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THE MECHANICAL IMPEDANCE OF THE HUMAN BODY IN SITTING AND STANDING POSITION AT LOW FREQUENCIES

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FOREWORD

The research reported here was conducted in support of Project 7231, Acoustic Energy Control, Task 71786, Biological Aspects of Vibratory and Acoustic Energy, administered by the Bioacoustic Branch, Biomedical Laboratory of the Aerospace Medical Laboratory. Tests were performed in the laboratories of Aeronautical Systems Division. The development of the special instrumentation was accomplished by personnel of the Aeronautical Research Laboratory, Kentucky Research Foundation, University of Kentucky, under the direction of Prof. Dr. K. O. Lange. Meritorious work has been done by Messrs. A.L. Wittwer, V.C. Currens, T.D. Sharp, Aeronautical Research Laboratory, under Contract No. AF 33(616)-5335. Much help was given during the evaluation of the test data by the University of Dayton, under Contract No. AF 33(616) 5465. Medical assistance was given by E. B. Magid, Capt, USAF, MC. The author thanks all contributors for their help without which this research would not have been possible.

The following papers present some of the results from this research:

Roman, J.A., R.R. Coermann, and G. Ziengenruecker, "Vibration, Buffeting and Impact Research," paper presented at the 29th Annual Meeting of the Aero Medical Association, Washington, D.C., 24-26 March 1958.

Coermann, R., "The Mechanical Impedance of the Human Body in the Sitting and Standing Position and its Significance for the Subjective Tolerance to Vibrations," paper presented at the 3rd Annual Meeting of the Biophysical Society, Pittsburgh, Pennsylvania, February 1959.

Coermann, R.R., "The Dynamic Properties of the Human Body in the Low Frequency Range and Their Significance upon the Subjective Tolerance to Vibrations," paper presented at the Third International Congress on Acoustics, Stuttgart, Germany, 1-8 September 1959.

Coermann, R.R., "The Response of the Human Body to Low Frequency Vibrations," paper presented at the 1959 Annual Meeting of the Society of Experimental Stress Analysis, Detroit, Michigan, 21-23 October 1959.

Coermann, R.R., G. Ziegenruecker, A.L. Wittwer, and H.E. vonGierke, "The Passive Dynamic Mechanical Properties of Human Thorax-Abdomen System and of the Whole Body System," paper presented at the 30th Annual Meeting of the Aerospace Medical Association, Los Angeles, California, 27 April 1959.

The first investigator using the impedance measurement on the human body was G. von Bèkèsy, in determining the threshold of vibration feeling for humans. He used a small table supported by load cells with quartz crystals as sensing elements.[1]

At the same time as our experiments were performed, D. Dieckmann made similar tests.[2] The seat used for his measurements was guided by a shaft in a tube and the force was measured by an elastic beam with strain gauges. The results obtained generally coincide with ours, but the measurements were taken only at a few frequencies sitting and standing without modifications of posture or restraint.

ABSTRACT

The theory of the mechanical impedance of systems with one or more degrees of freedom is applied to the human body. A method of measuring mechanical impedance and determining the parameters of the vibrating systems is developed. Impedance curves for longitudinal vibrations of a sitting and standing subject are established for the frequency range of 1 to 20 cps. The influence of varied posture and restraining systems is investigated. Dynamic movements of body parts are measured, directly or indirectly, and compared with the impedance curves. The responsible elements in the body for the apparent resonances are identified. Correlations between the impedance function of the body and the subjective tolerance curve to vibration are found and the reasons for the tolerance limits are elucidated. The variability of subjective tolerances due to varying posture, restraining systems, cushions, duration of exposure and vibrations are discussed, and conclusions for the development of protective devices are drawn. The correlation between the steady state response of the human body system and the effects of impact is discussed.

PUBLICATION REVIEW

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION I

INTRODUCTION

The human being in the environment of modern technology has to endure stresses of many and varied kinds of dynamic forces. When riding in vehicles on the ground, in the air or in space, he undergoes vibrations, buffetings, transient accelerations and decelerations, and, especially in cases of accidents, very hard impacts. All these stresses have first a mechanical effect on the human body which depends upon the dynamic properties of the body with respect to these special excitations. As a consequence of these mechanical effects, physiological effects will occur depending upon the magnitude and duration of the stress. The same consideration has to be applied if the forces act on certain parts of the body. For instance, if a blast wave hits the thorax of a man or if the man is exposed to rapid decompression of the surrounding air, the effects of these rapidly changing air pressures on the different parts of the body will be a function of the dynamic properties of the involved parts. Thus, before investigating any physiological effect of dynamic stresses, it is necessary to determine first the dynamic properties of the effected body system with regard to these specific stresses.

As a first approach, the human body may be considered as a very complicated system of masses, elasticities and viscous dampers, each connected to the others. It would be a hopeless attempt to describe this complex system exactly with respect to any excitation over the whole frequency range which may occur in the environment of modern technology. However, we can distinguish between three different kinds of alternating forces effective on the body: (1) the steady state vibrations and buffetings, transmitted by structures, (2) the impacts, transmitted by structures and air, and (3) the noise transmitted by air. Though our considerations may be applied principally to all three kinds of stresses as long as the induced wave has a wave length much larger than the length of the body, we will restrict this paper mainly to case (1) and only outline the basic application to case (2). For case (1) we can limit the considered frequency range from one cycle per second (cps) up to about twenty cps, because under 1 cps other physiological effects become more and more important and over 20 cps it is relatively easy to protect the human body against vibrations by means of mechanical damping systems.

In this report the human body will be considered in the sitting and standing positions, and only the force effective in the direction of the longitudinal axis of the body will be treated. This directional force as applied to a man in the sitting position is the most important aspect of this study. The lateral and transversal movements of the seat are less effective, since the compliances of the body in these directions are much higher. The mechano-dynamic properties of the human body will be described under the above limitations. The question is, then, how many degrees of freedom the system has or, in other words, how many lumped parameters have to be determined to predict the response of the system to the excitation considered.

SECTION II

THEORY OF THE MECHANICAL IMPEDANCE

A simplified model for the human body is a system consisting of a single mass (m), one spring with the elasticity (k) and one damper with the damping constant (c) connected to a circuit as shown in Figure 1. Assuming that the mass (m) is constant during the whole motion, the characteristic of the elasticity (k) is linear and the damping constant (c) is viscous, i.e., the damping force is proportional to the relative velocity $(\dot{x}_1 - \dot{x})$ of the connectors on the dashpot, then a sinusoidal motion of the seat $(x = x_0 \sin \omega t)$ will produce two different kinds of sinusoidal motions of the mass (m): the damped free vibration and the forced vibration. After a short time the damped free vibration disappears so that the forced vibration persists alone. For random motions of the seat and for impacts, the free vibrations may be of great importance, particularly if the damping of the system is low. However, first we will consider only the sustained vibrations and neglect the transient motions.

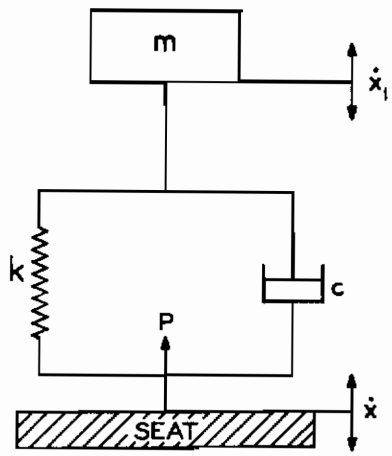


Figure 1. The Simplest Model of the Human Body is a One-Mass-Spring System with Damping

The equations which describe the motion of the mass (m), depending upon the motion of the seat, are well known. Since the motion of the mass is generally out of phase with the motion of the seat, these equations have to be complex, i.e., for each frequency not only the magnitude (called "Modulus") but also the phase (called "Argument") between the variables changes.

Since the sum of all forces in the system must be zero, the relative displacement $(x_1 - x)$ of the mass (m) can be calculated from the equation

$$m\ddot{x}_1 + c(\dot{x}_1 - \dot{x}) + k(x_1 - x) = 0$$

or $$m(\ddot{x}_1 - \ddot{x}) + c(\dot{x}_1 - \dot{x}) + k(x_1 - x) = -m\ddot{x} \qquad (1)$$

For a sinusoidal motion of x, $(-m\ddot{x})$ can be substituted by $m\omega^2 x_o$, where x_o = amplitude of x. Then equation (1) has the same form as the fundamental equation for the forced vibration with viscous damping, i.e., in $m\ddot{x} + c\dot{x} = kx = P_o(\sin \omega t)$ merely substitute $(x_1 - x)$ for x and $m\omega^2 x_o$ for the force. The solution of that equation is well known[3] so that the modulus and the phase angle of equation (1) can be written immediately as:

$$(x_1 - x) = \frac{m\omega^2 x}{\sqrt{(c\omega)^2 + (k - m\omega^2)^2}} \tag{2}$$

and

$$\tan\phi \frac{(x_1 - x)}{x} = \frac{c\omega}{k - m\omega^2} \quad ; \quad \phi = \text{phase between } (x_1 - x) \text{ and } x. \tag{3}$$

Introducing the abbreviations

$$n = \frac{f}{f_o} = \frac{\text{Frequency of the forced vibration}}{\text{Undamped natural frequency of the system}} = \frac{\omega}{\omega_o}$$

$$\omega = 2\pi f; \qquad \omega_o = 2\pi f_o$$

$$\delta = \frac{c}{\sqrt{km}} = \frac{\omega_o c}{k} = \frac{c}{\omega_o m} = \text{damping factor of the system}$$

$$\omega_o = \sqrt{\frac{k}{m}} = \text{undamped natural frequency of the system,}$$

the equations (2) and (3) can be changed into:

$$\frac{(x_1 - x)}{x} = \frac{n^2}{\sqrt{(1 - n^2)^2 + n^2\delta^2}} \tag{4}$$

and

$$\tan\phi \frac{(x_1 - x)}{x} = \frac{n\delta}{1 - n^2} \tag{5}$$

The force P transmitted from the seat to the model is composed of the force of the spring $k(x_1 - x)$ and the force of the damper $c\omega(x_1 - x)$. The phase angle between these forces is always 90° so that the combined force has the magnitude

$$P = (x_1 - x)\sqrt{k^2 + (c\omega)^2} = (x\omega)\, m\omega \sqrt{\frac{\delta^2 n^2 + 1}{(1 - n^2)^2 + n^2\delta^2}}$$

$$= x m \omega \sqrt{\frac{\delta^2 n^2 + 1}{(1 - n^2)^2 + n^2\delta^2}} \tag{6}$$

All equations are functions of the ratios of the three factors m, k and c.

Unfortunately, we cannot measure these ratios on the human body directly; and, moreover, the model of Figure 1 is undoubtedly oversimplified, which, in turn, actually requires the determination of even more parameters. However, there is another possibility of obtaining a good insight into the dynamic properties of a complex system. Similar to the measurement of a complex resistance, such as the so-called "impedance" of an electrical circuit which consists of inductivities, capacities and resistances, we can measure the mechanical impedance of the human body. According to the electrical impedance, which is the complex ratio of the voltage to the current going through the circuit ($Z = U/I$), we can define the mechanical impedance as the ratio of the transmitted force to the velocity of that point where the force is transmitted.

$$Z = \frac{P}{\dot{x}}$$

This quantity can be measured for the excitation considered by way of an experiment for each subject regardless of how complicated the system really is. For our problem the subject had to sit on a very stiff plate which is connected over force transducers to the vertically vibrating exciter (shake table) producing sinusoidal motions. For each frequency the transmitted force, the velocity of the shake table, and the phase between force and velocity have to be measured. Plotting the modulus of the impedance and the phase angle vs the applied frequencies, we obtain two curves from which we can derive under certain limitations the response of the subject to steady state vibrations.

Equation (6) gives immediately the modulus of the impedance for the simplified model of Figure 1.

$$Z = \frac{P}{\dot{x}} = m\omega\sqrt{\frac{\delta^2 n^2 + 1}{(1-n^2)^2 + n^2\delta^2}} \tag{7}$$

The phase angle of Z is given by the equation

$$\tan\phi(Z) = \frac{1 - n^2(1-\delta^2)}{n^3\delta} \tag{8}$$

The formula for the impedance is composed of the values of the impedance of a pure mass:

$$Z_{mass} = \frac{m\ddot{x}}{\dot{x}} = \frac{m x_o \omega^2}{x_o\omega} = m\omega \tag{9}$$

and the square root of a function consisting of the damping factor and the ratio of the forced vibration to the undamped natural frequency of the system. At resonance, i.e., if the frequency of the forced vibration is equal to the undamped natural frequency, $n = 1$ and thus the impedance

$$Z_{n=1} = m\omega_o\sqrt{1 + \frac{1}{\delta^2}} \tag{10}$$

Equation (8) gives the value of δ for $n = 1$:

$$\delta = \tan \phi_{\,n = 1} \tag{11}$$

This indicates that the phase of the impedance is not zero at the undamped natural frequency, but depends upon the damping factor, δ .

It would be incorrect to assume that the frequency at the peak of a response curve is always the undamped natural frequency (f_o). For the impedance curve of the one-mass-spring system as in Figure 1, the location of the resonant peak, depending on the damping factor δ , can be determined by setting $\dot{Z} = 0$.

This condition is fulfilled if

$$n^2 = \frac{\delta^2 + \sqrt{2\delta^2 + 1}}{2\delta^2 + 1 - \delta^4} \tag{12}$$

Curve a in Figure 2 demonstrates this function. Up to $\delta = 0.7$, the difference of the measured resonant peak from the undamped natural frequency is < 5%. For $\delta = 1.55$, n = infinity, i.e., the impedance curve has no peak. Above $\delta = 1.55$, n becomes imaginary.

