SUBSTRUCTURE AND THE FLOW STRESS OF POLYCRYSTALS

by

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ABSTRACT

From a study of polycrystalline alpha brasses, it is shown that the dislocation structure developed in the high Zn alloys (low stacking fault energy) during a strain ε gives a hardening $\chi\varepsilon + \beta\varepsilon^{1/2}\ell^{-1/2}$, where χ and β are constants and ℓ is the grain diameter. Thus, in the equation for the flow stress σ_f at constant strain, $\sigma_f = \sigma_o^f + k_f \ell^{-1/2}$, where σ_o^f is the friction stress and k is a constant, σ_o^f increases linearly with strain and k_f also increases with strain because of the inclusion in it of the strain hardening $\beta\varepsilon^{1/2}\ell^{-1/2}$. At high strains or low Zn contents, cross-slip and cell formation occurs; the significance of the grain size in the flow stress then diminishes and k_f falls.

The present paper describes a study of the effect of substructure on the flow stress of polycrystalline α-brasses. The term substructure is here used in a very general sense to denote the dislocation structure produced by the plastic deformation during the flow stress measurement. There is no attempt to restrict the term to mean the presence of definite subgrain networks.

The α-brass system was chosen for this work because it presents the possibility of a wide variation of dislocation locking strength and of stacking fault energy with composition, and because there is a long range of almost linear hardening, at least with high zinc contents. It was felt that these qualities might simplify the interpretation of the results and this proved to be the case.

First, the experimental facts. Prior to the present work, it had been shown (1) that the equation relating the lower yield stress σ_y of a polycrystalline material to the grain diameter ℓ, namely,

$$\sigma_y = \sigma_o + k\ell^{-1/2} \qquad \text{.......... (1)}$$

where σ_o and k are constants, has a counterpart in the equation relating the flow stress σ_f at a given strain with the grain diameter, so that

$$\sigma_f = \sigma_o^f + k_f \ell^{-1/2} \qquad \text{.......... (2)}$$

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This applies to the flow stress after the yield point and also to the flow stress in the absence of a yield point.

The present work has examined the effect of substructure through the application of equation (2). Figure 1, which plots the yield stress and the flow stresses at various strains for a 25% Zn alloy at 77°K as a function of grain size, illustrates the type of result obtained. It will be seen that equations (1) and (2) fit the measurements closely. Also, as the strain increases, $\sigma_o{}^f$ increases and there are alterations in k_f. This type of measurement was made on a series of brasses up to 35% Zn. The measurements extended to strains just below the onset of necking.

Figure 2 shows the variation with composition and strain of the slope k_f extracted from these measurements at 77° and 295°K. Three points are clear.

(a) The value of k at the yield point increases with zinc content, except for a final decrease at 35%.

(b) With the high zinc alloys, k_f increases with strain up to a maximum and then decreases. The maximum occurs at a strain of 0.1/0.15 for alloys with 25% Zn and above, but it occurs at lower strains with lower zinc contents, and at 10% Zn and below no initial increase in k_f was detected, only a progressive decrease.

(c) The value of k at the yield point is independent of temperature, but the increase of k_f with strain in the high zinc alloys is less at the higher temperature.

Values of σ_o were also extracted from the measurements. At the yield point, these showed an increase with zinc content and quite strong temperature-dependence, but the increase in $\sigma_o{}^f$ with strain is of greater interest in the present connection and this is plotted in Figure 3 for 77° and 295°K. With 20% Zn and above, the increase with strain is accurately linear at 77°K up to the highest strain used and the slope is practically independent of composition. Some departure from linearity occurs with the lower zinc alloys. The measurements at 295°K show that the linear slope is practically independent of temperature, but the departure from linearity is more marked at this temperature, and it occurs at smaller strains and in alloys of higher zinc content.

INTERPRETATION

In the simplest interpretation (1) of equation (2), derived from its relationship to equation (1), σ_f represents the "friction stress" in a slip band. That is, the stress a slip band could sustain if the displacement in it could move out freely at the grain boundary, so producing a step in the boundary. The other term $k_f\, l^{-1/2}$ then represents the limiting value of the additional stress the slip band can sustain because such step formation is opposed by the next grain. The limiting value of this additional stress arises when the stress generated at the end of the slip band induces plastic deformation in the next grain.

On this view, the increase of $\sigma_o{}^f$ with strain represents simply the effect of strain-hardening and the alterations in k_f must represent other complicated changes in the grain boundary resistance to slip band formation.

