$

where ξ is small and W is a positive definite weighting matrix, chosen by the designer.

The design improvement problem reduces to finding δb to minimize δJ of (4) subject to the constraints (10) and

$$\delta\tilde{\phi} = \Lambda^T \delta b \leq \Delta\tilde{\phi} .$$

The Kuhn-Tucker necessary conditions for this problem yield:

$$\delta b = - \frac{1}{2\nu} W^{-1} (\ell^J + \Lambda\mu),$$

where μ is a Lagrange multiplier vector and ν is a scalar multiplier. By the arguments explained in [25], the value of μ can be found from the following equation:

$$B\mu = -[2\nu\Delta\tilde{\phi} + \Lambda^T W^{-1} \ell^J] , \tag{11}$$

where

$$B = \Lambda^T W^{-1} \Lambda .$$

Substitution of μ in the expression for δb yields the following equation

$$\delta b = -\frac{1}{2\nu} W^{-1}[I - \Lambda B^{-1}\Lambda^T W^{-1}]\ell^J + W^{-1}\Lambda B^{-1}\Delta\tilde{\phi}$$

Define $\eta = \frac{1}{2\nu}$

$$\delta b^1 = W^{-1}[I - \Lambda B^{-1}\Lambda^T W^{-1}]\ell^J \tag{12}$$

and

$$\delta b^2 = W^{-1}\Lambda B^{-1}\Delta\tilde{\phi}. \tag{13}$$

Thus, δb may be written as

$$\delta b = -\eta\delta b^1 + \delta b^2, \tag{14}$$

where η is an arbitrarily small quantity, interpreted as a step size, which must be chosen by the designer. The following scheme for choosing η has been tried and has worked well. When all the constraints are satisfied, so that $\Delta\phi = 0$, η is chosen to yield a few percent reduction in J. One first chooses the desired change $\Delta J < 0$ in J. Then assuming $\Delta\tilde{\phi} = 0$, ΔJ is given by $\Delta J = -\eta\ell^{J^T}\delta b^1$, from which one obtains

$$\eta = -\Delta J/[\ell^{J^T}\delta b^1] . \tag{15}$$

Once η is chosen δb is determined by (14) and an improved design is obtained by adding δb to the preceding design estimate. The procedure is then repeated to form an iterative optimal design algorithm. Details of the algorithm are given in [25]. Computational results may be found in [25] and [29].

In order to implement the computational algorithm, two principal matrices ℓ^J and Λ must be assembled. These matrices are composed of various matrices such as,

$$\frac{\partial}{\partial b}[A(b)q],\ \frac{\partial}{\partial b}[K(b)v],\ \frac{\partial}{\partial b}[M(b)v],\ \frac{\partial\tilde{\phi}}{\partial q},\ \frac{\partial\tilde{\phi}}{\partial b},\ \frac{\partial\tilde{\phi}}{\partial\zeta}, \text{ etc.}$$

The matrices, $\frac{\partial}{\partial b}[A(b)q]$, $\frac{\partial}{\partial b}[K(b)v]$, and $\frac{\partial}{\partial b}[M(b)v]$ can be computed explicitly from the state equations of the problem. The displacement method of structural analysis is used to obtain the state equations of the problem and the various derivative matrices are computed from them. The matrices such as

$$\frac{\partial\tilde{\phi}}{\partial q},\ \frac{\partial\tilde{\phi}}{\partial b}$$

etc. can also be computed explicitly by considering the various types of constraints one by one. The procedure of computing these matrices can be automated. For details of these computations the reader is referred to [29].

Steepest-Descent Computational Algorithm

The above procedure of successively improving the best available design can be put in a computational form as follows:

Step 1. Obtain the best available engineering estimate of the optimum design variable vector $b^{(j)}$ and solve for $q^{(j)}$ from (1). Also compute the lowest eigenvalue $\zeta^{(j)}$ and the corresponding eigenvector $v^{(j)}$ from (2).

Step 2. Check the constraints (3) and form the constraint function $\tilde{\phi}$ of (6). Also choose the constraint error correction vector, $\Delta\tilde{\phi}$ of (7).

Step 3. Compute various matrices such as, $\frac{\partial\tilde{\phi}}{\partial q}$, $\frac{\partial\tilde{\phi}}{\partial b}$, $\frac{\partial\tilde{\phi}}{\partial \zeta}$, $\frac{\partial}{\partial b}[K(b)v]$, etc.

Step 4. Solve for λ^J and for $\lambda^{\tilde{\phi}}$ from (8) and (9).

Step 5. Assemble matrices ℓ^J and Λ of (4) and (5), respectively.

Step 6. Choose η from (15) and calculate the Lagrange multiplier vector μ from (11).

Step 7. Check the algebraic sign of each component of μ. If some components of μ are negative, remove the corresponding columns from Λ matrix. Also delete the corresponding elements from the vector $\Delta\tilde{\phi}$ and return to Step 6.

Step 8. Compute δb from (14) and put

$$b^{(j+1)} = b^{(j)} + \delta b$$

Step 9. Make a check for convergence. If $||\delta b^1||$ is sufficiently small, all constraints are satisfied and no further reduction in cost function is possible, terminate the process or return to Step 1 with $b^{(j+1)}$ as the best available estimate of the optimum design.

Interactive Structural Design Using Sensitivity Data

The steepest descent optimization method outlined in the preceding section has been used to solve a number of relatively large scale structural optimization problems with good success [25,29]. All these problems, however, have been well formulated mathematically and have involved structures with a predetermined form. Difficulties have occurred when certain structural elements tend toward zero cross section. Further, no universal method has been found to determine the best step-size η in the optimization algorithm. These and other inherent difficulties in automated optimization lead one to consider interjecting an experienced designer into the computational, optimization algorithm. The result is a hybrid structural optimization technique.