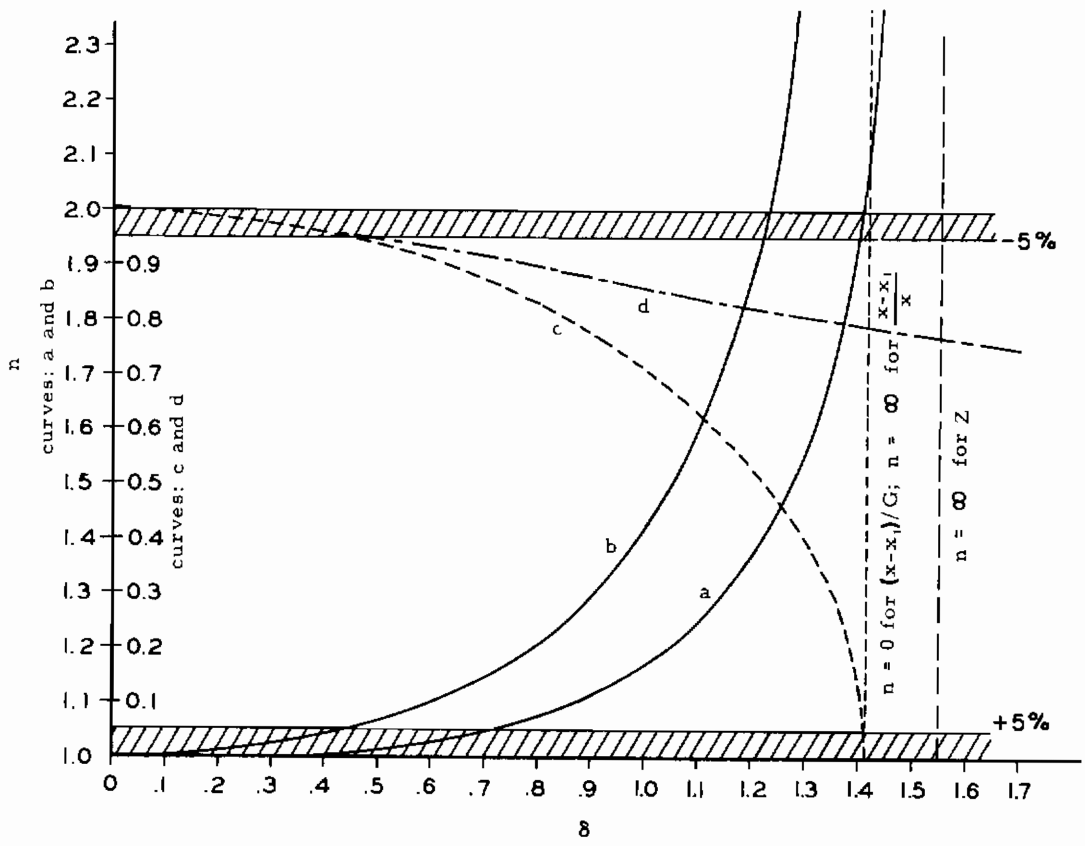


Figure 2. Change of the Resonant Peak of a One-Mass-Spring System with Increasing Damping Factor δ : (a) for Z, (b) for $\frac{x - x_1}{x}$, (c) for $\frac{x - x_1}{G}$, (d) for $\frac{x_1}{x}$

The maximum of the relative displacement of the mass related to the displacement of the point of excitation ($\frac{x_1 - x}{x}$) is located where

$$n^2 = \frac{1}{1 - \frac{\delta^2}{2}} \qquad (13)$$

and related to the acceleration of the point of excitation ($\frac{x_1 - x}{G}$) is located where

$$n^2 = 1 - \frac{\delta^2}{2} \qquad (14)$$

Both functions are also plotted in Figure 2 (curves b and c). The transmission factor $\frac{x_1}{x}$ has its maximum if

$$n^2 = \frac{\sqrt{1 + 2\delta^2} - 1}{\delta^2} \qquad (15)$$

This function is demonstrated by curve d in Figure 2. The three curves b, c, and d exceed the $\pm 5\%$ error range at $\delta = 0.44$. This indicates that the damping factor δ has the least influence on the location of the maximum of the impedance curve. Allowing for an error of 5%, the measured peak of the impedance curve can be taken as the undamped natural frequency of this system if $\delta < 0.7$. In the case that $\delta > 0.7$ or that higher accuracy is required, a step-by-step procedure must be undertaken to approach the correct value of ω_o. In most cases, two steps will be enough, because for a small δ the phase changes rapidly near resonance but the influence at the location of the peak is small and for a high δ the phase around the resonance changes only a little.

Substituting equation (11) for equation (10), we obtain the effective mass of the system at the resonant frequency

$$m_{n=1} = \frac{Z_{n=1}}{\omega_o} \frac{1}{\sqrt{1 + \frac{1}{\delta^2}}} \qquad (16)$$

Hence, from the measured values of Z and ϕ at ω_o, the percentage of the whole body mass participating in the vibrations at resonant frequency can be determined. If the body responds as a one-degree-of-freedom system, then 100% of the body mass will move. A value less than 100% indicates that some body parts have a smaller displacement and a value of more than 100% indicates that some body parts have a larger displacement than the center of gravity of the body.

However, the above mentioned method has the disadvantage of not being very accurate, because around the resonant frequency the phase changes very fast, making correct measurements difficult. Fortunately, there is another possibility of obtaining the magnitude of the parameters from the measured impedance curve. Setting $n = \sqrt{2}$ in equation (7), the radicand becomes one, so that

$$Z_{n=\sqrt{2}} = m\omega \qquad (17)$$

Having the undamped natural frequency ω_o of the system, we can determine the effective mass by measuring the impedance at the frequency $\sqrt{2}\,\omega_o$; then,

$$m = \frac{Z_{\sqrt{2}}}{\sqrt{2}\,\omega_o} \tag{18}$$

From the definition of $\omega_o = \sqrt{\frac{k}{m}}$, the elasticity of the system can be determined by

$$k = m\,\omega_o^2 \tag{19}$$

Then, according to equation (10)

$$\delta = \frac{1}{\sqrt{\left(\frac{Z_{n=1}}{m\,\omega_o}\right)^2 - 1}} \tag{20}$$

and from the definition of δ :

$$c = \omega_o m \delta = \frac{\delta k}{\omega_o} = \delta\sqrt{km} \tag{21}$$

To obtain the three parameters which describe the dynamic response of a one-mass-spring system completely, it is only necessary to measure ω_o, the modulus of the impedance of the system at ω_o and at $\sqrt{2}\,\omega_o$. From these three values, the impedance and the phase angle curve of the system over the whole frequency range can be calculated by the two equations (7) and (8).

Probably the measured impedance curve will show another peak at a higher frequency. For this peak we can conduct the same procedure as above and calculate ω_{o_2} from the measured moduli of the impedance at the second resonant peak ω_{o_2} and at a frequency $\sqrt{2}$ times higher the values of the three parameters m_2, k_2, and c_2. Then the simplified model of Figure 1 can be replaced by an improved model consisting of one system with the parameters m_1, k_1, and c_1, the resonance at ω_{o_1}, and the impedance Z_1, and another system with the parameters m_2, k_2, c_2, the resonance at ω_{o_2} and the impedance at Z_2.

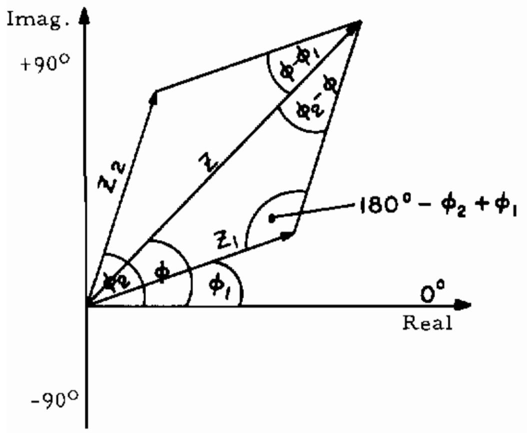
However, when the two resonant peaks lie too closely together, the influence of the adjacent system at ω_o and at $\sqrt{2}\,\omega_o$ cannot be neglected. Then a step-by-step method must be applied, first taking the impedances at ω_{o_1} and $\sqrt{2}\,\omega_{o_1}$ of the highest peak and calculating m_1, k_1, and δ_1. Figure 2 gives then the correction for ω_{o_1} and thus the improved values for m_1, k_1 and δ_1. With equations (7) and (8), Z_1 and ϕ_1 can be calculated for the frequency range considered. These values have to be deducted vectorially from the measured Z and ϕ by the equations

$$Z_2^2 = Z_1^2 + Z^2 - 2\,Z\,Z_1 \cos(\phi - \phi_1) \tag{22}$$

and

$$\tan \phi_2 = \frac{\frac{Z}{Z_1} \sin \phi - \sin \phi_1}{\frac{Z}{Z_1} \cos \phi - \cos \phi_1} \qquad (23)$$

over the frequency range studied in order to get the impedance and phase angle curve of the remaining system. The equations (22) and (23) can be derived from the vector diagram in Figure 3.



$$Z_2^2 = Z^2 + Z_1^2 - 2 Z Z_1 \cos(\phi - \phi_1)$$

$$\frac{Z}{Z_1} = \frac{\sin(180° - \phi_2 + \phi_1)}{\sin(\phi_2 - \phi)}$$

$$= \frac{\sin(180° + \phi_1) \cos \phi_2 - \cos(180° + \phi_1) \sin \phi_2}{\sin \phi_2 \cos \phi - \cos \phi_2 \sin \phi}$$

$$\frac{\sin \phi_2}{\cos \phi_2} = \tan \phi_2 = \frac{\frac{Z}{Z_1} \sin \phi - \sin \phi_1}{\frac{Z}{Z_1} \cos \phi - \cos \phi_1}$$

Figure 3. Vector Diagram for the Subtraction of Z_1 from measured Z

From this second impedance curve, the parameters m_2, k_2, and δ_2 can be calculated in the same manner as before and the theoretical curves of Z_2 and ϕ_2 can be calculated with equations (7) and (8). The vectorial deduction of the Z_2 and ϕ_2 curves from the second impedance curve gives the third impedance curve demonstrating the impedance of the residual system of the model. This procedure can be continued until the remaining impedance curve is negligible in its magnitude compared with the originally measured impedance curve. Finally, an impedance curve will remain which no longer has a peak but is ascending and descending over the entire frequency range. This means that the next higher system does not consist of a mass-spring system but only of elements of a pure mass or a single spring or a single damper or a combination of mass and damper or spring and damper. The characteristics of the impedance and the phase of such elements are shown in Figure 4. By comparing these characteristics with the remaining impedance and phase curves, it can be decided which kind of substituting element should be used to obtain a better approach to the measured impedance and phase curves. In this way it is possible to find a model which can be substituted for the human body which has, with fair accuracy, the same dynamic response to the studied excitation. Revealing conclusions about the mechanism of the effect of vibration in the body can be made from this model.

By utilizing complex quantities, it is relatively easy to calculate the impedance of a model consisting of two or three damped mass-spring systems and simple elements, such as a pure mass or spring or damper, etc., if the values for all parameters are acquired by the method described above. The formula of the impedance of the system as in Figure 1 in complex form is

$$\overline{Z} = m\omega \frac{1 + jn\delta}{n\delta + j(n^2 - 1)} = m\omega \left[\frac{n^3 \delta}{(n^2 - 1)^2 + n^2 \delta^2} + j \frac{1 - n^2 + n^2 \delta^2}{(n^2 - 1)^2 + n^2 \delta^2} \right] \qquad (24)$$

the impedance of a pure mass is

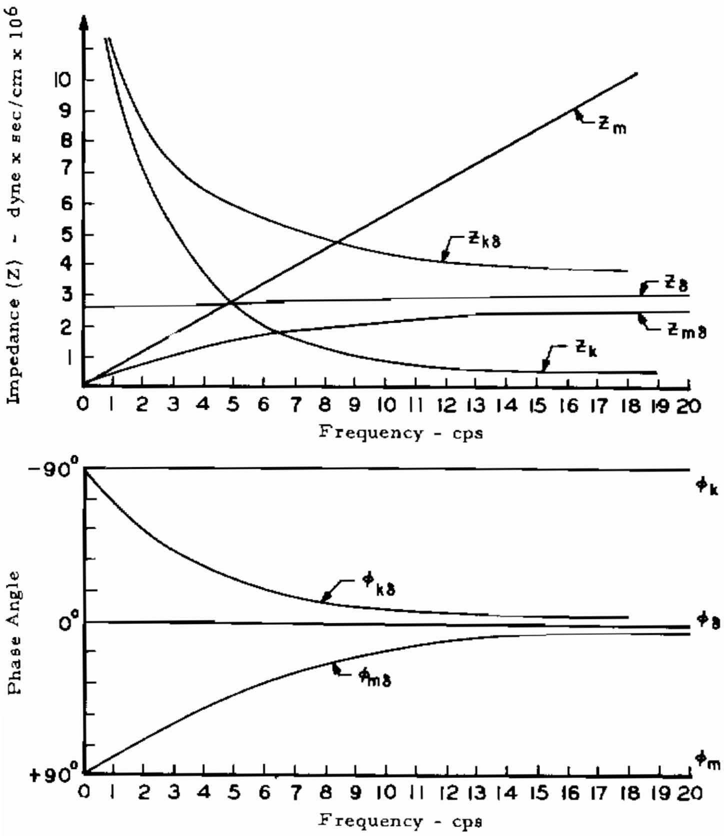
$$\overline{Z} = jm\omega \qquad (25)$$

of a pure spring

$$\overline{Z} = -j\frac{k}{\omega} \qquad (26)$$

of a pure damper

$$\overline{Z} = c \qquad (27)$$



Pure mass: $Z_m = m\omega$, $\phi_m = +90^\circ$

Pure spring: $Z_k = \frac{k}{\omega}$, $\phi_k = -90^\circ$

Pure damper: $Z_\delta = c$, $\phi_\delta = 0^\circ$

Mass-damper system: $Z_{m\delta} = m\omega\sqrt{\frac{c^2}{(m\omega)^2 + c^2}}$, $\phi_{m\delta} = \tan^{-1}\frac{c}{\omega m}$

Spring-damper system: $Z_{k\delta} = \sqrt{c^2 + (\frac{k}{\omega})^2}$, $\phi_{k\delta} = -\tan^{-1}\frac{k}{\omega c}$

Curves plotted for:

$m = 86200 \frac{\text{Dyne x sec}^2}{\text{cm}}$

$k = 85.25 \frac{\text{Dyne}}{\text{cm}}$

$\delta = 0.636$

$c = 1.72 \times 10^6 \frac{\text{Dyne x sec}}{\text{cm}}$

Figure 4. Impedance and Phase Characteristics of a Pure Mass, Pure Spring, Pure Damper, Mass-Damper and Spring-Damper System

Since the sum of the impedance of elements connected in series is

$$\overline{Z} = \overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3 + \ldots\ldots \tag{28}$$

and of elements connected parallel is

$$\overline{Z} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots\ldots} \tag{29}$$

the expression for any combination of systems and elements can be written in complex form and, after separating the real factors ΣA from the imaginary factors ΣjB, the equation for the modulus and the phase angle of the total system can be determined according to the principle

$$Z = \sqrt{A^2 + B^2} \tag{30}$$

and

$$\tan \phi = \frac{A}{B} \tag{31}$$

Principles for combining sub-systems into a complete diagram are explained in publications about vibration analysis of machinery.[4] Hence, by computing the impedance of the model, it can be proved that the body can be substituted for that model with sufficient accuracy.