To maintain compliance with equation (2) and yet retain this simple interpretation, it will be noted that the rate of strain-hardening must be independent of grain size, so that all sizes have the same friction stress at a given strain. It is not obvious that this should be true. Unless the average distance a dislocation travels is the same for all grain sizes, the dislocation structure at a given strain will depend upon grain size and this may well give rise to a variation in the friction stress.

In the present paper, it is suggested that such a variation does happen in the α-brasses. The argument is that the strain-hardening has a grain-size-dependent component, so that part of the friction stress becomes incorporated in the $k_f \ell^{-1/2}$ term in equation (2) and this accounts for the increase in k_f with strain.

To develop this argument, the details of the strain-hardening have to be considered.

It is clear from Figure 1 that the principal strain-hardening effect appears in $\sigma_o{}^f$. Figure 3 shows that it is a linear hardening. Further, it is, of course, independent of grain size (since grain size effects are extrapolated out in obtaining $\sigma_o{}^f$), and it is independent of temperature and of composition, except that linearity breaks down at smaller strains the higher the temperature and the lower the zinc content. The rate of hardening is in the range $\mu/230$ - $\mu/290$, where μ is the rigidity modulus. All this suggests close similarity to Stage II hardening in f.c.c. single crystals. Enough has been argued about the mechanism of such hardening to make further comment unnecessary, and possibly unwise here.

Consider now the possibility of a grain-size-dependent strain-hardening. In the high zinc alloys, the stacking fault energy is low, so the wide separation of the partials confines them to the slip plane and inhibits cross-slip. In this circumstance, grain boundary pile-ups rather than dislocation networks should form in the strained condition. Consequently, L, the average slip distance of a dislocation, should be comparable to the grain diameter ℓ.

If ρ is the dislocation density and $\underline{b}$ is the Burgers vector, then at a shear strain γ,

$$\gamma = \rho \, b \, L.$$

Thus, where $L \sim \ell$, a fine grain specimen must have a higher dislocation density at a given strain than a coarse grain one. This will affect the strain-hardening due to dislocation intersection.

Taking the shear stress τ' required for intersection as

$$\tau' = \alpha \mu b \rho^{1/2}$$

where α is a constant, it follows that

$$\tau' = \alpha \mu b^{1/2} \gamma^{1/2} L^{-1/2}$$

Using the Taylor orientation factor m to convert to tensile stresses and strains (1) and putting L $= \ell$,

$$\sigma' = \alpha m^{3/2} \mu b^{1/2} \varepsilon^{1/2} \ell^{-1/2} \qquad \text{..........} \; (3)$$

Thus, on this model, which depends upon having $L \sim \ell$, there should be a parabolic strain hardening term that is proportional to $\ell^{-1/2}$

The total strain-hardening at ε will then be $(\chi\varepsilon + \beta\varepsilon^{1/2}\ell^{-1/2})$ where χ and β are constants and $\chi\varepsilon$ is the linear hardening that is extracted into $\sigma_o{}^f$ in equation (2) and $\beta\varepsilon^{1/2}\ell^{-1/2}$ is the parabolic hardening that will appear in k_f.

We now add to this a belief that the limiting grain boundary resistance to slip band formation arises when dislocations are forced out from the grain boundary at the end of a slip band and that the stress required depends principally on interaction with alloy atoms segregated in the boundary. This resistance should then remain fairly close, whatever the strain, to the original value $k\ell^{-1/2}$ given by equation (1) for the yield point.

With these conclusions and provided the strain does not affect the process responsible for the initial friction stress σ_o, the flow stress at should be given by

$$\sigma_f = \sigma_o + \chi\varepsilon + (k' + \beta\varepsilon^{1/2})\ell^{-1/2} \qquad \text{..........} \; (4)$$

Comparison with equation (2) gives

$$\sigma_o^f = \sigma_o + \chi\varepsilon$$

$$k_f = (k + \beta\varepsilon^{1/2})$$

Thus, it is concluded that the increase in k_f with strain observed with the high zinc alloys arises from the incorporation in $k_f\ \ell^{-1/2}$ of the parabolic strain hardening $\beta\varepsilon^{1/2}\ell^{-1/2}$ in addition to the grain boundary resistance $k\ \ell^{-1/2}$

The value of β from (3) is $\alpha m^{3/2} b^{1/2} \mu$, which gives 1.9 kg mm$^{-3/2}$, using α = 0.2 from Bailey and Hirsch (2). The present measurements on k_f for the 25, 30 and 35% Zn alloys at 77°K agree with equation (4), and give a value of β = 1.6 kg mm$^{-3/2}$

Since the parabolic hardening is attributed here to dislocation intersection, temperature-dependence is a possibility and the smaller increase in k_f with strain at 295°K compared to 77°K, shown in Figure 2, is consistent with this.