Reconsidering the design improvement step of the optimization algorithm, given by (14), one might draw a vector picture in design space, as is depicted in Figure 2. Here, $-\eta\delta b^1$ is the direction which will yield the greatest reduction in J subject to the required constraints and δb^2 is the design change required to give the desired constraint error correction. While useful in this form, there is a better display of this data for use by the experienced structural designer. The scalar components of $-\delta b^1$ and δb^2 tell the designer whether he should increase or decrease his individual design variables to obtain desirable changes in overall structural response. Further, relative importance of design variable changes is given. For this reason, δb^1 may be interpreted as a vector of design sensitivity coefficients which relate individual design parameter changes to overall structural characteristics. It is extremely important to note, at this point, that these sensitivity coefficients account for constraints

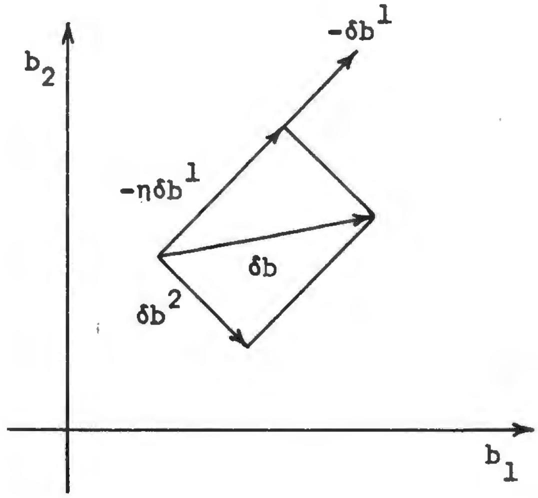

Figure 2. Vector Change in Design Space

implicitly. That is, the direction of change indicated in the design parameters will not cause significant violation in specified performance constraints such as stress limits, deflection limits, etc.

To illustrate these ideas, consider again the simple structural design problem in Figure 1. The cost function here is structural weight. If, for example, the stress in member 1 is at its allowable limit under one of the loads, then the indicated changes in design $(-\delta b_1^1, -\delta b_2^1, -\delta b_3^1)$ will not increase the stress in member 1. To make the design sensitivity data of maximum use to the designer, consider the graphical display in Figure 3. In this display, σ_i are the stresses in the various members. This display gives the experienced designer a clear picture of the manner in which he should change his design parameters to reduce total weight, subject to stress constraints. He can now choose the desired reduction ΔJ in weight and take the resulting design change δb given by (14). Or, if he wishes, he can input modified design changes through an iteractive computer terminal.

There are a number of other respects in which this mode of designer interaction with the computer algorithm is beneficial. First, it often happens in the automated use of the algorithm that oscillation of admissible designs occurs because too large a design improvement has been requested. Such oscillation can often be identified by the designer after only a few iterations and the step size can be reduced to prevent loss of computer time, which can be significant in large scale problems. Conversely, if an estimate quite far from the optimum is chosen to initiate the algorithm, it often happens that the designer chooses far too small a step size. The result is a very small improvement in the design which can be sensed by the designer and improved before excessive computation time is expended.

A second important benefit from designer interaction with the algorithm arises due to the occurrence of local minima and singularities in the analytical formulation of the design problem. The problem of local minima is illustrated by Figure 4. Virtually all optimization methods seek local optima and do not solve the global optimization problem. It is easy for an optimization technique to get hung up at point B and not get to point A, which is the global minima, so the designer must try different starting points to obtain the global solution. This is a very time consuming and indefinite technique with very few analytical aids to the designer. Part of the difficulty here arises because Figure 4 is the wrong display for the designer, in that it does not utilize his knowledge and experience with structures.

A much better approach for the designer is to look at a display such as Figure 3. He can use his experience to restart the optimization algorithm at a meaningful distribution of design variables which may be quite different from the design which resulted from previous calculations. His experience, thus, aids him in starting with different trial designs.

Perhaps even more important than trying various distributions of design variables, the designer can utilize the display of Figure 3 to change the configuration of the structure based on information he accumulates during iterative design and based on his experience. For example, he might try taking member number two out of the structure and optimize based on the modified configuration. Very often, significant gains are made in this manner. Precisely this behavior

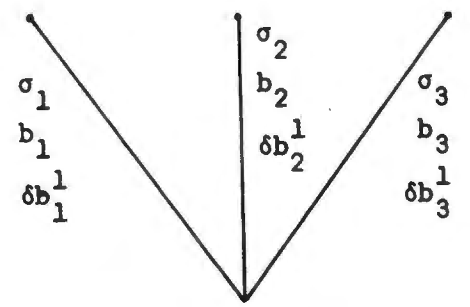

Figure 3. Display of Design Sensitivity Data

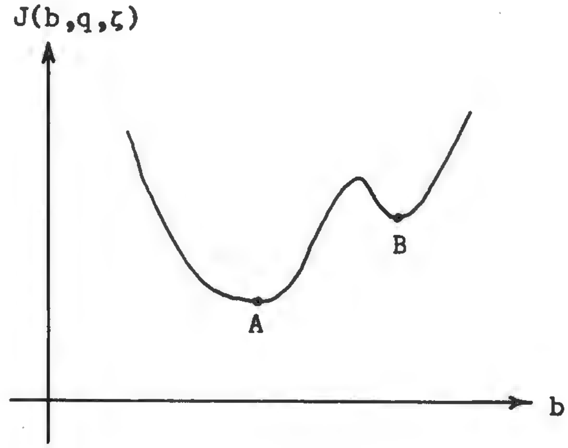

Figure 4. Local Optima

occurs in the three member truss being considered, as will be seen later when this problem is solved in detail.

There are actually compelling mathematical reasons for allowing the designer to make changes in configuration as outlined above. There are no general optimization methods, to date, which will remove a member during iterative design. The reason is that as a member cross section goes toward zero, as is required to remove a member, the equations of structural mechanics and stress constraints become singular. This sort of behavior is typical when the configuration of a system is changed and a different set of equations is required to describe the behavior. At the present time, allowing the designer to make changes in configuration appears to be the most feasible approach, which requires that he play an active role in the iterative optimization algorithm.

Example 1. A Three Member Truss

As an illustrative example of the technique presented above, an elementary optimal design problem will be solved under a number of loading conditions and a variety of constraints. The effect of designer-computer interaction on rate of convergence is examined as well as the effect of changing structural configuration. Results for a wide variety of loading and constraints are compared with solutions of similar problems solved in the literature.

Figure 5 shows the geometry and dimensions of the structure being considered. This structure has been studied by Schmit [30], Sved and Ginos [31] and Corcoran [32]. Three independent loading conditions are applied to the structure. These are as follows: 40K at 45°; 30K at 90°; 20K at 135°. The allowable stress level for members one and three is ± 5 KSI and for member two it is ± 20 KSI. The density of the material is taken as 0.10 pounds/cu. in., and Young's Modulus as 10^4 KSI. Starting from the feasible solution, $b_1 = 8.0$, $b_2 = 2.4$, $b_3 = 3.2$, Schmit [30] arrives at the solution $b_1 = 7.099$, $b_2 = 1.849$, $b_3 = 2.897$, for which $J = 15.986$ pounds. Sved and Ginos [31] have shown that this is only a local minima and by omitting member three, they obtained the solution as $b_1 = 8.5$, $b_2 = 1.5$ with $W = 12.812$ pounds. They have also shown that it is impossible to reach this minimum by an iterative optimization method unless member three is omitted from the calculations by the designer.

