

WADD TECHNICAL REPORT 60-901

**ION PROPULSION WORKING FLUID  
REQUIREMENTS FOR USAF**

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## FOREWORD

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ABSTRACT

Some of the anticipated requirements for ion engine working fluids are derived by the consideration of missions expected to be of interest, the probable systems which will produce the power, and handling needs.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

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## LIST OF SYMBOLS\*

$F'$	= Force thrust in dynes = $(4.4482 \times 10^5 F)$
$F$	= Force thrust in lbs = $2.2481 \times 10^6 F'$
$I_{sp}$	= Specific impulse in lbs force/lbs per sec mass flow
$V$	= Acceleration voltage
$A$	= Atomic weight or molecular weight of propellant ion species in Amus (= $6.023 \times 10^{23} M'$ )
$n$	= Number of charges per ion
$I$	= Beam current amperes
$P'$	= Total beam power kw (Note $P$ = power supply power times efficiency, the efficiency assumed to be 100 percent for this paper)
$P$	= Beam power in watts
$C$	= Exhaust velocity ft/sec
$C'$	= Exhaust velocity cm/sec
$\bar{C}$	= Average velocity cm/sec
$M$	= Mass of particles in lbs
$M'$	= Mass of particles in gms = $M(454) = \frac{A}{6.023 \times 10^{23}}$
$N$	= Number of ions per second exhausted
$Q$	= Charge on one electron (= $1.6 \times 10^{-19}$ coulombs)
$E$	= Field intensity = $V/(\text{distance thru which } V \text{ acts})^2 = V/x^2$
$X'$	= Distance in cm
$v$	= Vehicle velocity gain in ft/sec
$M'_O$	= Space system total mass, K gm
$M'_p$	= Propellant mass, K gm

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\* Note: A primed symbol usually means cgs units.

## LIST OF SYMBOLS (Cont'd)

$M'_g$	= Power Plant mass, K gm
$M'_L$	= Payload plus structure plus engine, K gm
$M'_E$	= Exhausted propellant mass
$t$	= Time of mission
$M_{wt}$	= Gram or lb atomic weight
$\eta_p$	= Propellant efficiency
$\bar{P}$	= Equivalent power in efficiency
$cm$	= Velocity of average mass particle
$\beta$	= Ionization fraction or ionization coefficient which is the number of ionized particles per total number incident
$T$	= Temperature °K
$\phi$	= Work function in ev
$V_1$	= Ionization potential in ev
$\alpha$	= Power Plant specific weight, lbs/K <sub>w</sub>
$f_i$	= Mass fraction of the ith species of ion

## MISSION PARAMETERS

At the present time there are four missions that appear feasible for using ion propulsion. These are: (1) Earth low altitude satellite sustainer, (2) Interorbital shuttle, (3) Lunar trips, and (4) Interplanetary trips such as to Mars or Venus.

To determine what working fluids are needed in these missions, it is necessary to perform some mission analyses. This has been done by several other authors; therefore, only a minimum treatment will be presented in this report.

Reference 1 states that

$$F = 3.24 \times 10^{-5} \sqrt{\frac{VA}{n}} I \quad (1)$$

$$F = 3.24 \times 10^{-2} P \sqrt{\frac{A}{nV}} \quad (2)$$

$$F = 1.476 \times 10^3 P/C \quad (3)$$

$$F = 45.91 P/I_{sp} \quad (4)$$

$$I_{sp} = 1.417 \times 10^3 \sqrt{\frac{nV}{A}} \quad (5)$$

$$I_{sp} = 3.10 \times 10^{-2} C \quad (6)$$

A quick derivation of some of these equations may be found in Appendix I.

# EARTH ORBIT SUSTAINER MISSIONS

The optimum system for a low altitude orbit with thrust to overcome drag has been determined to be one that starts its mission with mass fraction  $M'_p/M'_o = 1/2$ ,  $M'_g/M'_o = 1/4$ , and  $M'_L/M'_o = 1/4$ . Thus the system is 1/2 propellant, 1/4 powerplant, and 1/4 payload by weight. The impulse of a system depends on the desired time of mission; the specific powerplant weight, ( $\alpha$ ); and the system efficiency. The total system depends also on altitude of orbit but the specific impulse, (Isp), does not. Isp for several mission times is shown plotted for parameters of  $\alpha$  in figure 1. The trade-off time for chemical rockets vs electrical devices is about 40 days total mission time; thus for missions of less than 40 days, ion engine systems are heavier and probably would not be used. Electrical systems for missions beyond 40 days require less total weight and probably would be used.