Current theoretical ideas on the theory of strain-hardening are rather fluid and it may be that developments in this field will require a modification of the present attribution of parabolic hardening to dislocation intersection. The point that we wish to emphasize is not so much this attribution as the existence of the parabolic grain-size-dependent term.

Consider now the drop in k_f with increasing strain that occurs with the low zinc alloys and even with the high zinc ones at larger strains (Figure 2).

The low zinc alloys have a high stacking fault energy, so cross-slip and the development of a dislocation cell structure can be expected at an early stage in the plastic deformation. Even with the low stacking fault alloys, cell formation should eventually occur.

Once cell structure is present, it is improbable that the slip distance is fixed by the grain size, so the basis for equation (4) and for the $\beta \varepsilon^{1/2} \ell^{-1/2}$ term disappears. With a strongly developed cell structure, this, rather than the grain structure, may become the significant unit in plastic deformation. Thus, the reduced significance of grain size shown by a drop in k_f in Figure 2 is thought to be due to cell formation when cross slip occurs. This condition is also associated with the departure from linear hardening of $\sigma_o{}^f$.

The conclusion from this study is that when the dislocations are confined to their slip planes, the dislocation structure developed on deformation gives a flow stress that follows equation (4), but once cross-slip, leading to cell formation, occurs, the significance of the grain size decreases and $\sigma_o{}^f$ shows departure from linear hardening.

REFERENCES

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2. J.E. Bailey and P.B. Hirsch, Phil. Mag., Vol. 5, 485, 1960.