Considering an excitation with a steady state and sinusoidal motion of the seat, we obtain for the transmitted force (vector)

$$P = Z_1 \dot{x} = x_o \,\omega\, Z \tag{32}$$

which we measure directly during the experiments or which we can calculate if the impedance curve is given.

In order to obtain the transmitted force over the considered frequency range at constant acceleration of the seat, equation (32) can be changed to

$$P = x_o \frac{\omega^2}{\omega} Z = A \frac{Z}{\omega} \tag{32a}$$

A = vector of acceleration

The energy which is transmitted from the seat to the body may be important, too. The active or real energy which is dissipated in the body has to be distinguished from the reactive energy which alternates during each oscillation between latent energy and kinetic energy.

The total energy is

$$\overline{N} = \dot{x}^2 \overline{Z} = x_o^2 \omega^2 \overline{Z} \tag{33}$$

of which the active energy is:

$$N = x_o^2\ \omega^2 Z \cos\phi\ = x_o A Z \cos\ \phi\ = (\frac{A}{\omega})^2 Z \cos\phi \qquad (34)$$

and the reactive energy is:

$$(N) = x_o^2\ \omega^2 Z \sin\phi\ = \ (\frac{A}{\omega})^2 Z \sin\phi \qquad (35)$$

For the simple system of Figure 1, the phase will turn from +90° at low frequencies to 0° after the resonance and then to negative angles. This means that the active energy will be zero at low frequencies, will be a maximum around the resonance, and will decrease at higher frequencies, but will never again become zero as long as the damper is effective.

The reactive energy turns from its maximum at low frequencies to zero around the resonance to a negative value above the resonance, which indicates that the elastic forces then dominate the mass forces. For a more complicated system, the phase may or may not cross the zero line or may even cross the zero line several times. Each time this occurs the active energy has a maximum, or, in other words, a maximum amount of energy will be dissipated in the body. But it is still to be determined whether this dissipated energy has a significant effect on the body or not.

SECTION III

INSTRUMENTATION AND TEST METHOD

In order to measure the impedance of a sitting or standing man, the subject has to sit or stand on a very stiff plate connected to a shake table over force-transducers. The natural frequency of the system consisting of the mass of the subject and the elasticity of the plate must be at least 6 times higher than the highest frequency to be studied (in our case 20 cps), assuming that the damping factor of the system is $\delta > 0.2$. Furthermore, the natural frequency of the system consisting of the mass of the subject plus the mass of the plate and the elasticity of the supporting force-transducers must be in the same range.

According to equations (7) and (8), the error of the impedance measurement will be < 2% and < 0.1° for the phase angle under these conditions.

However, at 20 cps only a small part of the body mass will be effective and the error in the impedance measurement reduces to < 1%.

A. The Plate

The plate was designed as an equilateral triangle of 24 inches long on each side, constructed of aluminum U-beams 3 inches high, and welded together. The triangle was reinforced internally by three radial I-beams. The weight of the plate including one-third of the weight of the force transducers was 22 lbs. With the support of three force-transducers on the corners of the triangle and the load of 175 lbs., the deflection of the place in the center was 0.0005 inches which means that the plate had an elasticity of $k_{Pl} = 35 \times 10^4$ lbs/inch and with a total load of 200 lbs. for the subject, a natural frequency of

$$f_{o_{Pl}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 130 \text{ cps}$$

assuming that the load is concentrated at the center of the plate. Thus, the lowest natural frequency was > 6.5 times higher than the highest frequency studied, since the load was actually distributed over a larger area of the triangle plate.

B. Force Transducers

Figure 5 shows the force cells for sensing a force in the vertical direction. They are constructed of nichrome steel and have an overall diameter of 5 inches and a width of 1-1/2 inches. An oval shape machined in the center of the force cell leaves 0.250 inches of wall thickness perpendicular to the axis of load. The machined oval also serves as an internal mounting place for the sensing element. Two symmetrical flattenings on the outside of the ring are used for the connection to the triangular plate and the shake table. The method of calculating the effective elasticity of the rings is shown in the appendix.

The rings employed had an elasticity of

$$k_R = 10^6 \frac{\text{lbs}}{\text{inch}}$$

and 14,000 lbs. could be applied before the elastic limit was exceeded.

Thus, the total elasticity of all three force-cells was

$$k_{FC} = 3 \times 10^6 \frac{\text{lbs}}{\text{inch}}$$

and the natural frequency with a mass of 222 lbs.(200 lbs. subject + 22 lbs. table) was

$$f_{o_{FC}} = 364 \text{ cps}$$

or 18 times higher than the highest frequency studied. Therefore, the problem of constructing a force-cell table for higher frequencies lies more in the design of a stiff seat than in the stiffness of the force transducers.

C. Sensing Elements

The deflection of the rings at 1 G acceleration with a load of 220 lbs. was

$$\Delta_r = \frac{220}{3 \times 10^6} = 0.000073 \text{ inch} = 1.85 \, \mu\text{m}$$

A very sensitive element was necessary to measure such a very small displacement with an accuracy of at least ± 1%.

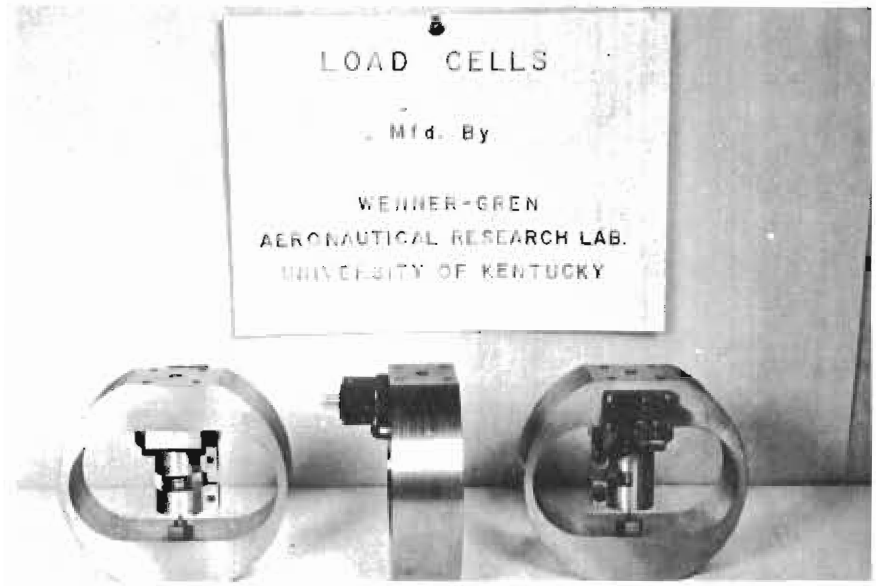


Figure 5. The Force Cell with Variable Reluctance Transducer

Among the different methods of measuring very small displacements (such as a strain gage or a variable capacity or variable reluctance transducer), the variable reluctance transducer was chosen but with two modifications:

1) the cross-armature system was employed, which gives about 30 times more sensitivity than the axial-core system (differential transformer);

2) both coils were double wound so that the full bridge was located in the cells. Compensation for changes of temperature of the coils was very good, since each branch of the bridge had the same resistance and reluctance.

The cross section of the coils with the armature and the circuit of the bridge is shown in Figure 6. The gap between the armature and the coils was about 0.3 of a millimeter. By changing the gap, the sensitivity of the transducer could be adjusted as required. To avoid shifting of the zero-line due to the changes of the temperature, the ring, the holder for the coils, and the rod to hold the armature were manufactured of materials with the same coefficient of expansion.

The three bridges were fed by 5 volts at 3000 cps from a CEC Type 1-118 Carrier Amplifier. The output leads were connected to three equal windings of a transformer from which the outlet was connected to the input of the Carrier Amplifier. In this way, the output voltages from the three transducers were added to a total signal and the total force could be recorded on one channel of the recorder.

However, the sensitivity of the individual force cells had to be adjusted to an equal value so that the total output was independent of the position of the center of gravity of the subject.

The mechanical shake table employed could not be expected to produce an absolutely pure sinusoidal motion. Therefore, in order to suppress the extraneous superimposed accelerations or forces, a bandpass-filter had to be placed between the Carrier Amplifier and the recorder. The attenuation and the phase shift produced by this filter had to be considered during the evaluation of the recordings.

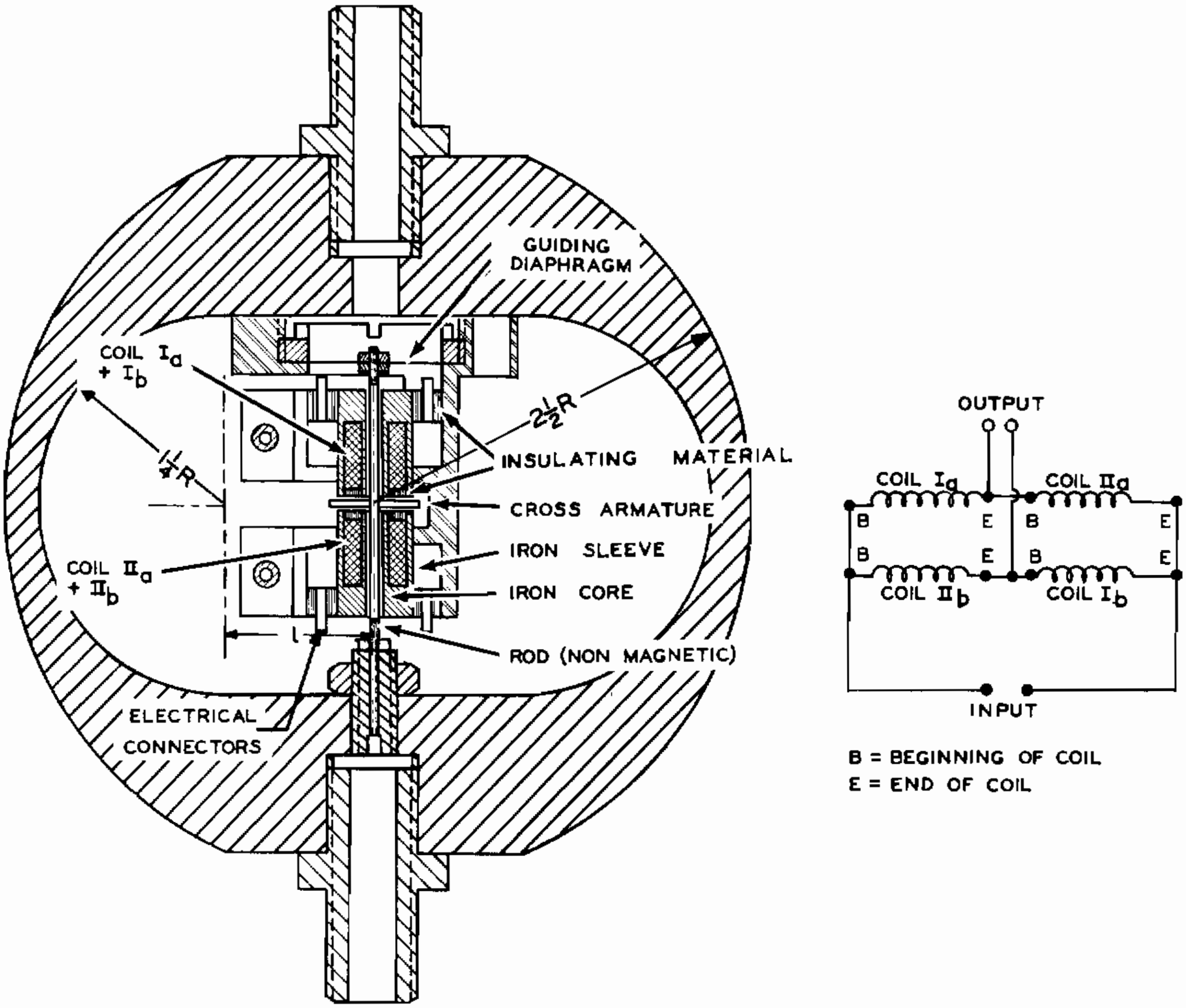


Figure 6. Cross Section of the Force Cell and the Circuit of the Bridge Used as Force Transducer

For the same reason, the displacement, not the velocity, of the shake table was recorded by means of a variable reluctance type displacement meter. The amplitude of the velocity (v_o) for the sinusoidal motion can be calculated from the displacement amplitude (x_o) by $v_o = x_o \omega$ and the phase of the velocity is always 90^o ahead of the displacement.