In the present work, considerable experimentation was done with this problem. Starting from a feasible point $b_1 = 10$, $b_2 = 5$, $b_3 = 5$, the solution obtained without interaction was $b_1 = 7.064$, $b_2 = 1.971$, $b_3 = 2.835$ and the minimum was $J = 15.97$ pounds. The variation of weight with respect to iteration number is shown by Curve 1, Figure 6. Next, by adjusting the step size in interactive computing, the solution was obtained in only five iterations. This is shown by Curve 2, Figure 6. It was observed that member two never reached its allowable stress level. As a second starting point, the area of member two was initially chosen to bring its stress to the allowable limit. The minimum reached in this case was the same as before (Curve 3, Figure 6). Another solution was obtained by starting for an infeasible point $b_1 = 5.0$, $b_2 = 1.5$, $b_3 = 0.10$. The solution in this case was $b_1 = 6.98$, $b_2 = 2.30$, $b_3 = 2.68$ with $J = 15.97$ (Curve 4, Figure 6).

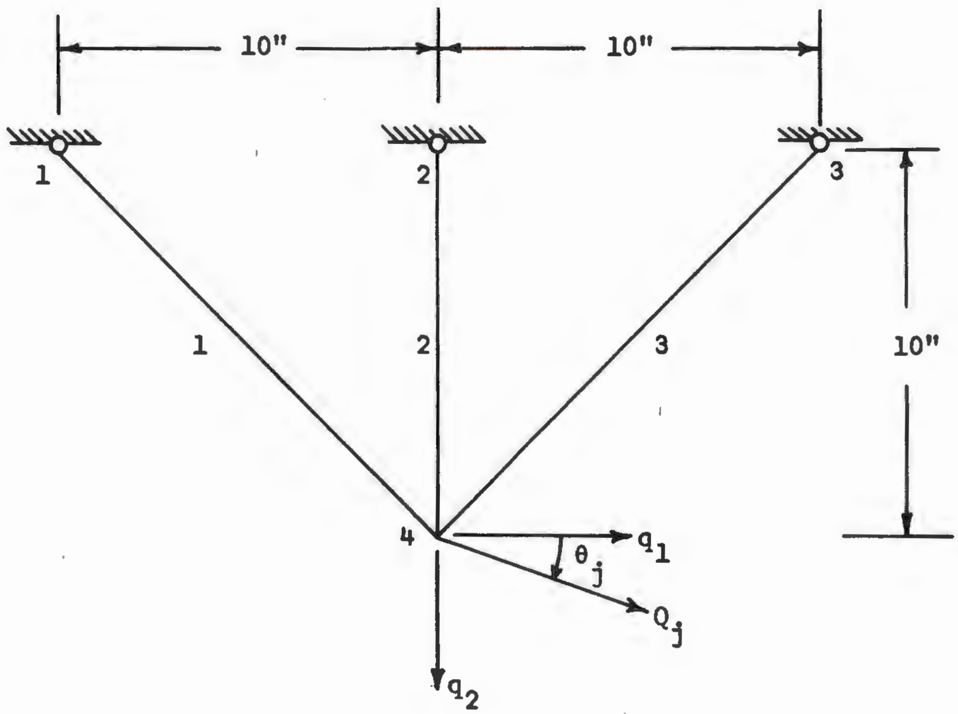

Figure 5 Three-Bar Truss

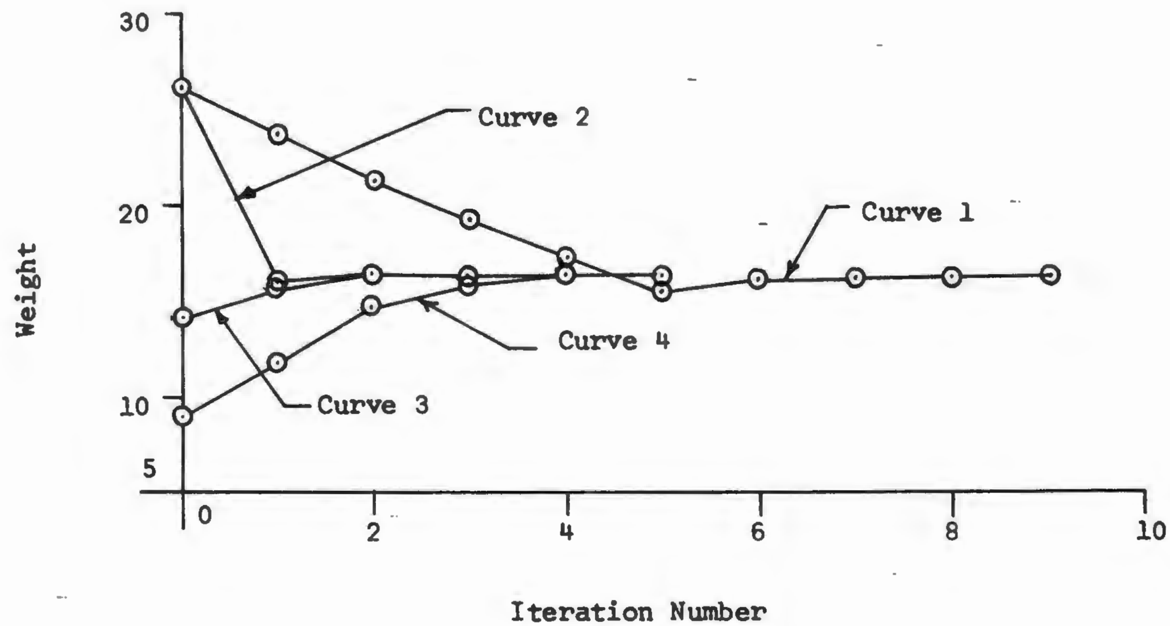

Figure 6. Weight Versus Iteration Curves for Three-Bar Truss (Stress Constraints)

Next, member three was omitted from the structure. Starting from a point $b_1 = 10$, $b_2 = 5$, the solution obtained was $b_1 = 8.0$, $b_2 = 1.5$ with $J = 12.812$ pounds (Curve 1, Figure 7) which is same as reported in [31]. At this point an interesting observation was made. The maximum horizontal and the vertical deflections of node four were as follows: with three bars, $q_1 = 0.689 \times 10^{-2}$ $q_2 = 0.595 \times 10^{-2}$ in.; with two bars, $q_1 = 0.239 \times 10^{-1}$ in., and $q_2 = 0.20 \times 10^{-1}$ in. Thus, although the optimum weight obtained by omitting member three is approximately 24 percent lower than the weight obtained by including member three, the deflections of node four in the former case were approximately four times greater than in the latter.