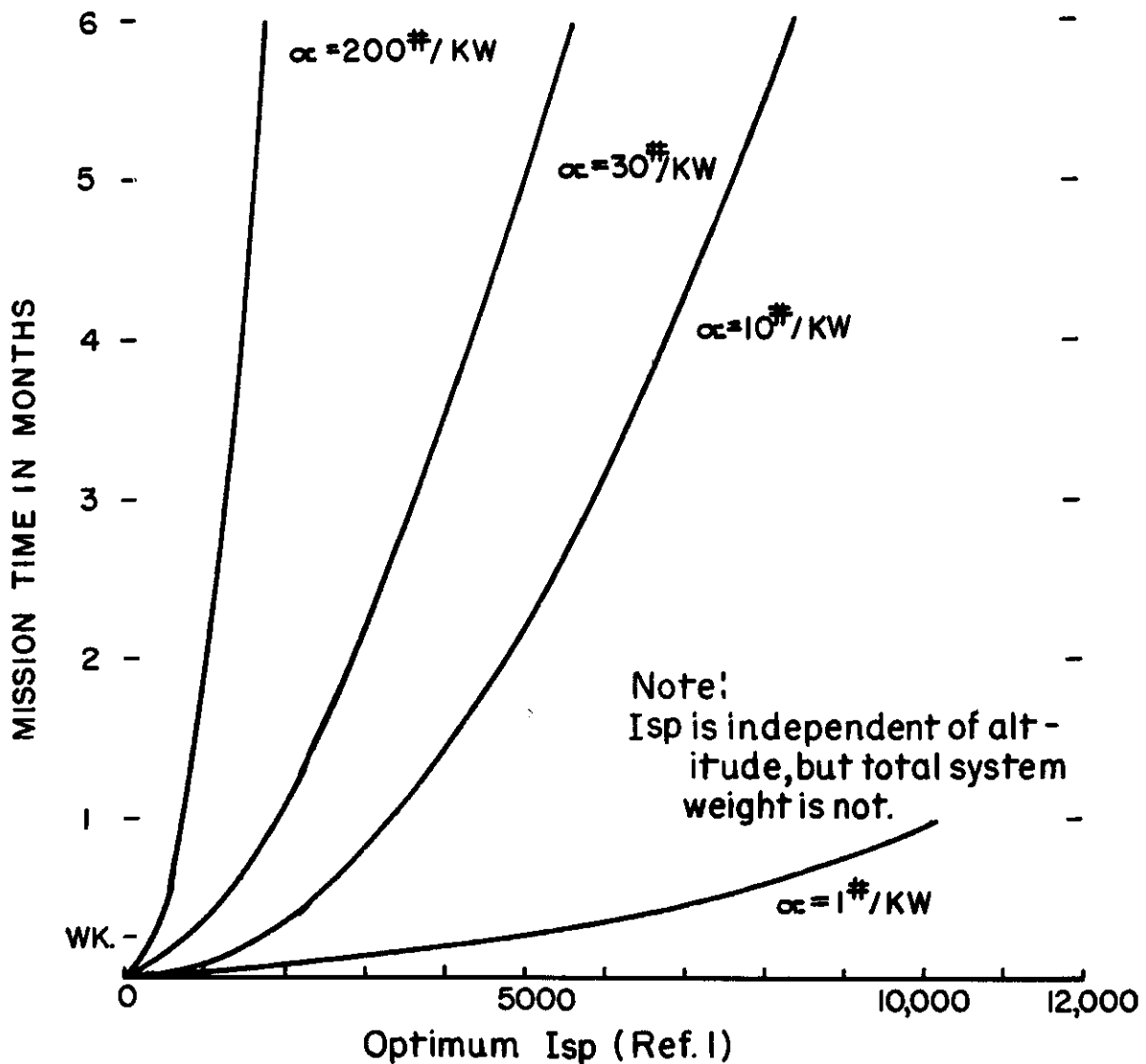


Figure 1. Low Altitude Sustainer Mission



## SHUTTLE MISSIONS

Shuttle missions presently can be achieved much faster with chemical propellants; however, ion propulsion will carry much larger payloads given the time. Several different missions are considered in ref. 1, and one is shown graphically as a function of Isp vs. time for optimized payload fractions and parameters of  $\alpha$  in figure 2. Rough trade-off values of electrical propulsion show that chemical rockets will perform an orbital transfer in half a day or so with about 30 percent payload where ion engine systems would require 10 to 20 days carrying 80 percent payload.