D. Data and Analysis

Before measurements were taken with a subject, the equipment was calibrated. The displacement meter was calibrated statically. To obtain the gage factor of the force-cell table and its frequency response, two lead bars, each weighing 75 lbs., were bolted rigidly to the force cell table and shaken at various amplitudes from 1 to 20 cps. The calculated acceleration times the mass of the lead plus the mass of the triangle table was the applied force. The force cells were linear and had a flat frequency response up to 20 cps. Furthermore, the phase shift between force and amplitude produced by the electrical filter could be analyzed from this record. The paper speed of the recorder had to be fast enough to measure accurately the time between the peaks of the amplitude tracing and the force tracing. The phase shift between force and table amplitude due to the electrical circuit is

$$\phi = 360^{\circ} \, af \qquad (36)$$

a = time between the peak of the amplitude trace and the force trace (sec)

f = applied frequency (1/sec)

In the evaluation of the records, it had to be considered that the force (F_T) to accelerate the mass of the triangular table (including 1/2 of the mass of the rings) has a different phase to the displacement of the shake table than the force to accelerate the mass of the subject (F_S). Therefore, the force for the table had to be subtracted vectorially from the measured force. From the vector diagram in Figure 7, the magnitudes can be derived immediately:

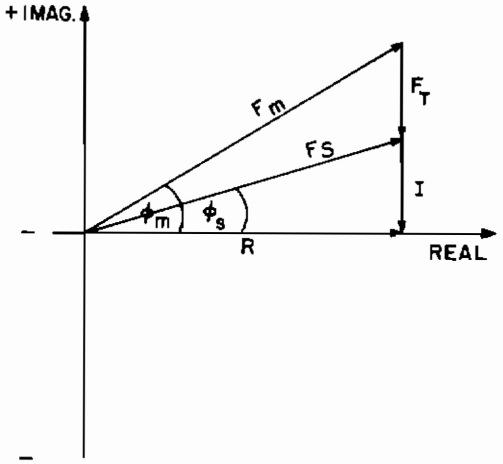


Figure 7. Vector Diagram for the Subtraction of the Mass Force of the Plate from the Measured Force

The imaginary part of F_s

$$I = \sin \phi_m F_m - F_T \qquad (37)$$

The real part of F_m and F_S

$$R = \cos \phi_m F_m \qquad (38)$$

Thus

$$F_s = \sqrt{R^2 + I^2} \qquad (39)$$

$$\phi_s = \tan^{-1} \frac{I}{R} \qquad (40)$$

F_m = measured force (Dyne)

ϕ_m = measured phase angle ϕ

F_T = $x_o \omega^2 \times 22 \times 454 = x_o \omega^2 \times 10000$ (Dyne)

x_o is measured in cm

The phase angle ϕ between the force and the velocity of the shake table is 90° ahead of the displacement.

$$\phi = \phi_s + 90^\circ \qquad (41)$$

With the subject on the table, the amplitude of the shake table was kept at a level comfortable to the subject. The man's legs hung down freely over the side of the shake table. The hands lay on the upper thigh and all belting was removed from the waist. There was no padding between the force cell table and the man except for clothing.

E. Test Program and Procedure

Records were taken at erect and relaxed postures of the sitting subject and at an erect posture standing with stiff knees. In order to check the influence of the mobility of the soft body parts, such as viscera, diaphragm, heart and lungs, and of the elasticity of the pelvis, a semi-rigid envelope was applied to the abdomen and the impedance measurements were repeated. In the same way, the influence of an Air Force Type MC3 Pressure Suit was tested. The linearity of the impedance, i. e., the changes of the impedance with increasing velocity of the seat, was also an important question. Due to the limited subjective tolerance to vibrations, this test could be performed only up to 0.5 G over the frequency range studied.

At the moment before the records were taken, the subject breathed out completely. The exposure time to the vibration was an average of one minute for each record. About fifty records were taken for each condition to cover the frequency range studied. From 1 to 14 cps frequency intervals of 1/2 cps and above 14 cps intervals of 1 cps were chosen. After the highest frequency was recorded, 20 records were taken with frequencies decreasing between the frequencies of the first series. This acted as a control on the first measurements. About 2000 records were necessary for the entire program.

Another test made was the measurement of the transmission of vibrations through the body to the head. The transmission factor is defined as the ratio between the acceleration on the head to the acceleration on the seat. This indicates the stretching and compression of the skeleton at various frequencies. At resonances due to the elasticity of the skeleton, the transmission factor should also show a resonance peak. Lightweight accelerometers (2.5 gr.) were mounted on the top of the head with an elastic bandage to measure the acceleration of the head.

It is of major interest to know which part of the skeleton undergoes the most stretching at the resonant frequency. Since it is not possible to connect an accelerometer or a displacement meter directly on a bone, means have to be employed to record from points on the body with a minimum shift relative to the skeleton. A flexible aluminum girdle with two molds covering the hip bone was tightened around the pelvis. Another flexible aluminum plate was mounted by elastic belts over the area of the first thoracic vertebra. The relative displacements between the shake table and the hip girdle and between the hip girdle and the neck plate were measured by elastic capillary tubes filled with mercury and connected to a resistance bridge. A stretching of the tube changed the resistance of the mercury column giving a proportional voltage output of the bridge. The system was calibrated statically, proving that the linearity was sufficient in the required range. Transversal vibrations of the tube could be suppressed by adhesive tape. These measurements were accomplished only tentatively on one subject, since a larger program to measure relative body displacements under vibration and impact will be started in the near future.

Eight subjects with different weights, heights, and ages were employed in the measurements.

TABLE 1

Data of the Employed Subject Panel

Subject	Weight - lbs.	Height	Age
W. B.	198	6'0"	34
R. C.	185	6'2"	47
B. D.	219	6'4"	30
W. E.	155	5'9"	29
W. G.	180	5'11"	40
R. H.	200	5'11"	35
E. M.	208	6'0"	29
G. Z.	158	5'7"	29

The subjects were in healthy condition without any abnormality.

SECTION IV

RESULTS

A. The Unrestrained Body

Figure 8 shows the impedance vs. frequency of one subject (RC) sitting erect, sitting relaxed, and standing erect. Also, the $m\omega$ line (impedance of a pure mass with the same weight as of the subject) and the impedance of a simple one mass-spring-system, as shown in Figure 1, are plotted in this diagram. The same mass and a damping factor calculated from the first peak of the "sitting-erect" curve were used. The frequency of this peak was taken as the undamped natural frequency of the system. Up to 2 cps the body responds at any posture like a pure mass, i.e., the body moves as a compact unit without remarkable relative displacements of body parts. But then the first resonance becomes more and more effective culminating in a peak of which amplitude and resonant frequency depend on the posture of the subject. In the sitting-erect posture, the body has the highest impedance peak and the highest natural frequency and, therefore, the lowest damping factor. By relaxing the muscles, the first resonant frequency goes down and the damping factor up. Standing erect with stiff knees, the body has about the same impedance peak and a resonant frequency between the resonances of the two other measured sitting postures. With the legs bent, the body has such a low natural frequency that above 2 cps practically only the feet move and the body is not affected by the vibrations. Calculating from these peaks and the mass of the subject, the body had the following parameters at the three measured postures in the frequency range up to the resonance:

TABLE II

Parameters for One Subject (R. C.) at First Resonance

	Dimension	Sitting Erect	Sitting Relaxed	Standing Erect
Resonant frequency f_o	cps	6.3	5.2	5.9
Impedance at resonance $Z_n = 1$	$\frac{\text{Dyne x sec}}{\text{cm}}$	6.7×10^6	5.05×10^6	5.2×10^6
Damping factor δ	---	0.57	0.65	0.74
Damping constant c	$\frac{\text{Dyne x sec}}{\text{cm}}$	1.9×10^6	1.73×10^6	2.31×10^6
Elasticity k	$\frac{\text{Dyne}}{\text{cm}}$	131×10^6	84×10^6	116×10^6

Up to the resonant frequency, the body responds very closely to the single mass-spring-system at any posture. Above resonance the measured impedance diverges from the impedance of the single system. While sitting erect, a small peak appears at about 10 cps and another at about 15 cps. When sitting relaxed, the second peak becomes more evident and moves to about 11.5 cps, while the third peak flattens. By standing erect, the second peak is also at about 11.5 cps and becomes much higher, but the third peak practically disappears. However, the general trend of all these curves is similar to the impedance of the simple system which is true for n>>1 to

$$Z_{n \to \infty} = \delta m \omega_o$$

a constant value, depending on the magnitude of the mass, damping factor and the natural frequency of the system. For the sitting-erect posture of Figure 8, this value is

$$Z_{n \to \infty} = 1.88 \times 10^6 \frac{\text{Dyne x sec}}{\text{cm}}$$

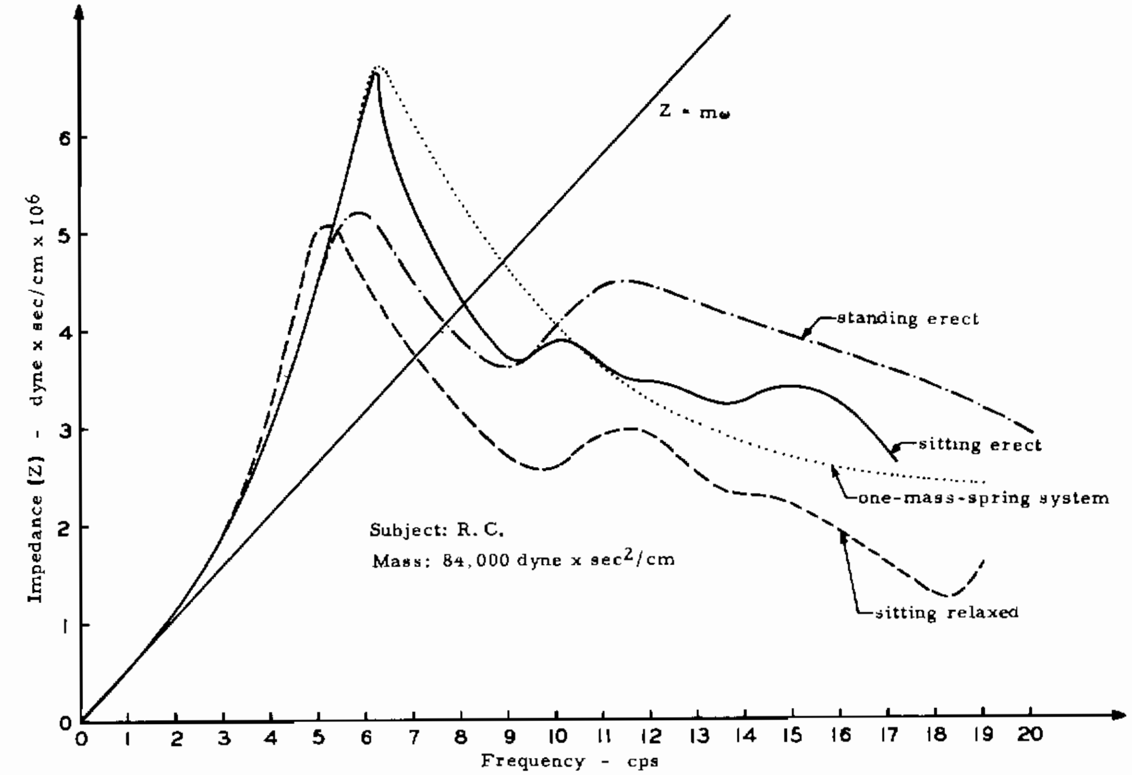


Figure 8. The Modulus of the Impedance of One Subject at Varied Body Postures Compared with the Impedance of a Pure Mass (mω) and of a One-Mass-Spring System with Damping

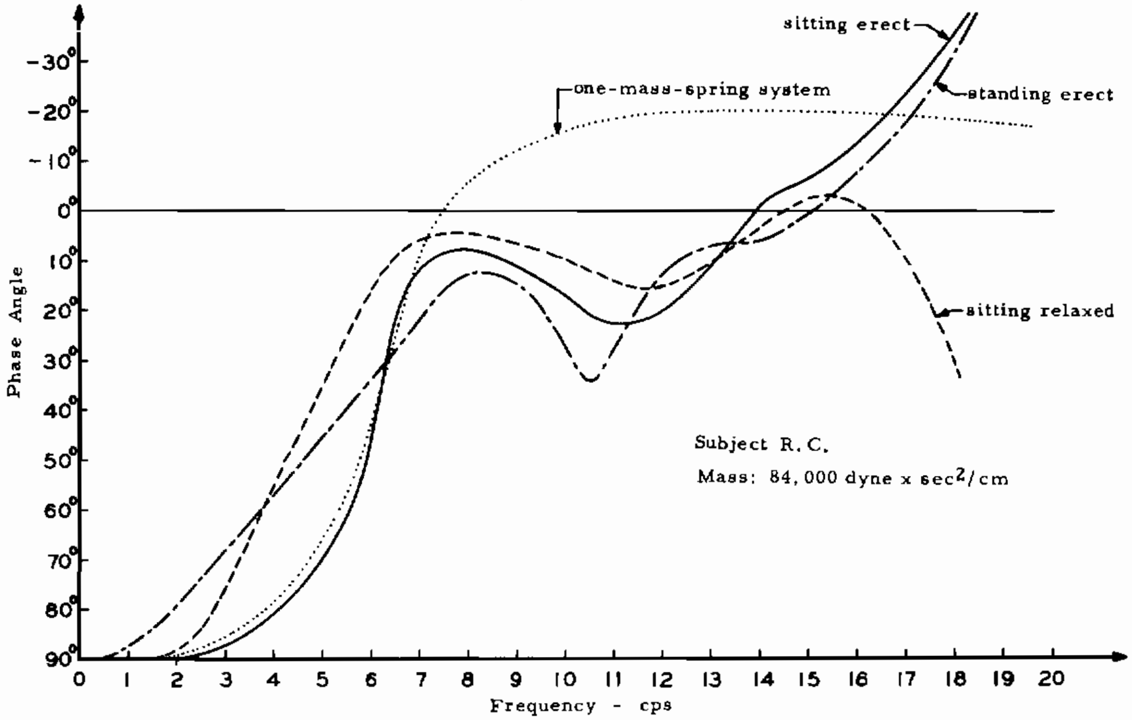


Figure 9. Phase Angle of the Impedance for One Subject at Varied Body Postures Compared with Phase Angle of a One-Mass-Spring System with Damping

The phase between force and velocity of the seat at the three postures is shown in Figure 9. The curve for the sitting-erect posture follows very closely the calculated phase curve of the assumed one-mass-spring system up to 7 cps. But then the phase is reversed by the second and third resonance. Above 14 cps the phase angle becomes negative, indicating that the impedance of the elasticity dominates the impedance of the mass.