One might be led to believe that if deflection or frequency constraints were enforced, then the optimum structure might not be statically determinate. To investigate this possibility, displacement as well as buckling and natural frequency constraints were imposed. The deflection limits were taken as, $q_1 = \pm 0.005$ in. and $q_2 = \pm 0.005$, and the lower limit on natural frequency was taken as 3830 cps. With the starting point $b_1 = 10$, $b_2 = 5$, $b_3 = 5$, the solution obtained was $b_1 = 9.18$, $b_2 = 2.16$, $b_3 = 3.85$, and $J = 20.59$ pounds (Curves 2 and 3, Figure 7). When member three was omitted, the starting point was taken as $b_1 = 10$, $b_2 = 10$ (Curve 4, Figure 7) and as $b_1 = 18$, $b_2 = 10$ (Curve 5, Figure 7). The solution obtained in this case was $b_1 = 16.0$, $b_2 = 11.31$, and $J = 33.94$ pounds. Thus, the optimum weight obtained for the statically determinate case is approximately 70 percent higher than the optimum weight obtained for the statically indeterminate case.

It was found that interactive computing yielded convergence more rapidly than was the case in the batch mode. It is expected that even more significant reduction in computing time will occur in large scale problems.

The key point in the solution is that the configuration of the optimum design is not obvious from analytical considerations. A designers experience and insight is required to select candidate configurations and then obtain the optimum design analytically. The global solution in this case must be chosen by comparing relative minima. It may be expected, in structures with greater redundancy, that certain members may be reduced during interactive computation when they are observed to approach their allowable lower limits.

An interesting point, illustrated by Table 1, is that a statically determinate truss is optimum when only stress constraints are imposed. Quite the contrary, when the full range of constraints are imposed, a statically indeterminate truss is optimum.

Example 2. Transmission Tower

Figure 8 shows the geometry and dimensions of the transmission tower to be studied. This problem has been considered by Venkayya and others [7, other references are discussed in this report]. The tower has 25 members, 10 joints, 18 degrees of freedom and is designed for six loading conditions. The structure is indeterminate with a degree of indeterminacy of seven.

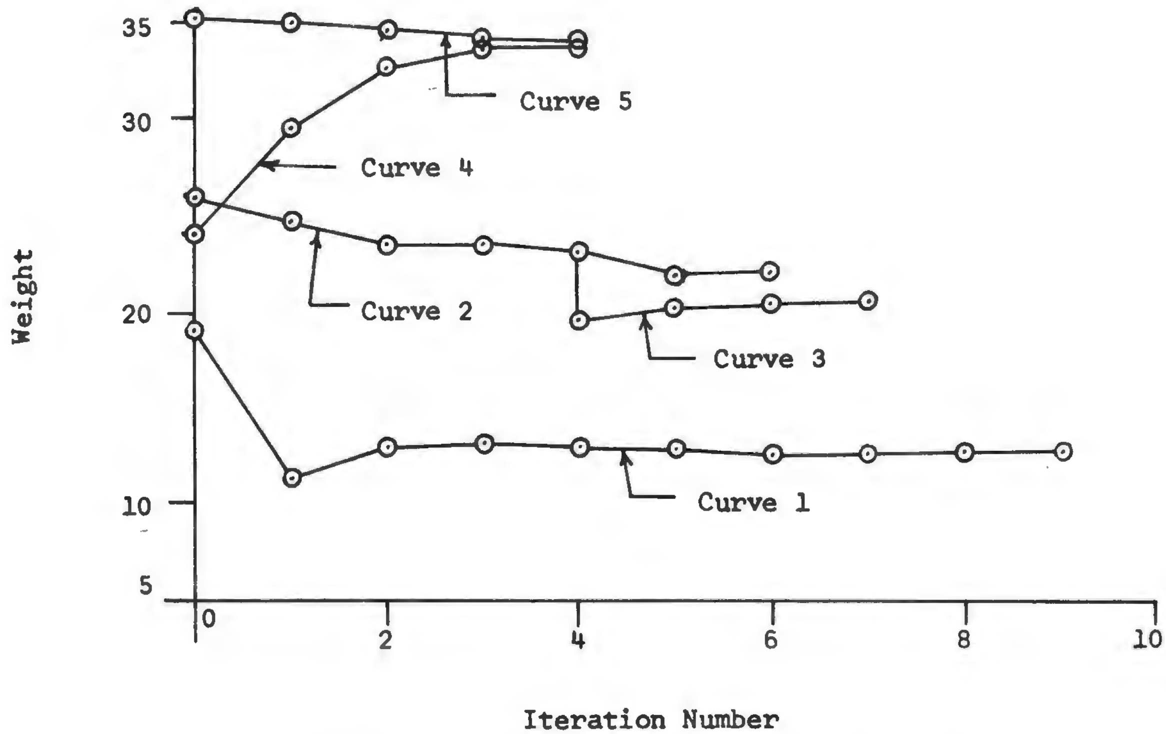

Figure 7. Weight Versus Iteration Curves for Three-Bar Truss (All Constraints)