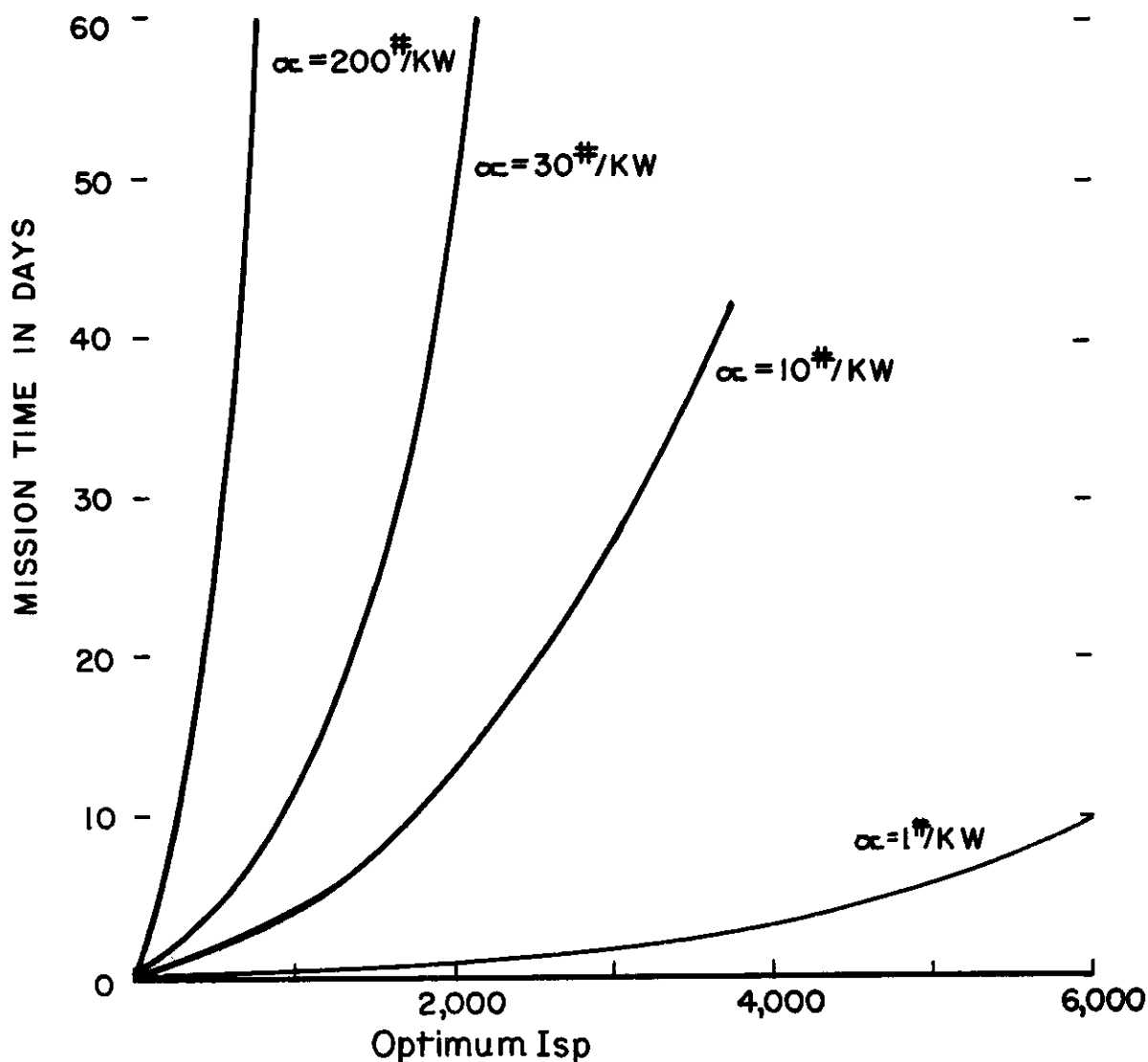


Figure 2. 100 Mile Orbit to 22,000 Miles

# LUNAR MISSIONS

Lunar landing and take-off will probably be by chemical systems; however, the orbital missions can be accomplished by means of either chemical, ion, or plasma devices. With electrical devices the time required is again longer than with chemical devices but the payload fraction is higher, and if a reactor is to be transported to start a base, it may be considered part of the payload. Figure 3 shows some calculated mission times vs. Isp for optimized values of payload as parameters of  $\alpha$ . The powerplant is not considered payload in this figure.