The phase angle of the one-mass-spring system has a maximum at

$$n = \sqrt{\frac{3}{1 - \delta^2}} = 2.12 \text{ or } f = 13.4 \text{ cps for } \delta = 0.57$$

and goes to 0^{o} for $n \rightarrow \infty$

In the relaxed posture, the phase angle curve has a similar course up to 15 cps but then turns downward to the positive angles indicating that the mass impedance dominates. At the standing-erect posture, the curve takes a similar course as at the sitting-erect posture, except in the low frequency range, where the phase angle rises faster due to the higher damping.

The modulus and the phase angle of the impedance depend on the mass, the elasticity and the damping of the body. All three parameters change with the weight, size, and posture of the subject. The median, the twentieth, and the eightieth percentiles of the impedance of 8 different subjects sitting erect and relaxed are shown in Figure 10. Once again the $m\omega$ line and the impedance curve of a one-mass-spring system with damping, based on the average mass of the subjects (86200 $\frac{\text{Dyne x sec}^2}{\text{cm}}$), the undamped natural frequency at 5.0 cps and a damping factor δ = 0.636, are also presented in the figure. The corresponding curves for the phase angle are

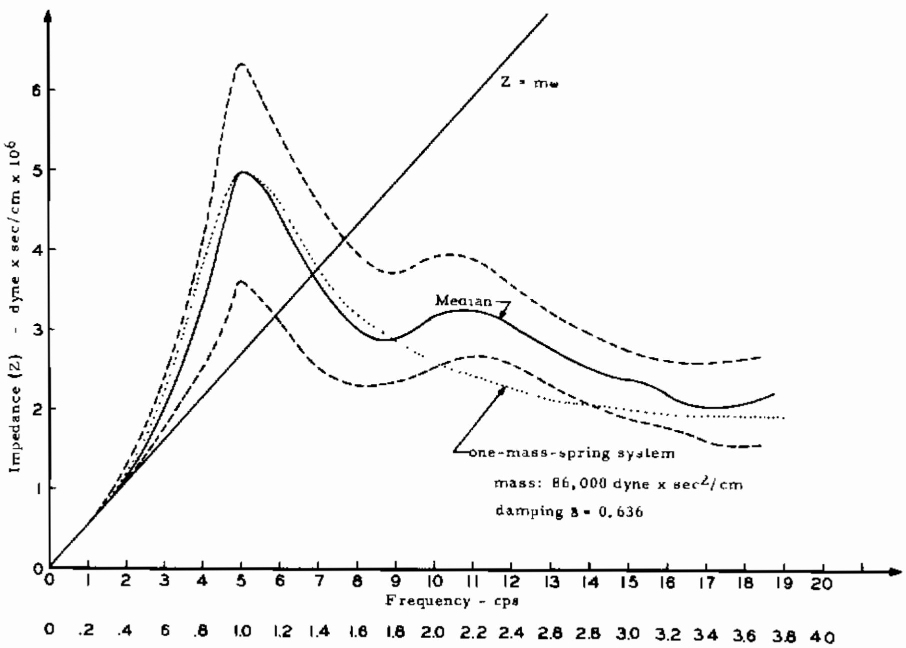


Figure 10. The Median and the Twentieth and Eightieth Percentile of the Modulus of Impedance of 8 Different Subjects Sitting Erect, Compared with the Impedance of a Pure Mass and a One-Mass-Spring System with Damping

plotted in Figure 11. Median curves may be considered as the average values of the impedance and the phase angle of the human subject sitting erect without any restraint of the body and without seat cushions.

In order to design a mechanical model with about the same mechano-dynamic properties as the human body in the frequency range studied, it is possible, utilizing equations (22) and (23), to calculate the impedance and phase angle curves of the second effective system. The curve Z_2 in Figure 12 is the vectorial difference of the calculated impedance of the one mass-spring system in Figure 11 ($m_1 = 86200 \frac{\text{Dyne x sec}^2}{\text{cm}}$, $\delta_1 = 0.636, f_{o1} = 5.0$ cps) and the measured median impedance. The ϕ_2 curve is the phase angle of that calculated difference. From the resonant peak of Z_2 at f_{o2} = 10.5 cps and the magnitude of Z_2 at $\sqrt{2}\ f_{o2}$, the parameters of a second substituting system can be determined with the aid of equations (18), (19), (20), and (21). The results are:

$$m_2 = 7520 \ \frac{\text{Dyne x sec}^2}{\text{cm}}$$

$$k_2 = 32.7 \times 10^6 \ \frac{\text{Dyne}}{\text{cm}}$$

$$\delta_2 = 0.258$$

$$c_2 = 128{,}000 \ \frac{\text{Dyne x sec}}{\text{cm}}$$

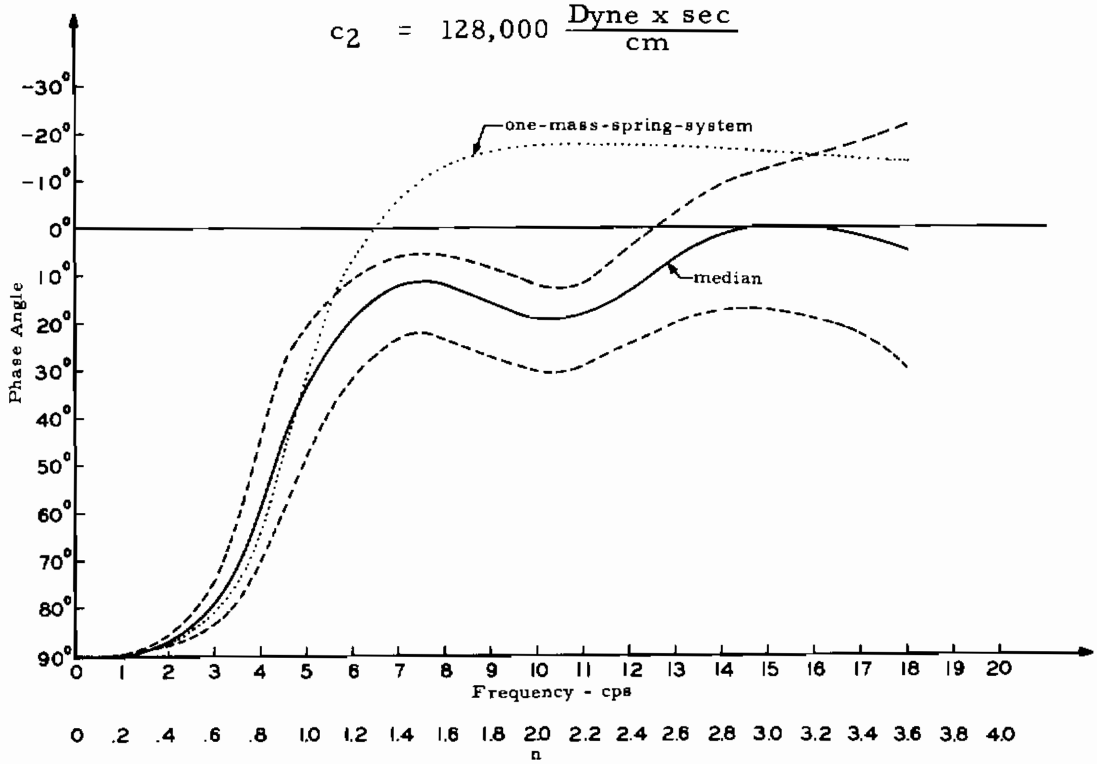


Figure 11. The Median and the Twentieth and Eightieth Percentile of the Phase Angle of the Impedance Measured from 8 Different Subjects Compared with the Phase Angle of a One-Mass-Spring System with Damping

Curve Z_2' in Figure 12 is the impedance of this substituting system. Of course, this curve does not fit the Z_2 curve as was done by the first substituting system in Figure 11. However, the maximum deviation between 7 and 8 cps is only about

10% of the total impedance. Curve ϕ_2' in Figure 12 demonstrates the phase angle of Z_2'. But the phase angle curve ϕ_2 of Z_2 is quite different from ϕ_2'. This indicates that the influence of the third system on the phase angle is well above 9 cps. Considering the changes of the impedance and phase angle in the higher frequency range by varied posture of the subject, it is not worthwhile to calculate the parameters of that third system. For the purpose in question, it is enough to know that the human body in the sitting-erect position can be replaced with fair accuracy by the two-mass-spring systems using the above mentioned parameters.

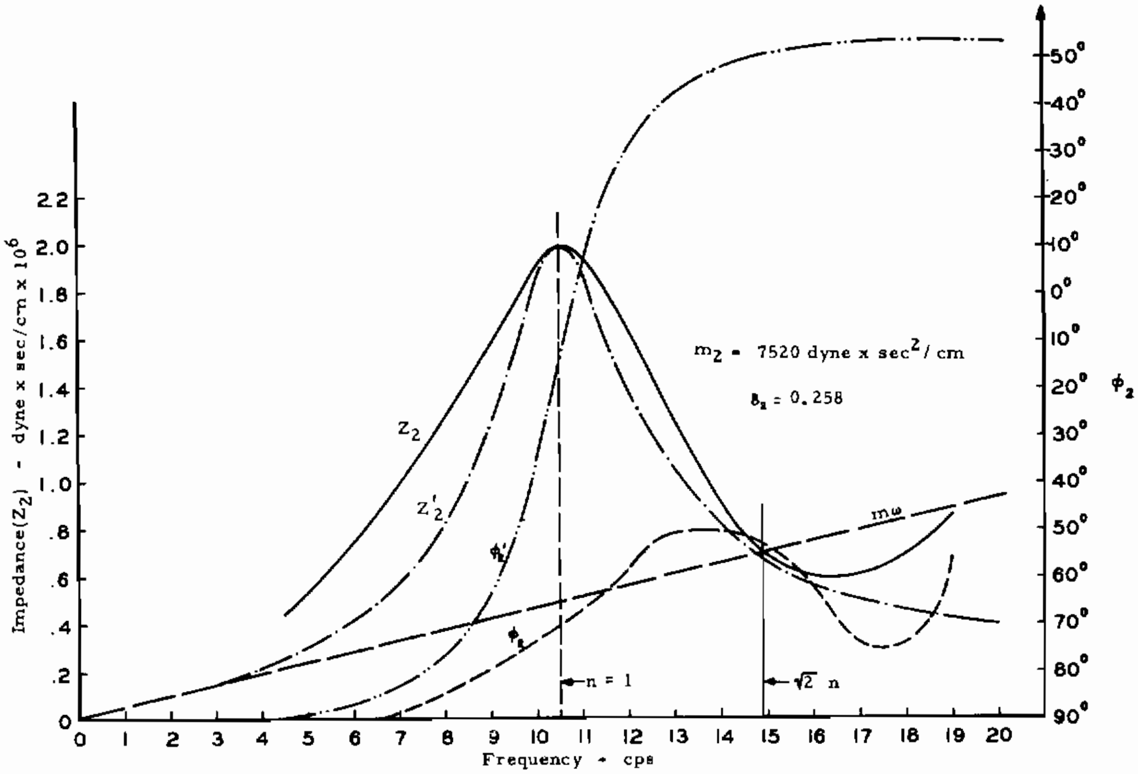


Figure 12. Impedance and Phase Angle of the Second Mass-Spring System Derived from the Median Curves in Figures 11 and 12, Compared with the Impedance Z_2 and Phase Angle ϕ_2' of a Substituting System

B. The Restrained Body

A semi-rigid envelope around the pelvis and abdomen changes the impedance of the sitting subject considerably. Two kinds of envelopes were used: 1) a corset with stays; 2) two or three layers of one-way elastic cloth. The envelope reached from a line extending from the symphysis to the coccyx up to the level of the diaphragm. The envelope slightly inhibited both breathing and venous blood flow. The pain in the torso was intolerable after 40 to 60 minutes and the envelope had to be removed.

Typical changes of the impedance curves of one subject at erect and relaxed postures are shown in Figure 13. The most evident effects are the suppression of the first resonant peak and a slight enhancement of the second peak around 9 cps. It seems that the envelope has no remarkable effect at higher frequencies. Because the envelope increases the elasticity and the damping of the lower torso, the phase angle decreases to only about +20° and does not cross the 0° line in the frequency range studied. These effects are more evident in slender than in stout subjects.

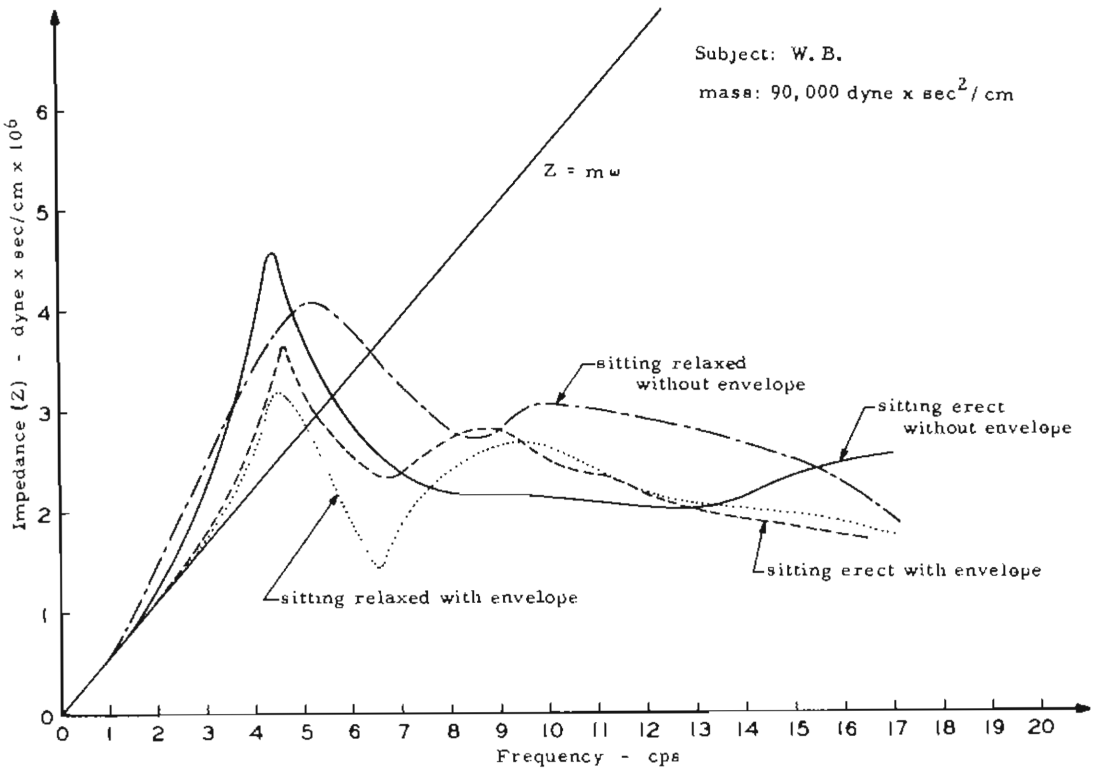


Figure 13. The Effect of a Semi-Rigid Envelope Around the Abdomen on the Impedance of One Sitting Subject at Erect and Relaxed Postures

The Air Force Type MC3 Pressure Suit was used as a restraining garment for the whole body. The Pressure Suit, made of almost inextensible cloth, fits the body by cross strings and is tightened by air pressure tubes along the arms, back and legs. A bladder is built in the front of the suit covering the abdominal wall and the lower part of the chest. A subject wearing this pressure suit and sitting on the force cell table is shown in Figure 14.