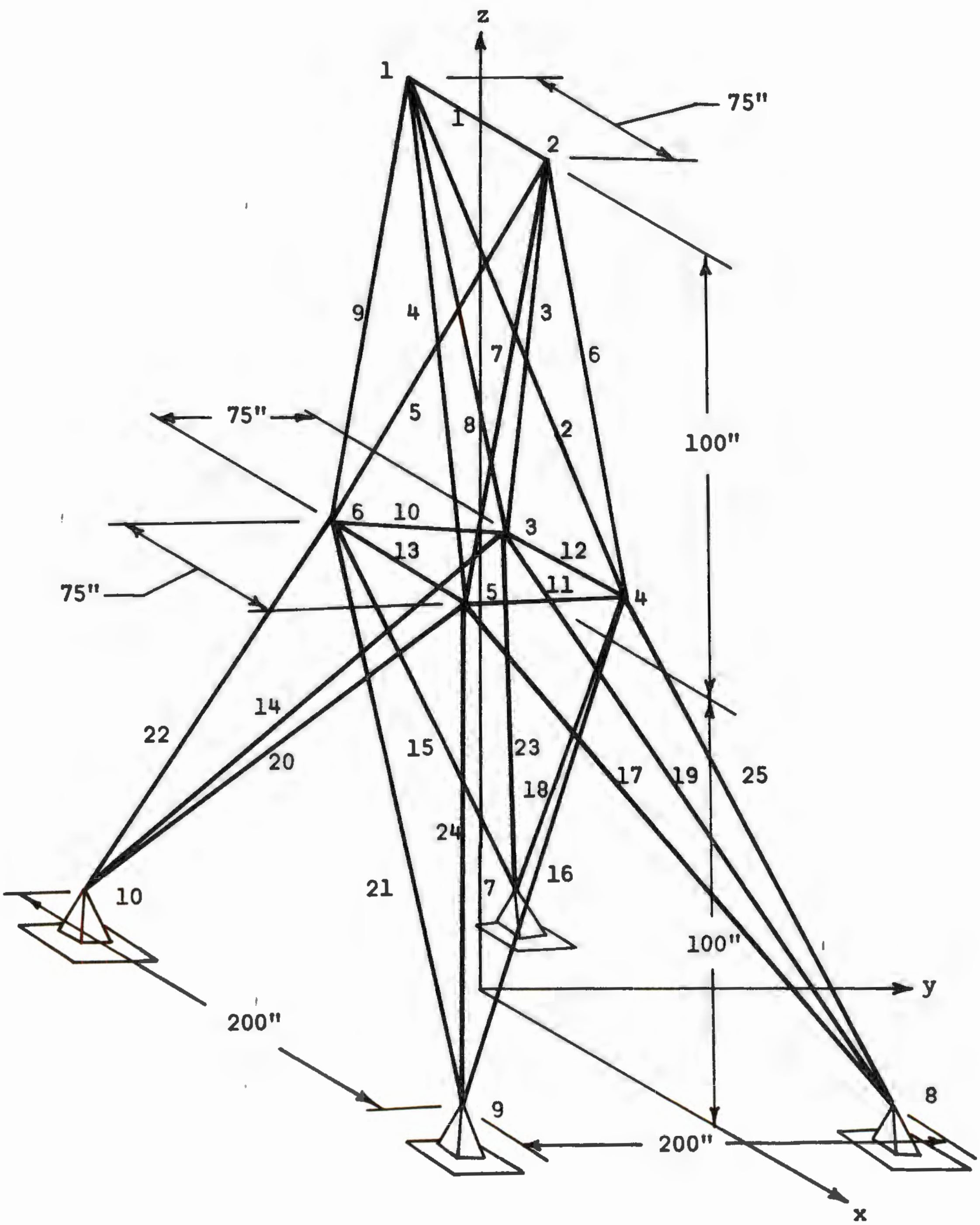

Figure 8 Transmission Tower

Table 1 Optimum Three Member Trusses

El. No.	With Stress Constraints Only Final Area in Sq. In.		With All Constraints Final Area in Sq. In.	
1	7.064	8.500	9.180	16.000
2	1.971	1.500	2.160	11.310
3	2.835	- -	3.850	- -
Wt. in lbs.	15.970	12.812	20.59	33.94
Max. Defl.	0.00689	0.02390	0.005	0.005

The tower was designed by first imposing only stress constraints, and then by imposing stress, displacement, buckling, and natural frequency constraints. Design information is given in Table 2 and the final results obtained are shown in Table 3. In this table, Columns 1, 2, 3, and 4 give the optimal designs when only stress constraints were considered and Columns 5, 6, 7, and 8 give results when all the constraints were imposed. For results given in Column 1 of Table 3, all the members of tower were included in the computation and the Curve 1 of Figure 9 shows the variation of cost function with the number of iterations. The computations of this case were monitored to determine which cross-sections went to their lower bounds.

One set of members which attained their lower limits of cross-sectional area were numbers 10, 11, 12, and 13. It was observed that these members carried small forces and could be removed without causing collapse of the tower, so they were removed from the tower. The final values of areas of cross section of the resulting structure are given in Column 2 of Table 3. Curve 2 of Figure 9 shows the variation of cost function with respect to the design cycle. The final weight in this case was slightly less than the previous case.

The next member that reached its lower limit was number one. So, it was also removed from the structure. The results of this case are given in Column 3 of Table 3 and Curve 3 of Figure 9. The final weight in this case was 86.94 pounds which is given slightly less than the previous case. Finally, members 14, 15, 16, and 17 were at their lower limits of cross-sectional area. Removal of any of these members, however, would cause collapse of the structure. Members 2 and 5 or 3 and 4 could be removed to make the structure determinate. The results for a statically determinate structure, obtained by removing members 2 and 5, are shown in Column 4 of Table 3. The final weight in this case was 106.97 pounds. It may be noted that this statically determinate structure yielded only a local optimum(Curve 4, Figure 9).

All these tower configurations were also optimized by imposing all constraints, i.e. stress, displacement, buckling, and natural frequency. The results of these cases are given in Columns 5, 6, 7, and 8 of Table 3 and the Curves 1, 2, 3, and 4 of Figure 10, respectively. It can be observed from the results of Table 3 that, for the case in which all constraints were imposed, the optimum weight of the tower increased as more redundant members were removed from the structure.

Table 2 Design Information for Transmission Tower

For each member of the structure, the modulus of elasticity, E_i, the specific weight, ρ_i, the constant, α_i (moment of inertia of ith member, $I_i = \alpha_i b_i^2$) and the stress limits are 10^4 kips/sq. in., 0.10 lbs/cu. in., 1.0 and $\pm$ 40.0 kips/sq. in. respectively. The lower limit on the area of cross section of each member is 0.10 sq. in. for the case with stress constraints only and 0.01 sq. in. for other cases. There is no upper limit on the member sizes. The resonant frequency for the structure is 173.92 cps and the displacement limits are 0.35 in. on all nodes and in all directions. There are six loading conditions and they are as follows (all loads are in kips):

Load Cond.	Node	Direction of Load			Load Cond.	Node	Direction of Load		
		x	y	z			x	y	z
1	1	1.0	10.00	-5.0	2	1	0	10.0	-5.0
	2	0	10.0	-5.0		2	-1.0	10.0	-5.0
	3	0.5	0	0		4	-0.5	0	0
	6	0.5	0	0		5	-0.5	0	0
3	1	1.0	-10.0	-5.0	4	1	0	-10.0	-5.0
	2	0	-10.0	-5.0		2	-1.0	-10.0	-5.0
	3	0.5	0	0		4	-0.5	0	0
	6	0.5	0	0		5	-0.5	0	0
5	1	0	20.0	-5.0	6	1	0	-20.0	-5.0
	2	0	-20.0	-5.0		2	0	20.0	-5.0