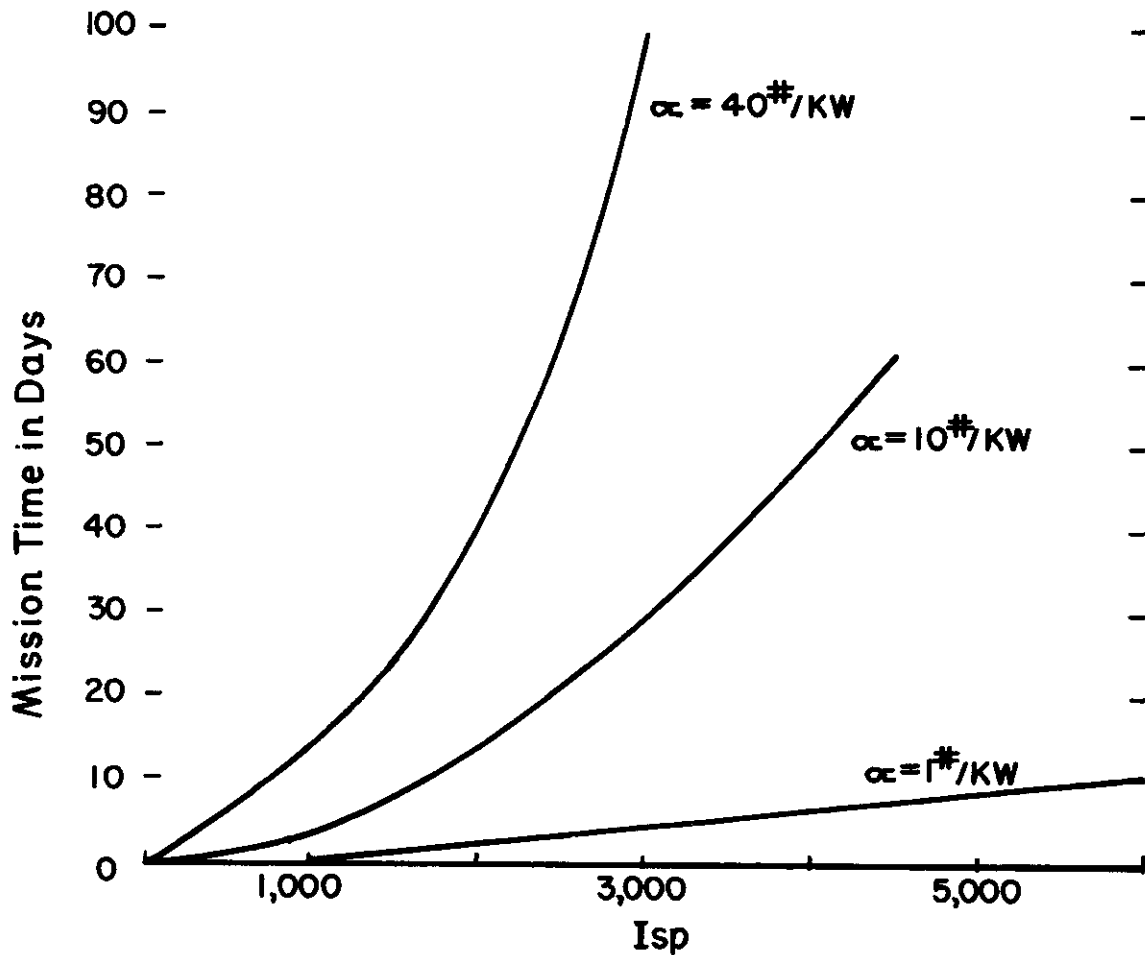


Figure 3. Lunar Orbit and Return (Carrying Optimum Payload)

## MARS AND VENUS MISSIONS

Mars and Venus missions are much more ambitious than any of the previous examples. They would require a round trip of 300 days or more in most cases, as shown in figure 4.

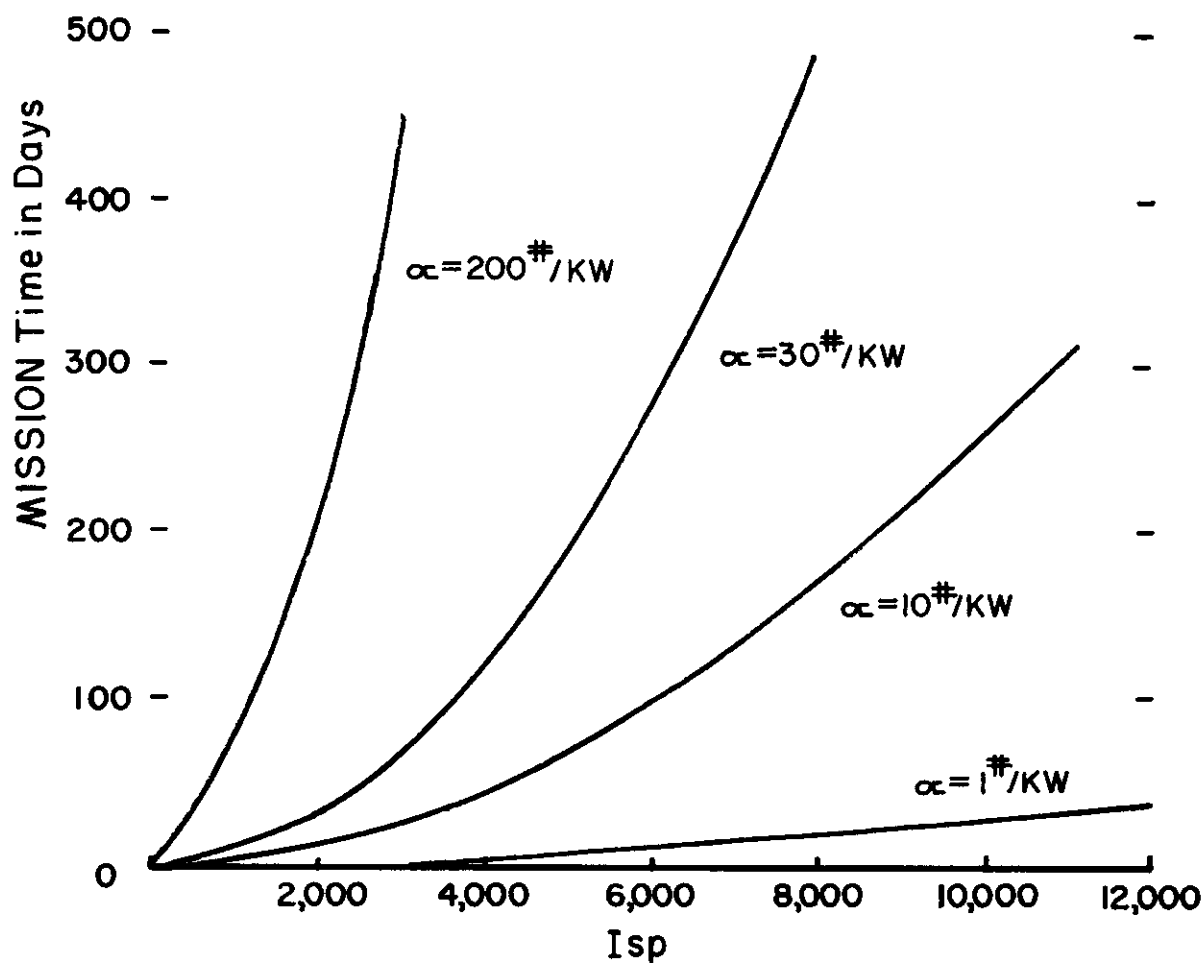


Figure 4. Mars or Venus Mission With Optimum Payload

## USEFUL ISP RANGE

For the four listed missions the range of impulse values necessary are dependent upon the mission, the power, and the desired voltage. For long duration missions with low voltage systems almost any propellant will do and the present "best" propellant, cesium, is satisfactory. For shorter missions with high voltage systems such as electrostatic generation, propellants with a molecular weight of up to 2000 are needed. Mars and Venus are in the 150-to-650 range and up to 2000 is desirable for a 200,000 volt system. For derivation of some of these values see Appendix II.