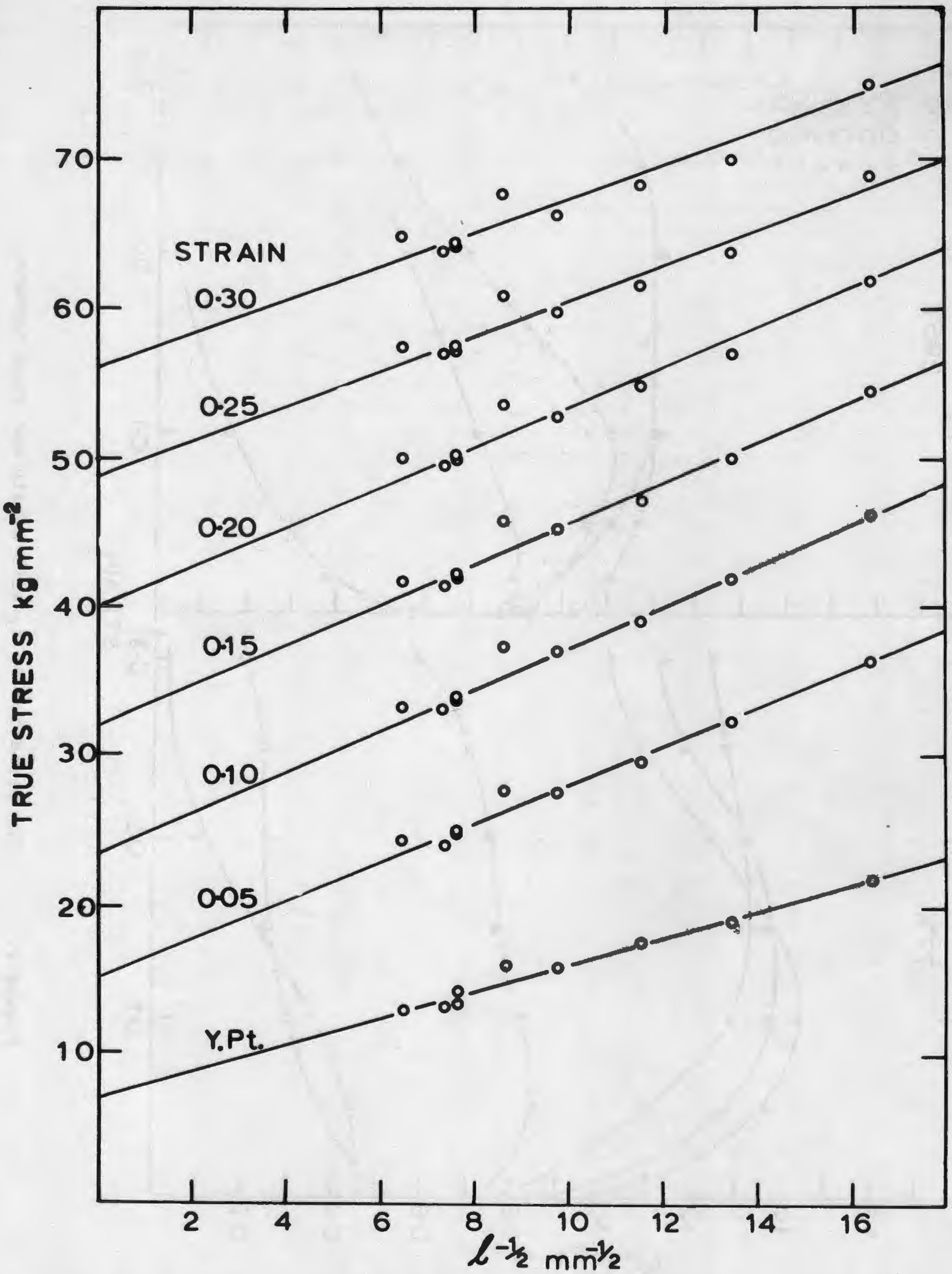

Figure 1. Yield and Flow Stress of 25% Zn alloy at 77°K as a function of grain size.