Table 3 Optimum Transmission Towers

EL. NO.	With Stress Constraints only — Final Areas in sq. in.				With All Constraints — Final Areas in sq. in.			
	1	2	3	4	5	6	7	8
1	0.100	0.100	- -	- -	0.010	0.010	- -	- -
2	0.376	0.377	0.346	- -	2.092	2.339	2.393	- -
3	0.376	0.377	0.346	0.100	2.075	2.386	2.404	0.548
4	0.376	0.377	0.346	0.100	2.095	2.339	2.393	0.548
5	0.376	0.377	0.346	- -	2.083	2.385	2.404	- -
6	0.471	0.470	0.494	0.779	2.357	2.085	2.076	7.132
7	0.471	0.470	0.494	0.779	2.354	2.084	2.076	6.857
8	0.471	0.470	0.494	0.779	2.350	2.113	2.083	6.895
9	0.471	0.470	0.494	0.779	2.335	2.112	2.082	7.101
10	0.100	- -	- -	- -	0.035	- -	- -	- -
11	0.100	- -	- -	- -	0.035	- -	- -	- -
12	0.100	- -	- -	- -	0.087	- -	- -	- -
13	0.100	- -	- -	- -	0.084	- -	- -	- -
14	0.100	0.100	0.100	0.165	1.113	1.114	1.139	1.785
15	0.100	0.100	0.100	0.165	1.113	1.114	1.139	1.735
16	0.100	0.100	0.100	0.165	1.112	1.117	1.146	1.727
17	0.100	0.100	0.100	0.165	1.112	1.117	1.146	1.798
18	0.277	0.279	0.292	0.413	2.056	2.047	2.027	4.317
19	0.277	0.279	0.292	0.413	2.058	2.034	2.022	4.390
20	0.277	0.279	0.292	0.413	2.046	2.047	2.027	4.400
21	0.277	0.279	0.292	0.413	2.058	2.034	2.022	4.328
22	0.380	0.374	0.363	0.547	2.822	2.878	2.886	5.655
23	0.380	0.374	0.363	0.547	2.808	2.878	2.886	5.730
24	0.380	0.374	0.363	0.547	2.803	2.926	2.895	5.743
25	0.380	0.374	0.363	0.547	2.785	2.926	2.895	5.648
Wt. in lbs.	91.13	87.90	86.94	106.97	590.32	596.64	597.82	1060.6
Max. Defl. in in.	2.288	2.305	2.311	3.489	0.350	0.350	0.350	0.350

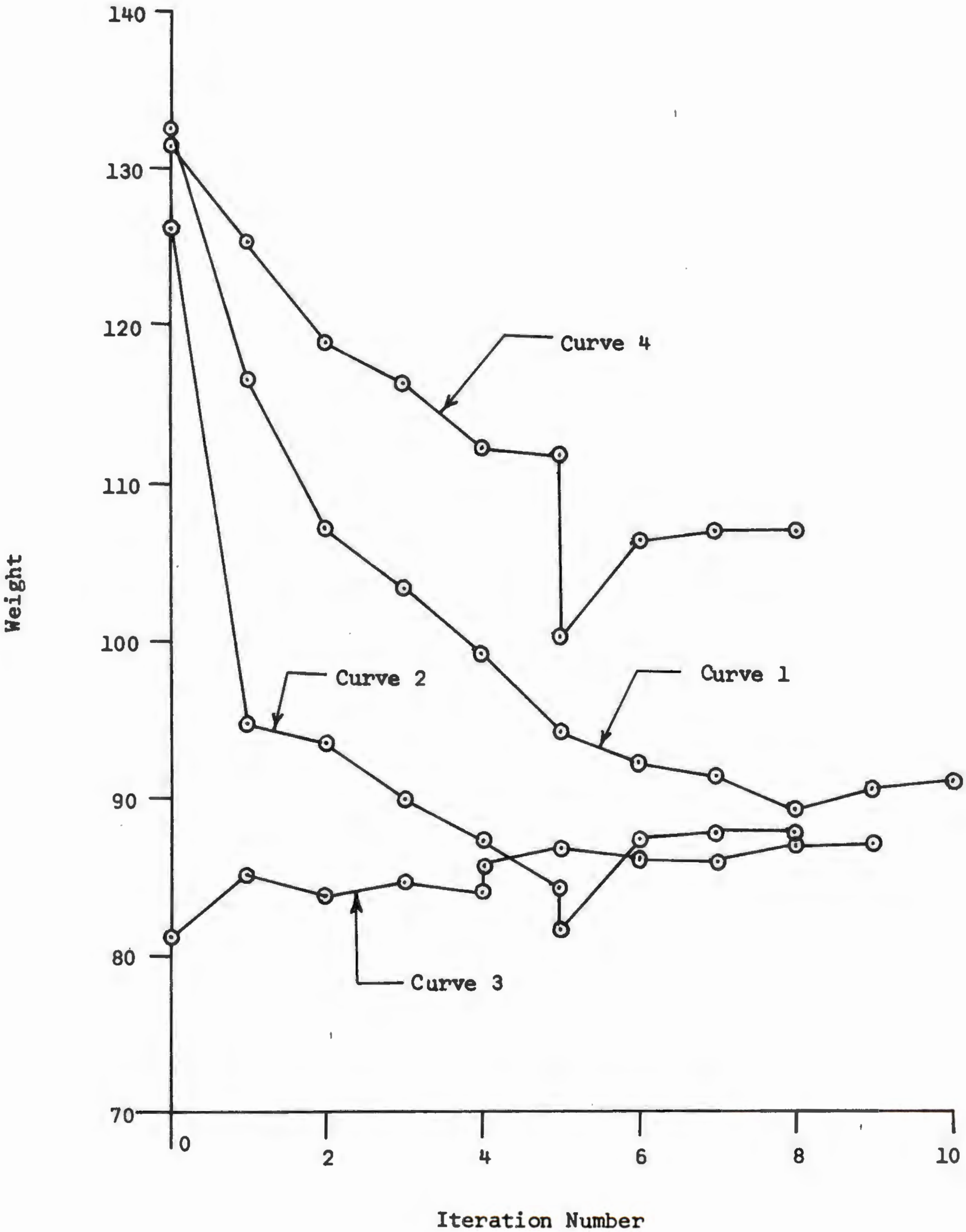

Figure 9. Weight Versus Iteration Curves for Transmission Tower (Stress Constraints)