## SINGLE ION SPECIES

It is necessary that only one species of ion be ejected. This is due to inefficiency if more than one species is present. If more than one velocity of ion is present in the exhausted stream, more power will be necessary to provide some constant thrust than would be consumed with any single velocity species. To state this another way, the mass flow that is necessary for constant thrust and constant power is greater for any mixed velocity propellant than for any single velocity propellant. There may be mission cases where a mixed ion species is desirable to increase thrust for a specific mission with an existing engine but mixed ion species should never be used to design an engine. For a more detailed derivation see Appendix III.

## PHYSICAL PROPERTIES

Properties of the ion propellant should be such that it may be readily ionized when desired and easily stored and handled prior to use. The range of these qualifications is wide, and it is probable that not all requirements can be met.

For storage a dense material is desirable to cut down on tankage weight. Storage in the liquid or solid state appears much more desirable than in gaseous state again due to tankage weight. The temperature at storage should be between 600°C and room temperature, which is the expected range of temperature that could be obtained in some system with radiation to space as coolant. A reactor, for example, could well use part of the radiated heat to maintain the working fluid at a fairly high temperature such as 600°C. The fuel could also be allowed to cool to the temperature of "space" at the Earth orbit distance from the sun. This assumes that an aluminum outer skin will cause the adsorption and radiation of heat of the system to be that of the aluminum and that the tank is rotating rapidly to allow the temperature to be uniform. Such a system would be about room temperature, i.e., 25°C. By causing the sun side to be reflective and the dark side radiative, - 50°C should not be impossible; however, room temperature is still a more practical estimate.

Because of the low flow rates the compound could best be metered as a low pressure gas; therefore, the boiling point should be at a reasonable temperature. Again reactor waste heat might be used to boil the propellant. A boiling point not higher than the temperature of the working fluid of the reactor is needed. If Na and K are used as working fluids for reactors as has been proposed, the temperature should not exceed

approximately 800°C. The boiling point of some other fluids may go up to 1000°C; hence, this could be considered the upper limit.

It is convenient to vaporize the propellant before it is ionized and in addition it is necessary to provide the energy of ionization. This energy of ionization should be as low as possible since it is lost. Recovery of this energy, even if possible, would probably be more costly weight-wise than adding enough extra power to overcome the loss. From the viewpoint of ionization energy, Cs is the best positive elemental ion, but molecular ions and negative ions may require less ionization energy. The latter are now being investigated.

With respect to storage, the fluid should be compatible with the tank material; moreover, it is desirable that it be compatible with as many other materials in the surrounding structure as possible to avoid corrosion in case of leakage. Lack of toxicity is not a necessity but it simplifies handling and is convenient.

The fuel may be part of a nuclear powered vehicle; hence radiation stability to neutron and gamma flux is strongly advisable. If the propellant is not stable shielding will be required. If it is relatively stable the working fluid may be used as an extra shield. Cesium is stable in this manner. Organics and most molecular ionic materials are generally not very stable.

#### PRESENT IONIZATION METHODS (CONTACT AND BOMBARDMENT)

The mode of ionization of ion working fluids can vary. Currently, contact ionization is the most popular; however, this method is limited to higher work function materials ionizing low ionization potential propellants. Tungsten, oxygenated tungsten, thorated tungsten, and platinum ionizing Cs were first investigated and are still considered the best surface contact materials. Other tested and usable surface contact elements include Fe, Mo, Ni, Os, Pt, Pd, Re, Rh and Ta, while Ag, Au, C, Co, Cr, Cu, Ce, Ir, Ru and Zr are untried possibilities. Of the usable group Re, Ta, Rh, Pt and Pd are very good; they are inert but expensive.

The working fluid to be used with the above surface contact elements should be of high Mwt and low ionization potential. Cs is the best element; Rb, Ba, K and Na are other possibilities which have been investigated.

The ionization coefficient ( $\beta$ ) has been found to be a function of temperature (T), work function of the contact material ( $\phi$ ), and the ionization potential of the incident propellant ( $V_i$ ).

$$\beta = \frac{\text{number of ions}}{\text{number of incident molecules}} \propto \frac{1}{\left(1 + \exp e \frac{v - \phi}{k T}\right)}$$

This simple equation is somewhat complicated by the fact that  $\phi$  is a function of temperature and the fact that the equation does not consider the "reflection coefficient" of the materials. A more complete treatment of this subject may be found in refs. 4 through 23.

It should be emphasized that while the atoms of any material are ionized more fully at low T they are not "desorbed" from the surface. Thus a maximum of useful ions per mass flow of propellant is produced at higher temperature. In the case of Cs on Pt or Cs on W this is in the range of 1400°K to 1600°K.

The bombardment type source may cause ionization of a molecule by attachment of an electron or by removal of an electron. In its simplest form the bombardment type source consists of an electron "gun" firing electrons through a vapor of the substance to be ionized. The ions thus produced are extracted and accelerated. The Von Ardenne or Pierre type geometry ion engine uses bombardment type sources.