Figure 14. Subject Wearing a Pressure Suit Sitting on the Force Cell Table Mounted on the Shake Table

The results obtained were opposite to the effect of the semi-rigid envelope around the pelvis; see Figure 15. In both postures, erect and relaxed, the first and the second resonant peaks were remarkably enhanced. Only above 18 cps was the effect negligible. This response of the body with this type of pressure suit is due to the elasticity of the air pressure tubes and bladder which was added to the elasticity of the body. Accordingly, the subjective tolerance to vibrations was lower than without the pressure suit.

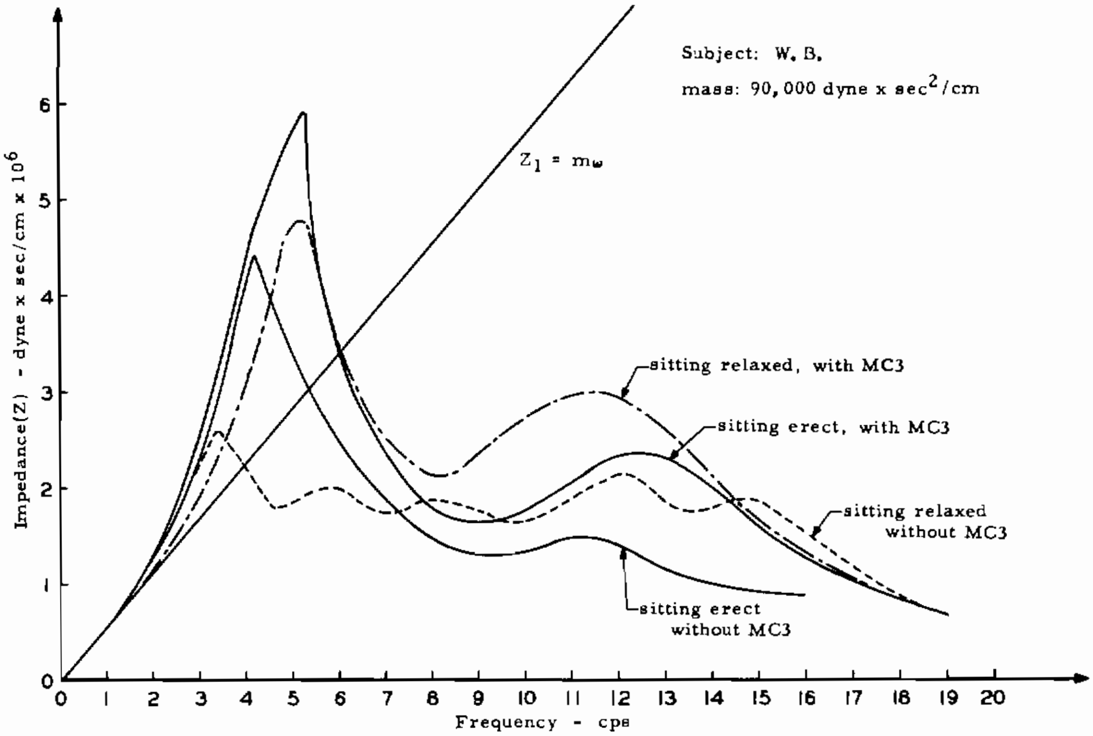


Figure 15. The Effect of the MC-3 Pressure Suit on the Impedance of One Subject Sitting at Varied Body Postures

It cannot be expected that the human body has a linear characteristic over a large range of acceleration, or, in other words, that the elasticity of the body is independent of the load employed. With increasing load the stiffness of the tissue probably increases and the body will respond more as a pure mass. In order to to test the influence of the magnitude of the applied acceleration, the impedance of one subject at .1 G, .3 G, and .5 G (vector) has been measured. A heavy man, subject B.D., was selected for this test because more influence was expected from soft tissue than from bone elasticity. No higher acceleration could be applied than .5 g since the subjective tolerance limit was reached in the frequency range between 4 and 10 cps for the time necessary to take the records.

The impedance obtained and phase angle curves remained in the range of $\pm 10\%$ (which is about the accuracy with which such impedance and phase angle measurements can be taken). This minor variation was probably due to the fact that the subject changed his posture slightly. At higher vibration stress, the muscle tension increases also; therefore, it is difficult to determine which changes of the impedance curve are due to an active or a passive change of the body elasticity. In the above test the subject was instructed to hold the same muscle tonus as nearly as possible. Nevertheless, in the applicable acceleration range the variation of the impedance was not evident. Since this determination is important for tolerance to vibrations during higher steady acceleration loads, as it occurs during a curved flight of an airplane and during launch and re-entry of space vehicles, another test method should be developed to investigate this factor.

C. Movement of Body Parts

The transmission of vibrations from the point of excitation to the mass of the one-mass-spring-system of Figure 1 is given by the equation:

$$\frac{\overline{x}_1}{x} = \frac{1 + j n \delta}{1 - n^2 + j n \delta} \quad (42)$$

from which the modulus is:

$$\frac{x_1}{x} = \frac{\sqrt{1 + n^2 \delta^2}}{\sqrt{(1 - n^2)^2 + n^2 \delta^2}} = \frac{Z}{m\omega} \quad (43)$$

and the phase angle between x_1 and x is:

$$\phi_{\frac{x_1}{x}} = \tan^{-1} \frac{n^3 \delta}{n^2 (1 - \delta^2) - 1} \quad (44)$$

The factor $\frac{x_1}{x} = \frac{\dot{x}_1}{\dot{x}} = \frac{\ddot{x}_1}{\ddot{x}}$ is the transmission factor for vibrations from the point of excitation to the mass. Having the impedance curve of a one-mass-spring-system, the transmission factor is given also by dividing the impedance at any frequency by $m\omega$. For a complex system consisting of several masses and springs, equation (42) can be applied only for each particular sub-system if the excitation x for this sub-system is known. In the human body the mass is distributed over the whole torso and there is no point where the displacement of the effective mass can be measured. However, in the frequency range where the body responds like a one-mass-spring system (see Figure 10), the displacement of the effective mass can be calculated by this equation.

Since the transmission of vibrations to the head is one of the most disturbing factors of the subjective tolerance to vibrations, this factor was measured on several subjects. Figure 16 demonstrates the transmission of vibrations from the seat to the head over the frequency range studied for the same subject as in Figure 8. The dotted curve indicates the transmission factor of the same one-mass-spring system from which the impedance curve is plotted in Figure 8. The curves indicate that the motions of the head are not the same as the motions of the effective mass of the body. In the sitting-erect posture, a first small peak appears at about 3 cps, probably produced by the resonance of the abdominal mass which has been determined in another test to be about 3 cps.[5] (The kinetic energy of abdominal mass transfers also impulsive forces to the skeleton in the vicinity of its resonances.) The second, and highest, peak has a frequency a little below the resonant frequency of the assumed one-mass-spring system and an amplitude considerably lower than at the dotted curve. The two resonances between 10 and 11 cps and at about 15 cps also appear.

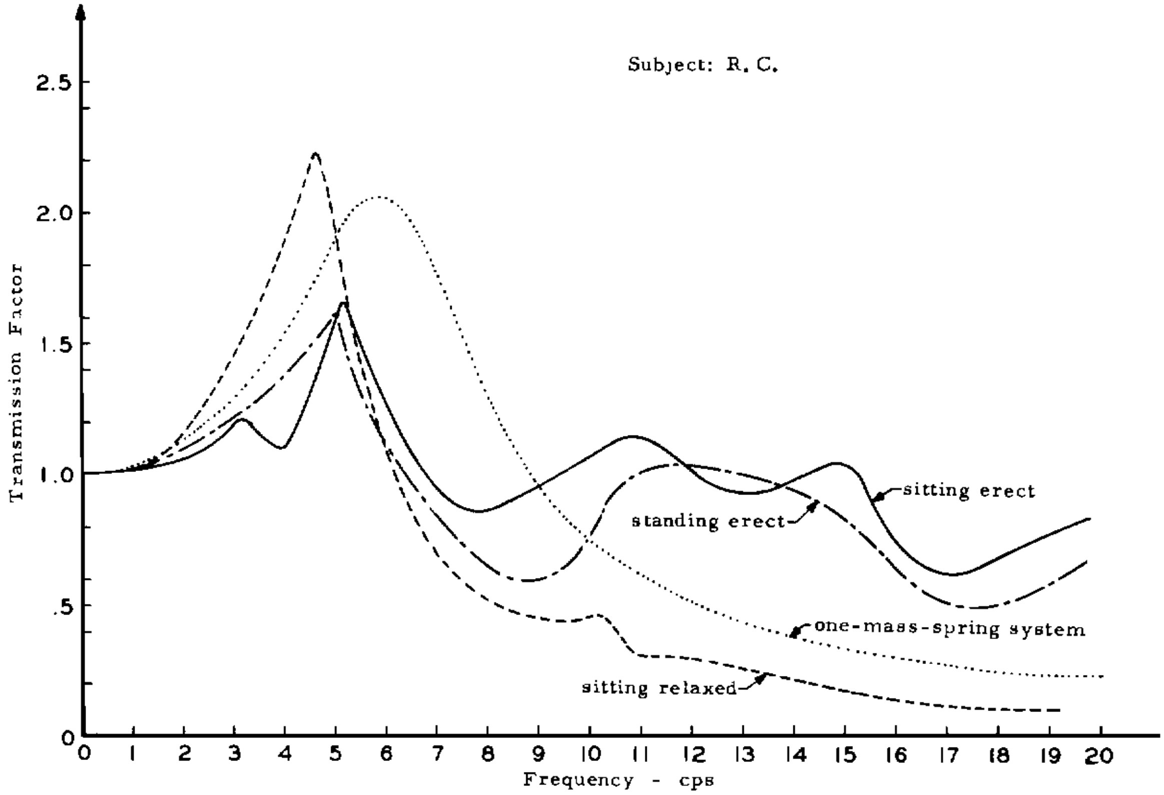


Figure 16. The Transmission of Vibrations from the Seat to the Head on One Subject at Varied Body Postures Compared with the Transmission Factor of a One-Mass-Spring System with Damping

If the subject takes the relaxed posture, the elasticity of the body increases, as indicated by the lower frequency at the peak, but the damping for the transmission of vibrations to the head decreases opposite to the total damping for the whole body. Above the main resonance the vibrations of the head are much more attenuated in the relaxed posture. Therefore, in order to protect the head against vibrations at frequencies below 5 cps, it is better to assume the erect posture, and at frequencies above 5 cps to relax the muscles and to bend the spine.

The transmission factor in the standing-erect posture is very similar to the sitting-erect posture, which proves that the stiff legs have practically no damping effect in the frequency range up to 20 cps.

Corresponding results were obtained from the other subjects tested, merely with less distinctness in stout subjects.

The next step in the investigation of the dynamic response of the human body was to measure the relative displacement of body parts. The skeleton as the essential structure in supporting the body will have probably the greatest importance in the elasticity of the torso. In the sitting and standing postures the alternating forces are first transferred through the pelvis to the body mass. From there the forces are transmitted over the sacrum and spinal column to the head. The masses of soft tissues, such as the abdominal mass, diaphragm, lungs and heart, the thorax and the arm-shoulder system, are hinged to this structure. Muscles are connected parallel to the different components of the skeleton and are effective as masses with varying elasticities. In order to determine whether the pelvis or the parts between pelvis and neck are responsible for the resonance peaks around 5 and 10 cps, displacement measurements between shake table and hip bones and between hip bones and neck were taken on one subject by the method previously described.

The results are presented in Figure 17. The curves plotted in this diagram indicate the relative displacement per G acceleration of the excitation point vs. frequency. Curve a is the relative displacement between the shake table and the hip bones and curve b between the hip bones and the neck, both related to one G on the shake table. Curve c is the relative displacement between hip bones and neck related to one G acceleration on the hip bones. Curves a and b show once again resonant frequencies at 5 cps and about 9 cps, but curve c has only one peak at 5 cps. Thus, very probably the pelvis has two resonances at 5 and 9 cps, and the spinal column only one at about 5 cps in the frequency range studied. The harmonics of this resonance are obviously so highly damped that they do not appear in the displacement curve.

The curves of Figure 17 indicate also that the body cannot be considered for these measurements as a one-mass-spring system with respect to the 5 cps resonance. With such a simple system the relative displacement of the mass per G excitation is given according to equation (4) by

$$\frac{x_1 - x}{G} = \frac{9810}{\omega^2} \frac{n^2}{\sqrt{(1-n^2)^2 + n^2\delta^2}} = \frac{9810}{\omega_o^2} \frac{1}{\sqrt{(1-n^2)^2 + n^2\delta^2}} \text{ mm} \qquad (45)$$

and the phase between x and x_1 by equation (5). Setting $n = 1$ and $f_o = 5$ cps ($\omega_o = 31.4$) in equation (45), one obtains with the magnitudes of $\frac{x_1 - x}{G}$ at the three peaks the damping factors for the curve a: $\delta = 0.8$, b: $\delta = 1.27$, and c: $\delta = 2.92$. The curves a', b', and c' demonstrate the mass displacements of one-mass-spring systems with $f_o = 5$ cps and the corresponding damping factors.