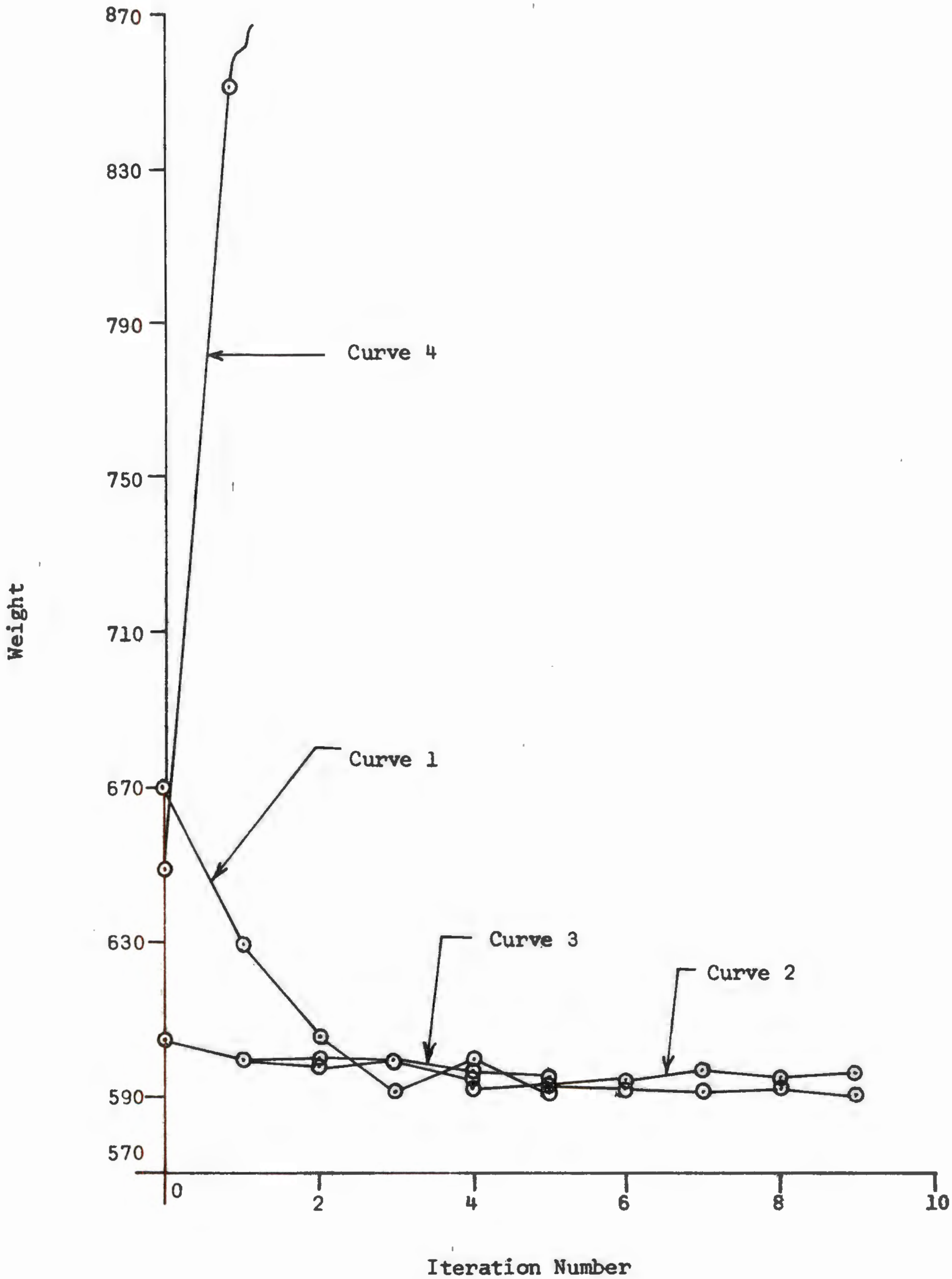

Figure 10. Weight Versus Iteration Curves for Transmission Tower (All Constraints)

Conclusions

Use of the steepest descent technique in the interactive mode, as developed in this paper has proved to be very effective from two points of view. First, computing times are considerably shorter than had been experienced when the same problems were solved in the batch mode. Second, and probably more significant, interactive computing allows the designer to alter the structural configuration in a systematic way to seek the global optimum design. This is not to say that a mathematically precise method of obtaining a global optimum has been found, for no such method is known to the authors. It appears, however, that the technique presented here makes strong use of the designers knowledge and intuition and gives him a tool with which to seek a global optimum in an organized way.

The results presented for the two examples solved in this paper are of interest in their own right. For the case when only stress constraints are imposed, results of Table 1 indicate that minimum weight designs for trusses with multiple loading may be statically determinate. However, the results of the second example given in Columns 1, 2, 3, and 4 of Table 3 indicate that all statically determinate trusses may not be lighter than the indeterminate trusses.

For the case when all constraints are imposed, results of Table 1 and 3 show that statically indeterminate trusses are lighter than the determinate trusses. Therefore, one is lead to believe that, in this case, redundancy is advantageous.

REFERENCES

[1] Douty, R., and Shore, S., "Technique for Interactive Computer Graphics in Design", Journal of the Structural Division, ASCE, Vol. 97, No. ST1, January, 1971, pp. 273-288.

[2] Moses, F., "Optimum Structural Design Using Linear Programming", Journal of the Structural Division, Proc. ASCE, Vol. 90, No. ST6, December, 1964, pp. 89-104.

[3] Romstad, K. M. and Wang, C. K., "Optimum Design of Framed Structures", Journal of the Structural Division, Proc. ASCE, Vol. 94, No ST12, December, 1968, pp. 2817-2845.

[4] Grierson, D. E. and Cohn, M. Z., "A General Formulation of the Optimal Frame Problem", Journal of Applied Mechanics, Trans. ASME, Vol. 37, No. 2, June, 1970, pp. 356-360.

[5] Kelley, J. E., "The Cutting-Plane Method for Solving Convex Programs", SIAM Journal on Applied Math., Vol. 8, No. 4, December, 1960, pp. 703-712.

[6] Johnson, D. and Brotton, D. M., "Optimum Elastic Design of Redundant Trusses", Journal of the Structural Division, Proc. ASCE, Vol. 95, No. ST12, December, 1969, pp. 2589-2610.

[7] Venkayya, V. B., Khot, N. S. and Reddy, V. S., "Energy Distribution in an Optimum Structural Design", Technical Report AFFDL-TR-68-158, Wright-Patterson Air Force Base, Ohio 45433, March, 1969.

[8] Fiacco, A. V. and McCormick, G. P., Nonlinear Programming: Sequential Unconstrained Minimization Techniques, John Wiley, New York, 1968.