With attachment of the bombarding electrons negative ions are formed; however, if the electron is attached to a molecule and not an atom there is a tendency to fragment. Several compounds do not exhibit these tendencies as much as others. These compounds are characterized by having usually one or more "acceptors" for the charge. The general requirement is for large electron capture cross section and non-fragmentation of the ions thus produced. Positive ions also fragment when formed. In general, the ring organic compounds are more stable since they distribute the charge and prevent undue "charge stress" upon any single bond. If there are attached alkyl groups, they tend to fragment from the ring. This indicates that the benzene and anthracene series of compounds are good possibilities. There is much more to be done in this area and work is progressing under present AF and NASA contracts to develop these propellants further. References 24 to 33 are concerned with this field.

The bombardment type source is potentially superior to the contact source as it can ionize any contact material and many materials that will not contact ionize. In current use, however, the contact source is better as it involves less energy loss per ion produced even with the large radiation losses in present contact sources. The compounds suggested to date for use in bombardment sources include sulfur hexafluoride ( $\text{SF}_6$ ), the interhalides,  $\text{BF}_3$ , and such organics as anthracene, the quinones, chrysene, and possibly organic metallic compounds such as ferrocene.

## PHOTOIONIZATION

Another method of producing ions is by photoionization. This method is simply that of shining light or X-rays into a gaseous propellant and extracting the produced ions. For most known systems the power to produce the X-rays or light is greater than that required for either contact or bombardment sources. For Cs this may be feasible and for other molecular compounds with very low ionization energy it should be equally feasible. At present very little has been done with this method.

## FREE RADICAL IONS

A variation from the foregoing methods is the "building" of a molecule which will split at some trigger condition to yield two charged molecular ions. Heat, photons, or electrons might be the fragmenting agent. One requirement is that some molecule must always break in exactly the same way. Another requirement is that the molecules must split efficiently upon "triggering" to yield a high percentage of ions.

## COLLOIDS AND AEROSOLS

It is possible that colloids or aerosols may prove useful in ion engines. Colloidal ions for example can be made in a fairly uniform size and in spherical shape. These particles can be produced with masses of about 1000 Amu; however, uniform charging of these "dust" particles has not been successful.

Aerosols or fine liquid sprays have been prepared by allowing an electric field to "tear" the liquid into small droplets. Unfortunately the size and charge-to-mass ratio of the aerosols prepared to date are far from ideal. Large numbers of uncharged droplets result and the charge-to-mass plot vs. percent is a probability curve with zero being maximum and with very wide limits. Such fluids as Octoil have been used. The dielectric properties of the fluid affect the dispersal and higher dielectric fluids are necessary.

The work in this area is progressing under Air Force, OSR, ARPA, and NASA sponsorship. References 35 to 38 cover work being done with colloids and aerosols.

## SUMMARY OF REQUIREMENTS

Summarizing we may state that a satisfactory ion propellant should meet the following requirements:

1. High mass-to-charge ratio (about 2000 AMU singly ionized)
2. One ion species
3. Liquefaction somewhere between 0° - 600°C
4. Vaporization under 1000°C
5. Low energy of ionization
6. Compatibility with normal lightweight structural materials
7. Lack of toxicity is convenient
8. Stability to radiation
9. Some easy method of ionization
  - a. Low ionization potential
  - b. Good electron cross section
  - c. Easy electron emission
  - d. Capability of splitting into two specific molecular ions at some stimulus
10. Easy desorption from some high work function material
11. Non-fragmentation

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## APPENDIX I

## ENGINE PARAMETERS AND MISSION PARAMETERS

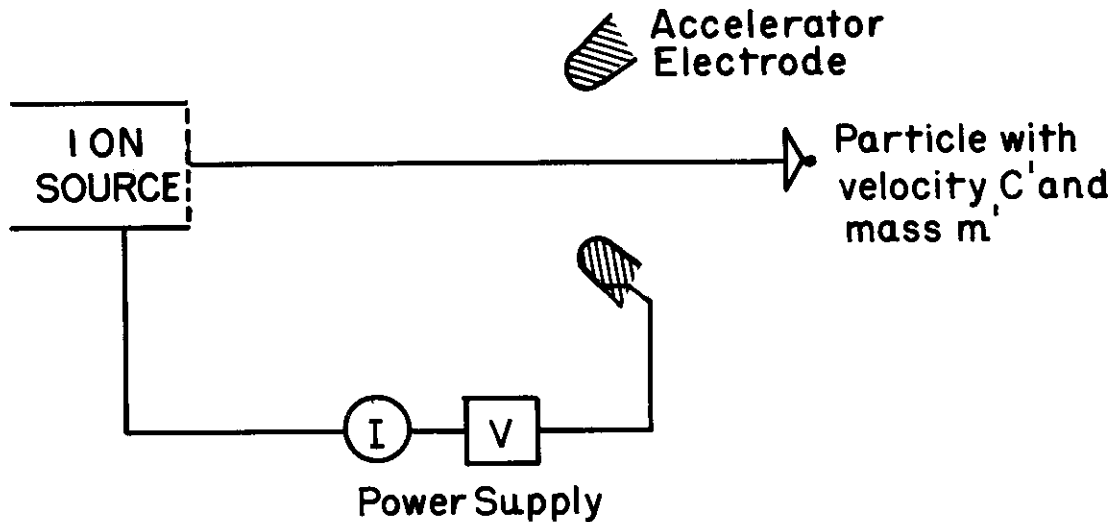


Figure 5. Simplified Ion Engine

The power that the beam will use is simply the voltage times the current, or

$$P = IV \text{ WATTS} \quad (7)$$

The force on the engine will be

$$F' = NM'C' \text{ DYNES} \quad (8)$$

which is the same as the force on the propellant.