At $n = 0$, the same natural frequency is

$$\frac{x_1 - x}{G} = 10 \text{ mm}$$

independent of the damping factor δ . All three curves do not approach this value for 0 cps. The comparison of the corresponding curves shows that another system is effective in the very low frequency range. Probably the transversal motions of the body which have a resonance at about 2 cps create these changes. Nevertheless, the peaks indicate in approximate terms the two main resonances of the pelvis and the one of the spinal column in this frequency range.

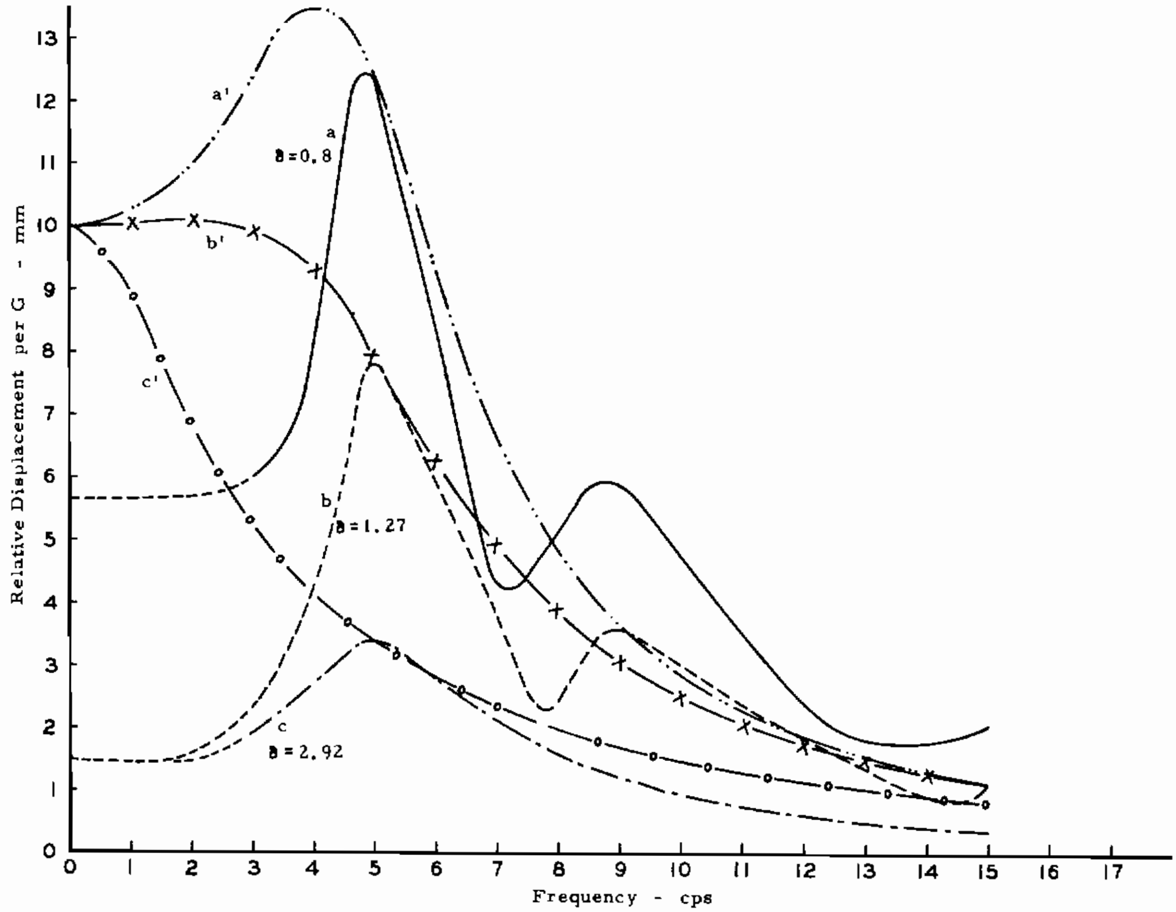


Figure 17. Relative Body Displacement on One Subject (a) between Shake Table and Pelvis, Relative to One G on the Shake Table; (b) between Pelvis and Neck, Relative to One G on the Shake Table; and (c) between Pelvis and Neck, Relative to One G at the Pelvis, Compared with the Relative Mass Displacement of a One-Mass-Spring System with a Natural Frequency of 5 cps and the Corresponding Damping Factors

D. Correlation Between Impedance and Subjective Tolerance

The impedance and displacement measurements on the human body in sitting and standing positions under vibration provided the fundamental dynamic parameters of the body in the low frequency range. For the design of manned vehicles, it is important to know the crew's subjective tolerance limits to vibration. This subjective tolerance has been determined for short-time excitation on 10 subjects sitting on a rigid seat and vibrated with sinusoidal vertical motions.[6] The indication that the tolerance limit had been reached was not moderate discomfort but the maximum endurable stress and pain. It would be of interest to find a correlation between this subjective tolerance curve and the mechanical impedance of the body. The question is: which of the physical factors (the transmitted force, the dissipated energy or the relative displacement of the most effective body masses), if any, determines the subjective tolerance.

According to equation (32a), the transmitted force per G of the peak seat acceleration with sinusoidal motion is

$$\frac{P}{G} = \frac{981 \times Z}{\omega} \tag{46}$$

and can be calculated directly from the impedance curve. If the tolerable transmitted force is constant over the frequency range studied, then in order to establish the overall level of the curve, one frequency can be chosen where the influence of the dynamic parameters of the body is very low. That is the case at 1 cps where the impedance of the body responds practically as a pure mass. Taking the tolerable acceleration at this frequency to 3.75 G, then the tolerable acceleration at the other frequencies due to the transmitted force would be:

$$A_P = 3.75 \frac{Z_{1\,cps} \times \omega}{Z \times 2\pi} \tag{47}$$

In a similar way the acceleration for a constant dissipated energy can be calculated according to equation (34). But in this case it would be difficult to relate these values to the tolerable acceleration at 1 cps, because at this low frequency the phase angle is very near to 90° or $\cos\phi \rightarrow 0$ and the tolerable acceleration A_N approaches infinity. Therefore, it is more sensible to shift the curve so that it best covers the experimentally defined tolerance curve. The equation for the tolerable acceleration at a constant dissipated energy level is:

$$A_N = \omega \sqrt{\frac{N}{Z \cos\phi}} \tag{48}$$

More difficulties arise in calculating the constant relative displacement of the effective body masses, since the really effective body masses are not known over the whole frequency range. The calculated model of the body evidenced two mass-spring systems with masses of 86700 and 7520 $\frac{\text{Dyne} \times \text{sec}^2}{\text{cm}}$. With equation (45) the relative displacement of the mass can be calculated for each system and then vectorially added to the combined relative displacement with the aid of the equation:

$$Y^2 = (x_1 - x)^2 + (x_2 - x)^2 + 2\,(x_1 - x)(x_2 - x) \cos(\phi_2 - \phi_1) \tag{49}$$

$(x_1 - x)$ = relative displacement of the first system,

$(x_2 - x)$ = relative displacement of the second system,

ϕ_1 = phase angle of $(x_1 - x)$,

ϕ_2 = phase angle of $(x_2 - x)$,

Y = combined relative displacements of the masses.

The results can be related again to the tolerable acceleration at 1 cps = 3.75 G, so that the tolerable acceleration for a constant relative displacement of the effective body masses will be

$$A_D = 3.75 \frac{Y_{1\,cps} \times \omega}{Y \times 6.28} \tag{50}$$

Figure 18 shows the subjective short-time tolerance curve plotted vs. frequency together with the curves for constant transmitted force, constant dissipated energy, and constant combined relative displacements of the effective body masses. The "force-curve" follows the "subjective curve" only about up to the first resonance, then rises much faster. This curve can be shifted also so that the best coincidence is attained in the higher frequency range (this has been done in a preliminary publication[7]), but then the curve crosses the 0-frequency line at about 2 G or, in other words, the maximum tolerable force would be only 2 times the weight of the subject, which is probably too low. The force curve in Figure 18 crosses the 0-frequency line at about 4 G, which is in approximate agreement with the tolerable short time accelerations on centrifuges. Therefore, it seems to be more sensible to start the force curve at 3.75 G for 1 cps.

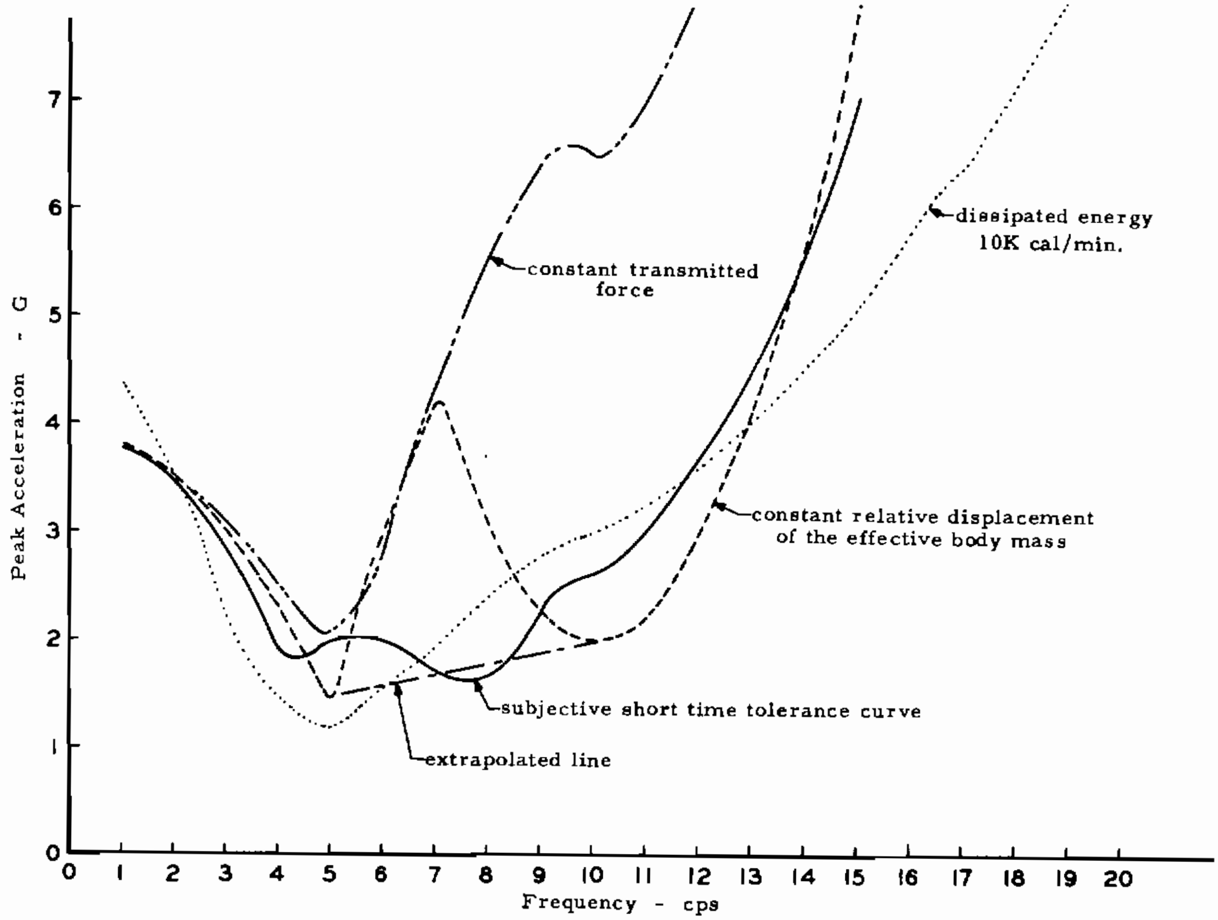


Figure 18. Human Tolerance Curve to Vertical Vibrations Compared to Curves for Constant Transmitted Force, Constant Relative Displacement of the Effective Body Mass, and Constant Dissipated Energy Derived from the Mechanical Impedance Measurements

The "energy curve" in Figure 18 follows the subjective curve very roughly up to 13 cps, but then it has a much lower slope. The absolute value of the dissipated energy at that level would be about 10 K cal/min. This value cannot be determinative for the short time tolerance to vibration, since up to 30 K cal/min. can be produced in the human body during very hard physical work.

The combined "displacement curve" covers the subjective curve only in the frequency ranges from 1 to 5 cps and from 10 to 15 cps. Between 5 and 10 cps the curve ascends to a peak at about 7 cps. However, the statements of the subjects in this frequency range indicate that another system is effective here. The tolerable acceleration between 5 and 10 cps is limited by pain in the chest with a maximum at frequencies around 7 cps. Apparently an organ in the chest, maybe the heart, has its resonance around 7 cps, bouncing up and down and stretching certain tissues up to intolerable pain. The mass of that organ, compared with the two other effective masses, is so small that its impedance has no measurable effect on the total body impedance. This means that, without the pain stimulation in the chest, the subjective tolerance curve between 5 and 10 cps should be much higher. But an extrapolated line between the two lowest points of the displacement curve will follow the subjective curve with sufficient accuracy. Thus, the relative displacements of the effective masses seem to be determinative for the subjective tolerance to vibrations, except in the frequency range between 5 and 10 cps where the movements of inner organs in the thorax set the limit. However, with the aid of the measured mechanical impedance of the body, a fair prediction of the subjective tolerance to vibration can be made in the low frequency range.

SECTION V

DISCUSSION AND CONCLUSIONS

There is no doubt that the reported results and theoretical considerations are only a beginning in the research on mechano-dynamic factors of the human body. Due to the high variability of the human body, the data obtained can present only an average of many values with great deviations. During the tests it is often very difficult to distinguish between natural deviation and measuring fault. It requires the experience of the investigator to extract the generally valid results from a large quantity of scattered values. Nevertheless, if the experiment is based on the correct fundamental theory, conclusions with broad applicability can be drawn. It is shown in this section how the presented results can contribute to elucidate the problem of the effects of vibrational stresses.

Figure 19 demonstrates that the short-time tolerance to vibration of a human subject sitting erect on a rigid chair can be considered as dependent upon the relative displacements of the effective body masses and that these relative displacements can be calculated with sufficient accuracy from the mechanical impedance of the subject. Now the questions arise:

1. How will the tolerance change by modifying

    a. the posture of the subject?

    b. the restraining system?

    c. the position of the subject relative to the direction of the transmitted force?