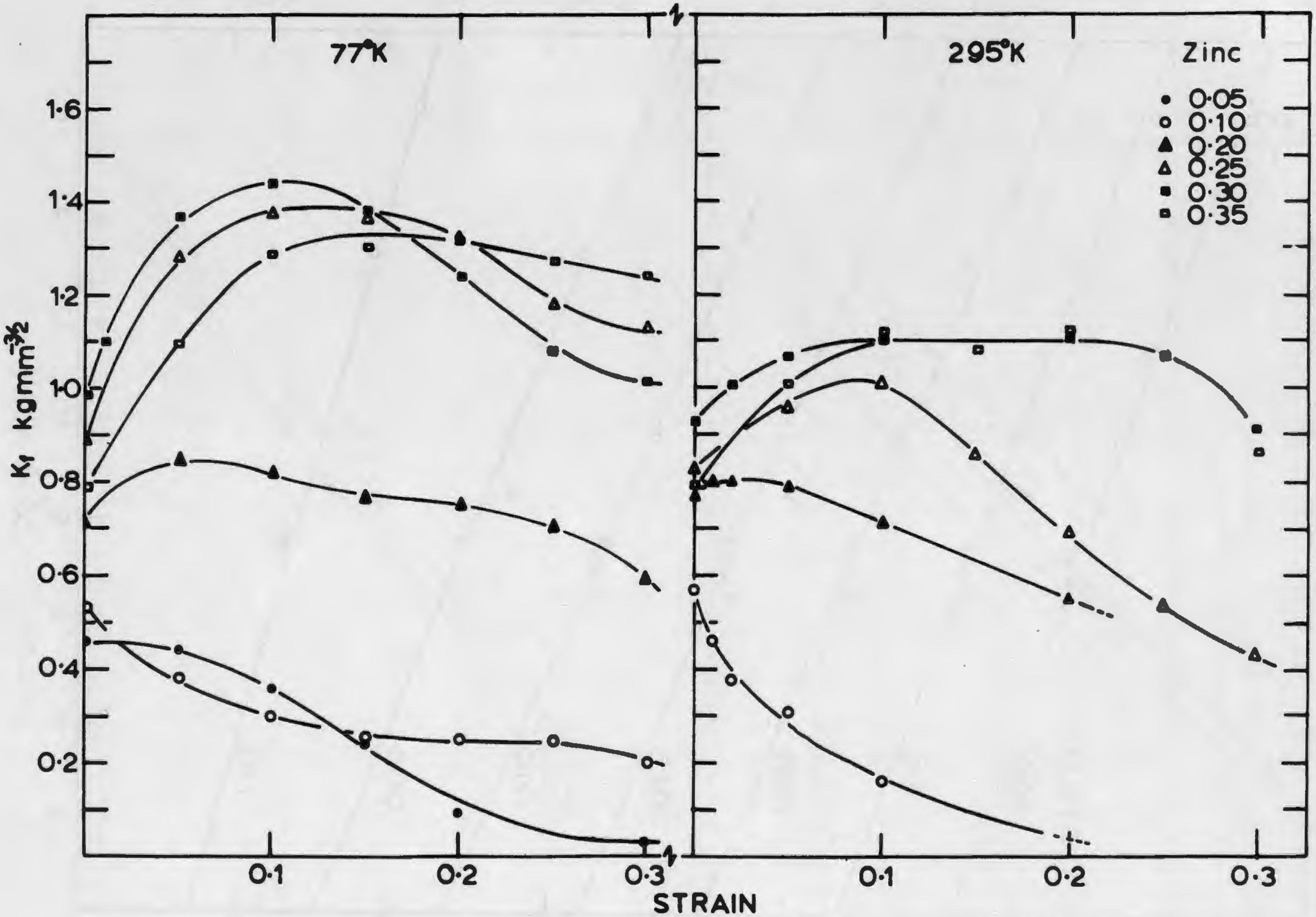

Figure 2. **The variation of k_f with strain and zinc content.**

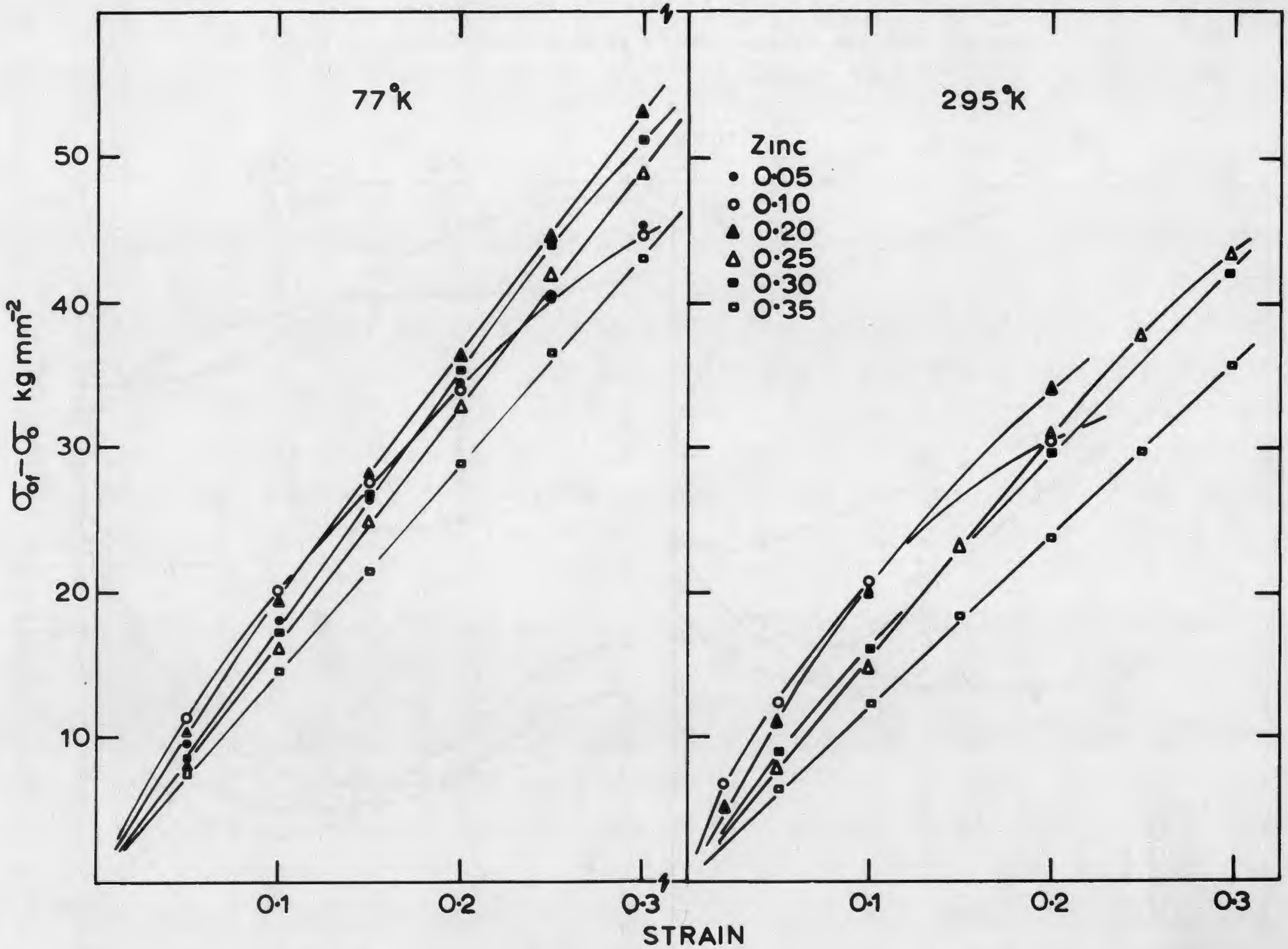

Figure 3. The variation of $\sigma_o{}^f$ with strain and zinc content.