The current will be

$$I = NQn \quad (9)$$

substituting eq.(9) in eq.(7),  $P = NQnV$

$$N = P/QnV \quad (10)$$

substituting (10) in (8),

$$F' = PM'C'/QnV \quad (11)$$

Writing an energy balance on one particle,

$$1/2 M'(C')^2 = kQE = kQV/(X)^2 \quad (12)$$

where  $k$  is a constant,  $9 \times 10^{18}$  dyne  $\text{cm}^2/\text{coulombs}^2$ , Solving for  $(C')^2$  and then  $C'$ .

$$C'^2 = 2kQV/m'(x)^2 \quad (12a)$$

$$C' = \sqrt{2kQV/m'x^2} \quad (13)$$

Substituting eq.(13) in eq.(11),

$$F' = \frac{PM'}{QnV} \sqrt{\frac{2kQV}{m'x^2}} \quad \text{or}$$

$$F' = \frac{P}{n} \sqrt{\frac{2kM'}{QVx^2}} \quad (14)$$

By definition  $I_{sp} = F/mN$ , or

$$I_{sp} = \left( \frac{F'}{2.242 \times 10^{-6}} \right) \left( \frac{1}{453.6 \text{ m}'N} \right) = 9.834 \times 10^3 \frac{F'}{m'N} \quad (15)$$

Substituting eq.(14) and eq.(10) in eq.(15),

$$I_{sp} = 9.834 \times 10^3 \frac{P}{n} \sqrt{\frac{2kM'}{QVx^2}} \left( \frac{QnV}{m'P} \right) \quad \text{or}$$

$$I_{sp} = 9.834 \times 10^3 \sqrt{\frac{2kQV}{m'x^2}} \quad (16)$$

by comparison with eq.(13)

$$I_{sp} = 9.834 \times 10^3 C' \quad (17)$$

By examination of (3),

$$F \propto P/C$$

it is obvious that the acceleration of any space system will depend on the power available to the engine. The precise value is not so obvious. It may be found by consideration of momentum of the ship and the exhausted propellant.

$$\Delta \text{momentum} = \int_0^P F dt = \int_0^V M'_0 dv + \int_0^{M'e} (c-v) dm = 0 \quad (18)$$

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if there is no gravitational acceleration.

$$\int F dt = \int M_e dt = \int_0^{\Delta V} M'_0 dv + \int_0^{M'_e} c dm \quad (19)$$

In a gravitational field where  $g$  is a variable,  $f$  of  $t$ ,  $M'_0$  is a function of  $t$ , and  $C$  may be solved for, given  $\Delta V$ . Equations (18) and (19) require specific missions for evaluation. These have been optimized for several specific missions in refs. 1 and 2.

*Contrails*

## APPENDIX II

## Isp RANGE CALCULATION

The specific impulse for various missions is very similar in all cases. It is developed in ref. 3 that

$$I_{sp} = 1990 \sqrt{t/\alpha} \quad (20)$$

The payload fraction will vary from mission to mission as will the total weight of the system but  $I_{sp}$  is a function of  $\alpha$  and "burn time" only. Burn time depends on the total velocity necessary, the payload fraction, and other mission parameters; thus,  $I_{sp}$  is again a function of the mission.

Typical values of  $\alpha$  for current power supplies are of the order of 30 lbs/kw for solar energy. Thermionic type power supplies may be able to achieve an  $\alpha$  of 10 lbs/kw and an unshielded reactor 5 lb/kw. An  $\alpha$  of 10 lbs/kw is a good assumption for the state of the art five to ten years from now. This means that  $I_{sp} = 629\sqrt{t}$ . The acceleration time for possible missions is as low as 10 days or as high as 500 days. This means that  $I_{sp}$ 's of from 2,000 to about 12,000 will be needed. This is checked by comparison with figures 2, 3, 4, and 5. The most probable  $I_{sp}$ 's are about 6,000, 2,000, 2,500, and 9,000, respectively, by very rough estimates.

This range of values will allow estimation of the molecular weight desired in an ionic fuel. By substitution of  $I_{sp}$  in eq.(5) and assuming that  $n = 1$  for single charged ions of a molecular type.

$$2000 = 1417 \sqrt{V/A} \quad \text{of } 2A = V. \quad (21)$$

A minimum of 100 volts is needed to allow control of the beam; thus an  $A$  of at least 50 is desired. Cesium, which is the current best propellant, has a Mwt of 133. This means that for short duration missions Cs is sufficient at minimum  $V$  values. For  $I_{sp}$  of 8,000 to 12,000 such as for a Mars or Venus mission,

$$\begin{aligned} 8000 &= 1417 \sqrt{V/A} & \text{or} \\ 31.9 A &= V \end{aligned} \quad (22)$$

which means that for 100 volts even helium would perform satisfactorily.

100 volts is approximately the minimum controllable voltage but it is desirable to use a much larger extraction voltage and thus cause the source to be smaller with higher current densities. Also power is produced much more efficiently by use of electrostatic generation than by electromagnetic generation. This means that a  $V$  of at least 5,000 to 20,000 volts will allow a much better and lighter system. The range of values near 2,000 Mwt is useful for such projects as a lunar mission. Longer missions to Mars or Venus would require  $I_{sp}$ s of 5,000 or greater; hence, Mwt of 156 will suffice with even 5,000 volts or 625 Mwt with 20,000 volts.