2. What influence will a soft elastic cushion have between subject and and vibrating seat?

3. What is the significance of the mechanical impedance to long-term tolerance to vibration?

4. Can the tolerance to a random motion with a certain frequency spectrum be predicted by virtue of the tolerance limits for sinusoidal motions?

5. Is there any correlation between the tolerance to steady sinusoidal vibration and to impact?

For the questions 1.a. and 1.b. it can be said that, if at a certain frequency, the impedance decreases by a varied posture or by a restraining system, then generally the tolerance for that frequency will increase and vice versa. For instance, the relaxed position of the subject in Figure 8 made him more comfortable over the frequency range studied, but the pressure suit in Figure 15 decreased the tolerance remarkably corresponding to the increase of impedance. Only at those frequencies where the resonances of smaller organs, such as the heart, bladder or rectum, are determinative for the tolerance limit, is it possible for tolerance not to be correlated with whole-body impedance. However, not enough data are yet available to confirm this statement.

It is much more difficult to make predictions for the tolerance at varied positions of the subject. In the supine or prone position the tolerance may be, in a certain frequency range, much lower than in the sitting position, especially if the vibrations are directly transmitted to the head, although the impedance is almost the same or even smaller. The tolerance curves in positions other than sitting or standing with stiff knees and their correlations to the impedance curves are still to be investigated.

The influence of a cushion or another damping system between subject and seat can be predicted with more accuracy if the frequency characteristic of the device is known. This characteristic can be determined experimentally by applying a dummy with solid mass property and with about the same load distribution on the cushion as produced by the living subject. In the simplest way a solid mass lying on the cushion or, better, an anthropomorphic, but not elastic, dummy with fixed extremities and head can be used as a substitute for the complex human body. The impedance of this mass-cushion combination can be measured and the impedance of the cushion alone can be calculated by vectorial subtraction of the pure mass impedance.

$$\overline{Z}\text{ cushion} = \overline{Z}(\text{mass + cushion}) - \overline{Z}\text{ mass} \qquad (51)$$

The impedance of the subject alone can then be calculated from the measured impedance of the subject on the chair by deducting vectorially the impedance of the cushion alone from this value.

$$\overline{Z}\text{ subject} = \overline{Z}\text{ cushion + subject} - \overline{Z}\text{ cushion} \qquad (52)$$

The attenuation of the vibration by the cushion will be

$$I = \frac{x\text{ subject}}{x\text{ shake table}} = 1 - \frac{Z\text{ cushion + subject}}{Z\text{ cushion}} \qquad (53)$$

and, if it is assumed that the subjective tolerance depends upon the relative displacement of the effective body masses, the tolerance with the cushion will be the tolerance without the cushion divided by I.

Since the repeated run of the same subject at tolerance limit may be dangerous for the subject and also expensive, the possibility of calculating the influence of body restraining systems and damping devices can be very important for the development of protective devices against vibrations.

The long-term tolerance to vibration has been determined by many investigators during the past three decades in order to acquire the comfort requirements in different types of vehicles. Due to the different test conditions and formulations of the problem, the results diverged sometimes by a factor of ten and more.[8] Recently[9] the one-minute and three-minute tolerance curves have been established using test conditions similar to those used in determining the short-time tolerance curve. These curves have a similar shape up to about 10 cps, but the one-minute tolerance curve has a course about 1.5 G below the short-time curve. Above 10 cps the one-minute curve seems to be less steep, ending at about 3.5 G for 20 cps. Apparently another physiological factor becomes more effective above 10 cps during long-time exposure, which indicates that the stimulation of other receptors in the nervous system dominates. However, the impedance curve can probably be utilized in the frequency range up to 12 or 13 cps to predict the tolerance to long-time exposure to vibration if this tolerance is known for about 1 cps. But to confirm this statement many more tests are necessary, especially since it can be expected that the central nervous system will be more and more involved in the physiological mechanism for long-time vibrational stresses.

The fourth question is brought up very often during discussions about the effect of vibrations on the crew of vehicles, because pure or even almost sinusoidal vibrations occur very seldom in practice. Theoretically, any periodic function $f(t)$ of fundamental frequency ω can be split up into a series of sine and cosine curves of frequencies ω , 2ω , 3ω , 4ω , etc.[10] This series can be written as

$$f(t) = a_1 \sin \omega t + a_2 \sin 2\omega t + \ldots + a_n \sin n\omega t + b_0 + b_1 \cos \omega t + b_2 \cos 2\omega t + \ldots + b_n \cos n\omega t \quad (54)$$

The factors $a_1, a_2, \ldots a_n$ and $b_1, b_2, \ldots b_n$ are the amplitudes of the different sine or cosine curves and can be determined by the equations

$$a_n = \frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} f(t) \sin n\omega t \; dt \quad (55a)$$

and

$$b_n = \frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} f(t) \cos n\omega t \; dt \quad (55b)$$

b_0 represents the average height of the function f(t) during one cycle. This factor can be neglected for the problem studied because a constant factor in the function has certainly no influence except that the function represents the acceleration of the seat and the superimposed constant acceleration has a great magnitude. But this condition was not investigated in this study.

The mathematical expression for the random motion is in most cases unknown, but the function is often available in the form of an electrical voltage or a graphical record. In both cases the integrals of equation (55) can be determined by electrical or mechanical "Harmonic Analyzers." To assure that the selected portion of the function contains all frequencies occurring in the function, the portion must be taken long enough, for instance one minute. After such a long period it can be assumed that the function is practically periodic. Then the amplitude of each harmonic can be related to the tolerance at that frequency. If the function f(t) is a measured acceleration of the seat, then the analyzed amplitudes $a_1, a_2 \ldots a_n$ and $b_1, b_2 \ldots b_n$ can be divided by the tolerable acceleration a_t at the same frequency as it has been determined experimentally or calculated from the impedance curve. These factors can be called the "danger factors" for each harmonic

$$D_n = \frac{1}{a_t} \sqrt{a_n^2 + b_n^2} \quad (56)$$

(The amplitudes of the sine curves are always perpendicular on the amplitudes of the cosine curves at the same frequency.) If $D_n < 1$, then that specific frequency in the random acceleration pattern is not dangerous. $D_n = 1$ means that at this frequency the tolerance is reached, and $D_n > 1$ means that the tolerance is exceeded. The combined effect of all harmonics will be maximum at that instant when all sine curves and all cosine curves are in phase. The danger factor will then be

$$D = \Sigma D_n$$

In practice this will happen very seldom, and in such a case the duration of the peak will be so short that it can be considered as an impact and can be compared with the tolerance limits of impacts with similar characteristics.[11] It is very likely that for most random patterns the total danger factor will be

$$D = \sqrt{\frac{D_1^2 + D_2^2 + D_3^2 + \ldots D_n^2}{n^2}} \tag{57}$$

At the present time not enough data are available to prove this assumption, but till such proof it seems to be sensible to use the equations (56) and (57) to rate random motion in view of the effect on humans.

To answer the fifth question, the calculations about the drop of a resiliently suspended mass on a rigid floor can be applied.[12] If a linear single mass-spring system without damping drops on a rigid floor with an impact velocity v_o, the maximum deceleration of the mass, during the time when the spring is still in contact with the floor, will be

$$\ddot{x}_1 = v_o \omega_o^2 \tag{58}$$

and the maximum compression of the spring, i.e., the maximum relative displacement of the mass, will be

$$(x_1 - x) = \frac{v_o}{\omega_o^2} \tag{59}$$

Both values depend on the square of the natural frequency of the system. The higher the natural frequency, the lower will be the relative displacement of the effective mass, but the higher will be the transmitted acceleration. Thus, soft tissue complexes, for instance, the abdomino-thorax system, should be restrained in order to avoid excessive organ displacement and rupture of ligaments due to hard impacts. Also, the restraining effects of a semi-rigid envelope around bony structures, as it is demonstrated in Figure 13, could be utilized to diminish their deformation during impact. However, for the sitting position with deceleration from pelvis toward head, such a measure would produce a higher transmission of deceleration to the upper portions of the body, i.e., vertebrae and head. Therefore, investigations must be conducted to determine which magnitude of compliance of the pelvis will be most effective in protecting the supporting bony structure as well as the upper skeleton and head. Since the subjective tolerance to sinusoidal vibrations is also due to excessive tissue displacement and head acceleration, it can be expected that any protective measure which improves the tolerance to vibration will also improve the tolerance to impact. This has been demonstrated by J.A. Roman et al.[3] on rabbits encased in a semi-rigid envelope around the abdomen and dropped from about a 30-foot height into a sand box. Without this envelope the control animals encountered severe injuries in the soft organs and multiple fractures in the skeleton. With envelope only separation of symphysis and, in a few animals, minor fractures in the pelvis were found. In both cases the test conditions have been kept constant as much as possible, producing about the same type of deceleration at each test. But the sand box did not respond as a rigid floor. Due to the deformation of the surface, it induced on the dropped animal a certain acceleration pattern with a relatively low frequency spectrum. For a harder impact, i.e., a deceleration pattern with higher frequency components, the effect could have been adverse producing excessive displacements in body systems with higher natural frequencies, such as the neck-head system. In the same manner as one protective device may be effective against vibrations in a certain frequency range but not at other frequencies, so different deceleration patterns require different types of protective systems.

One difficulty in the comparison of tolerance to vibration with tolerance to impact lies in the non-linearity of body systems for higher loads. At accelerations of 10 or more G's which may be tolerable for short-duration impacts, the elasticity of tissues is certainly not linear, resulting in a dependence of the natural frequency on the load. In this case, the load-displacement relation has to be determined experimentally for each system involved and a mathematical equation has to be found for this relation. Then the relative displacement of the effective mass for a system whose parameters have been found by sinusoidal vibrations can be calculated. The mechanical model of the body developed by virtue of steady-state parameters can also be helpful for impact research.

APPENDIX

CALCULATION OF THE FORCE CELL

The calculation of the elasticity of a ring may be done as follows:

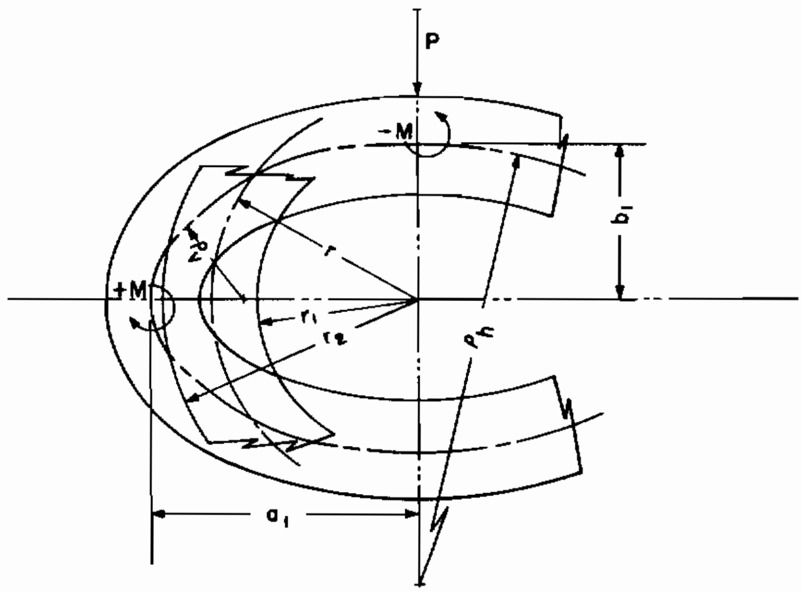


Figure 19. Principle for the Calculation of the Deflection of the Force Cell

The main deflection will occur in the sections with the small wall thickness. There the curved surfaces may be considered as parallel and the relative displacement due to the deformation of the ring to an elliptic oval between the two parts of the mounting planes may be calculated (Figure 19). The curvature of a ring with middle radius r will change to the curvature radius ρ_v of an ellipse if the ring is loaded with the momentum M. The relation between r and ρ_v will be:

$$\frac{1}{\rho_v} = \frac{1}{r} + \frac{M}{E\,x\,F\,r^2} \qquad (1A)$$

where:

E = Young's modulus of the material,

F = sectional area,

$x = \frac{I_o}{F\,r^2}$; I_o = moment of inertia of the sectional area.

For a rectangular section area $F = bh$ is:

$$x = \frac{1}{3}\left(\frac{h}{2r}\right)^2 + \frac{1}{5}\left(\frac{h}{2r}\right)^4 + \frac{1}{7}\left(\frac{h}{2r}\right)^6 + \ldots \qquad (2A)$$

If the lengths of elliptical axes are a_1 and b_1, then the curvature radius will be:

$$\rho_v = \frac{b_1^2}{a} \quad \text{and} \quad \rho_h = \frac{a_1^2}{b_1} \tag{3A}$$

According to equation (1A) we obtain:

$$\frac{b}{a_1^2} = \frac{1}{r} - \frac{M}{E \times F r^2} \tag{4A}$$

and

$$\frac{a_1}{b_1^2} = \frac{1}{r} + \frac{M}{E \times F r^2} \tag{5A}$$

or

$$b_1 = \eta r \sqrt[3]{\frac{1}{\eta^3 + M \eta^2 (1 - \frac{M}{\eta}) - M^3}} \tag{6A}$$

where $\eta = E \times F r$ and $M = \frac{Pr}{2}$ (P = load on the mounting planes) and the relative displacement between the mounting planes

$$f = 2r - 2b_1 \tag{7A}$$

In addition to this deflection, the longitudinal compressions (or extension) of the material occur:

$$\Delta l = \frac{\pi P}{4 b E} \cdot \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2} \tag{8A}$$

The maximum tension will be:

at the outside of the ring: $$\sigma_a = \frac{P}{2F} + \frac{M}{W} \left(1 - \frac{0.23 h}{r}\right) \tag{9A}$$

and at the inside: $$\sigma_i = \frac{P}{2F} + \frac{M}{W} \left(1 - \frac{0.25 h}{r - 0.45 h}\right) \tag{10A}$$

where:

W = section modulus of F.

The accuracy of the results obtained by the above calculation compared with the measured elasticity of the rings was in the range of 30%.

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