[9] Kavlie, D. and Moe, J., "Application of Nonlinear Programming to Optimum Grillage Design with Nonconvex Sets of Variables", International Journal for Numerical Methods in Engineering, Vol. 1, No. 4, 1969, pp. 351-378.

[10] Kavlie, D. and Moe, J., "Autmated Design of Frame Structures," Journal of Structural Division, Proc. ASCE, Vol. 97, No. ST1, January, 1971, pp. 33-62.

[11] LaPay, W. S. and Goble, G. G., "Optimum Design of Trusses for Ultimate Loads", Journal of the Structural Division, Proc. ASCE, Vol. 97, No. ST1, January, 1971, pp. 157-174.

[12] Schmit, L. A. and Fox, R. L., "An Integrated Approach to Structural Synthesis and Analysis", AIAA Journal, Vol. 3, No. 6, June, 1965, pp. 1104-1112.

[13] Schmit, L. A., Kicher, T. P. and Morrow, W. M., "Structural Synthesis Capability for Integrally Stiffened Waffle Plates", AIAA Journal, Vol. 1, No. 12, December, 1963, p. 2820.

[14] Gellatly, R. A. and Gallagher, R. H., "A Procedure for Automated Minimum Weight Structural Design, Part I: Theoretical Basis; Part II: Applications", Aeronautical Quarterly, Vol. 17, 1966, pp. 216-230 and pp. 332-342.

[15] Felton, L. P. and Hofmeister, L. D., "Optimized Components in Truss Synthesis", AIAA Journal, Vol. 6, No. 12, December, 1968, pp. 2434-2436.

[16] Ridha, R. A. and Wright, R. N., "Minimum Cost Design of Frames," Journal of the Structural Division, Proc. ASCE, Vol. 93, No. ST4, August, 1967, pp. 165-183.

[17] Fox, R. L. and Kapoor, M. P., "Structural Optimization in Dynamics Response Regime: A Computational Approach", AIAA Journal, Vol. 8, No. 10, October, 1970, pp. 1798-1804.

[18] Pope, G. G., "The Design of Optimum Structures of Specified Basic Configuration", International Journal of Mechanical Sciences, Vol. 10, October, 1968, pp. 251-263.

[19] Moses, F. and Onoda, S., "Minimum Weight Design of Structures With Application to Elastic Grillages", International Journal for Numerical Methods in Engineering, Vol. 1, No. 4, 1969, pp. 311-331.

[20] Best, G., "A Method of Structural Weight Minimization Suitable for High Speed Digital Computers", AIAA Journal, Vol. 1, No. 2, February, 1963, pp. 478-479.

[21] Rosen, J. B., "The Gradient Projection Method for Nonlinear Programming Part I - Linear Constraints", SIAM Journal on Applied Math., Vol. 8, No. 1, March, 1960, pp. 181-217; Part II - Nonlinear Constraints, Vol. 9, No. 4, December, 1961, pp. 514-532.

[22] Brown, D. M. and Ang, A. H. S., "Structural Optimization by Nonlinear Programming", Journal of the Structural Division, Proc. ASCE, Vol. 92, No. ST6, December, 1966, pp. 319-340 and Vol. 93, No. ST5, October, 1967, pp. 618-619.

[23] Seaburg, P. A. and Salmon, C. G., "Minimum Weight Design of Light Gage Steel Members", Journal of the Structural Division, Proc. ASCE, Vol. 97, No. ST1, January, 1971, pp. 203-222.

[24] Haug, Jr., E. J., "Optimal Design of Mechanical Systems", Unpublished class notes on Optimization of Structural Systems, University of Iowa, Iowa City, Iowa 52240, 1970-71.

[25] Haug, Jr., E. J., Pan, K. C. and Streeter, T. D., "A Computational Method for Optimal Structural Design: Piecewise Uniform Structures", to be published in the International Journal for Numerical Methods in Engineering, 1971.

[26] Bartel, D. L. and Rim, K., "Optimum Design of Spatial Structures", Project Themis Report, No. 12, University of Iowa, Iowa City, Iowa 52240, August, 1969.

[27] Bates, W. F., Jr., and Cox, W. M., "Structural Analysis by Computer Graphics", Journal of the Structural Division, ASCE, Vol. 95, No. ST11, November, 1969, pp. 2433-2448.

[28] Schiffman, R. L., and Stein, J. R., "Interactive Graphics: Settlement and Slope Stability", Journal of the Structural Division, ASCE, Vol. 97, No. ST1, January, 1971, pp. 253-272.

[29] Arora, J. S., "Optimal Design of Elastic Structures Under Multiple Constraint Conditions", Ph. D. Dissertation, Department of Mechanics and Hydraulics, University of Iowa, Iowa City, Iowa, August, 1971.

[30] Schmit, L. A., "Structural Design by Systematic Synthesis", Second Conference on Electronic Computation, Structural Division of ASCE, September, 1960, pp. 105-132.

[31] Sved, G. and Ginos, Z., "Structural Optimization Under Multiple Loading", International Journal of Mechanical Sciences, Vol. 10, 1968, pp. 803-805.

[32] Corcoran, P., "Configurational Optimization of Structures", International Journal of Mechanical Sciences, Vol. 12, 1970, pp. 459-462.

[33] Young, Jr., W. J. and Christiansen, H. N., "Synthesis of a Space Truss Based on Dynamic Criteria", Journal of the Structural Division, Proc. ASCE, Vol. 92, No. ST6, December, 1966, pp. 425-442.

[34] Reinschmidt, K. F., Cornell, C. A. and Brotchie, J. F., "Iterative Design and Structural Optimization", Journal of the Structural Division, Proc. ASCE, Vol. 92, No. ST6, December, 1966, pp. 281-318.

[35] Dorn, W. S., Gomory, R. E. and Greenberg, H. J., "Automatic Design of Optimal Structures", Journal de Mecanique, Vol. 3, No. 1, March, 1964, pp. 25-53.

[36] Dobbs, M. W. and Felton, L. P., "Optimization of Truss Geometry", Journal of the Structural Division, Proc. ASCE, Vol. 95, No. ST10, October, 1969, pp. 2105-2118.

[37] Rubin, C. P., "Minimum Weight Design of Complex Structures Subject to Frequency Constraints", AIAA Journal, Vol. 8, No. 5, May, 1970, pp. 923-927.

[38] Zarchamee, M. S., "Optimum Frequency Structures", AIAA Journal, Vol. 6, No. 4, April, 1968, pp. 749-750.