Further consideration of an electrostatic system will show that the higher the voltage the lighter the system for the same power up to about 200,000 volts, and ion extraction is more efficient and the engine is smaller because of the low currents. This indicates that larger and larger Mwt beams are desirable. A Mwt of 2000 is in the polymer range and at higher than 2000 Mwt precise control of Mwt is impossible. It is imperative that one ion species or at least a very small range be ejected.



## APPENDIX III

## SINGLE ION SPECIES

A single ion species is needed in the exhausted stream.

This conclusion may be reached by consideration of a group of ions moving at constant velocity. The power of this ith group of ions is

$$P_i = \dot{M}_e f_i v_i^2$$

where  $\dot{M}_e$  is the mass flow rate of all species,  $f_i$  is the mass fraction of the ith species, and  $v_i$  is the velocity of the ith species. The force exerted by this group is

$$F_i = \dot{M}_e f_i v_i$$

We may sum the above over all i groups

$$P = \frac{1}{2} \sum_i \dot{M}_e f_i v_i^2 \quad (23)$$

$$F = \sum_i \dot{M}_e f_i v_i \quad \text{or}$$

$$F = \dot{M}_e \sum_i f_i v_i \quad (24)$$

Since  $I_{sp}$  is defined as force per mass flow rate,

$$I_{sp} = \frac{F}{\dot{M}_e} = \sum_i f_i v_i \quad (25)$$

The power-to-force ratio of this propellant will be

$$\frac{P}{F} = \frac{1/2 \dot{M}_e \sum_i f_i v_i^2}{\dot{M}_e \sum_i f_i v_i}$$

The ideal power-to-force ratio will be

$$\frac{P}{F} \text{ Ideal} = I_{sp} (\text{Ideal})$$

from this we may define  $\eta_p$ .  $\eta_p$  will be the power-to-force ratio of some propellant divided by the power-to-force ratio of an ideal propellant, or

$$\eta_p = \frac{P/F \text{ real}}{P/F \text{ Ideal}} = \frac{1/2 \sum f_i v_i^2 / \sum f_i v_i}{1/2 I_{sp} (\text{Ideal})} \quad (26)$$

or substituting Eq.(25) into Eq.(26)

$$\eta_p = \frac{\sum f_i v_i^2}{\sum (f_i v_i)^2} \quad (27)$$

or

$$\eta_p = \frac{\bar{v}^2_{rms}}{\sum (v_i)^2} = \frac{(\bar{v}_{rms})^2}{(\bar{v}_{average})^2} \quad (28)$$

or since  $v$  is  $\propto$  to  $1/\sqrt{\text{Molecular weight}} = 1/\sqrt{\text{Mwt}}$

$$\eta_p = \frac{\sum f_i / \text{Mwt}_i}{\sum (f_i / \sqrt{\text{Mwt}_i})^2} = \frac{1 / \text{Mwt} (rms)}{1 / \text{Mwt} (average)}$$

It is clear from eqs.(26) through (28) that any single species will be more efficient as far as power is concerned than a multiple species propellant.

As an example, consider the case of propellant A of mass flow 1 unit per second molecular weight unity, power unity, and force unity in some artificial set of units. Next consider propellant B composed of 50 percent propellant A plus 50 percent of a propellant of Mass 4 times that of A.

	B				B	Normalizing to unit power
	A	50%	1	50%	4	
Mass flow/sec	1	1/2		1/2	1	8/5 or 1.6
Velocity	1	1		1/2	3/4 average	3/4 average
Force	1	1/2		1/4	3/4	6/5 or 1.2
Power	1	1/2		1/8	5/8	1

$$\text{Force} = (\text{mass flow/sec of fuel}) (\text{velocity of ions})$$

$$\text{Power} = (\text{mass flow/sec of fuel}) (\text{velocity of ions})^2$$

$$\text{Velocity} = 1/\sqrt{Mwt}$$

As shown above total force will be decreased by equal flow rates of A and B but the power will be cut also. Since the system is power limited, the flow must be increased to use all power available. The final result is that the force is increased to 1.2 units by use of fuel B but the consumption is increased to 1.6 units. This same force could be obtained by decreasing the velocity and increasing the flow rate of fuel A. The actual conditions for fuel A to obtain a thrust of 1.2 units would be a flow of 1.44 units of A and at velocity of .833 units.

Another approach to this problem is to state  $V$  times  $I$  is constant and the velocity,  $v$ , is proportional to  $\sqrt{V}$ , the mass flow is proportional to  $I$ , and force is proportional to  $I\sqrt{V}$  (or  $Iv$ ); thus  $I$  and  $V$  may be varied from design conditions to cause either a higher force and higher mass flow or a lower thrust at a lower mass flow.  $Isp$  is force per mass flow and as such is proportional to  $\sqrt{V}$ .

There may be some conditions where an existing engine must be used at fixed design voltage and current. A higher thrust may be obtained by the use of fuel B and a short duration mission could be performed more rapidly than with A alone, but not as efficiently. This mission could still be performed by a correctly designed engine using A alone for less total fluid expenditure. Thus, multi-species ion propellants should not be ignored but engines should not be designed using them either.