

## ANALYSIS OF THE LARGE URBAN FIRE ENVIRONMENT

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### ABSTRACT

An analysis describing the high temperature and velocity environment of a large urban area fire is presented. The boundary value problem treats the burning region in detail. A novel prescription of the boundary conditions at the fire periphery allows the burning-region analysis to be uncoupled from analyses of the free-convection column and the far field. The relationship between burning rate, buoyancy, pressure gradients, and the creation of high velocity fire winds is described. Sample results simulate the burning-region environment for the 1943 Hamburg firestorm.

### INTRODUCTION

The high-velocity inflow generated by an area fire and the characteristics of the initial free-convection flow are determined by the burning-region interactions. Formulation of an appropriate equation set to describe the flow physics depends on the scale of the heat addition and the size of the burning region. As opposed to weakly heated flows controlled by the diffusion of momentum and energy, the volume heat addition implies a strong coupling of buoyancy forces and inertia.

The size of the burning region governs the ordering of terms in the conservation equations. For a heat addition volume defined by a mean flame height  $H$  and a fuel bed radius  $R$ , conservation of mass implies

$$\frac{u}{v} \sim \frac{R}{H} .$$

If  $R/H \sim O(1)$ , the radial ( $u$ ) and axial ( $v$ ) velocities and the corresponding acceleration terms are of similar order. For  $R \gg H$ , the characteristic radial velocity is much greater than the mean axial velocity and the governing momentum equations may be simplified.

This paper considers the class of flows generated by an asymptotically large fire ( $R \gg H$ ). An analytical model for the axisymmetric, quasi-steady flow in and around the burning region is developed (1), and sample results are presented.

### EQUATIONS

For the large fires considered, the turbulent motion is expected to limit the flame heights (2) such that a more or less uniform heating-zone height  $H$  may be defined. A spatially-dependent volume heat function  $Q \times q(r, y)$  is used to model the combustion processes in that (finite) region.  $Q$  represents the mean rate of heat release and  $q(r, y)$  is an  $O(1)$  variable describing its spatial distribution. Since  $O(1)$  changes in temperature and density are expected, all density derivatives are retained.

The conservative equations are scaled using ambient (ground-level) thermodynamic values ( $P_a, \rho_a, T_a$ ) and the characteristic burning-region lengths,  $H$  and  $R$ . The asymptotically-large burning region is thus represented by an order-one domain with comparable radial ( $r/R$ ) and axial ( $y/H$ ) dimensions. The disparate scaling lengths introduce a small parameter,

$$\epsilon = \frac{H}{R}, \quad \epsilon \ll 1, \quad (2)$$

which can be used to order terms. The burning-region aspect ratio is  $\epsilon^{-1}$ .

Radial velocities are scaled with an arbitrary velocity  $U$  and, in order to preserve the two-dimensional structure of the continuity equation, axial velocities are scaled by  $\epsilon U$ . Since a subsonic flow is expected, the thermodynamic pressure  $\hat{P}$  is defined as

$$\frac{\hat{P}}{P_a} = 1 + \delta P, \quad \delta = \frac{U^2}{P_a/\rho_a}, \quad (3)$$

where  $P$  represents a perturbation pressure. In scaled variables, the leading-order set of conservation and state equations (3) is

$$\frac{\partial}{\partial r} (r\rho u) + \frac{\partial}{\partial y} (r\rho v) = 0, \quad (4a)$$

$$\rho \left( u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial r} + M_1 \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{u}{r^2} \right) + M_2 \frac{\partial^2 u}{\partial y^2}, \quad (4b)$$

$$\frac{\partial P}{\partial y} + A\rho = 0, \quad (4c)$$

$$\rho \left( u \frac{\partial T}{\partial r} + v \frac{\partial T}{\partial y} \right) = B \left( q(r, y) - \sigma(T^4 - 1) \right) + K_1 \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right) + K_2 \frac{\partial^2 T}{\partial y^2}, \quad (4d)$$

$$\rho T = 1, \quad (4e)$$

where

$$A = \frac{gH}{U^2}, \quad B = \frac{\gamma - 1}{\gamma} \left( \frac{QR}{P_a U} \right),$$

$$M_i = \frac{\epsilon^{3-2i} \mathcal{C}_i}{\rho_a UH}, \quad K_i = \frac{\epsilon^{3-2i} k_i}{\rho_a c_p UH},$$

$$\sigma = 4\pi \hat{\sigma} k^* \frac{T_a^4}{Q} = 4\pi \hat{\sigma} T_a^4 \left( \frac{k^* H}{QH} \right). \quad (5)$$

$\xi_i$  and  $k_i$  are dimensional mixing coefficients, the specific heat capacity  $c_p$  is assumed constant,  $\hat{\sigma}$  is Stefan's constant, and  $k^*$  is the reciprocal of the radiation mean free path (assumed constant). In this formulation, eddy viscosities are used to model the turbulent transport of momentum and energy, and the graybody approximation (4) is used to specify the radiative cooling of the hot gas/smoke mixture.

An appropriate value for the radial velocity scale  $U$  is found by balancing the terms for convective transport and heat addition in the energy equation so as to properly represent the physics of a flow driven by combusive heating. Accordingly, setting  $B = 1$ ,

$$U = \frac{\gamma - 1}{\gamma} \left( \frac{QR}{P_a} \right) \equiv \frac{\gamma - 1}{\gamma} \left( \frac{QH}{P_a \epsilon} \right). \quad (6)$$

#### BOUNDARY CONDITIONS

The type of boundary value problem to be solved depends on the relative magnitudes of the coefficients  $M_i$  and  $K_i$ ,  $i = 1, 2$ . Measurements defining the magnitudes of relative values of the turbulent exchange coefficients have not been performed. However, observations of experimental burns simulating large area fires (5, 6) indicate that the flow is highly turbulent and that the convection column thickness is comparable to the fuel bed radius.

Above several flame heights, the flow asymptotes to the weakly buoyant flow characteristic of the convection column, implying that  $M_2, K_2 \ll M_1, K_1$ . Radial shear should also characterize the flow near the center of the fire. Except in a thin sublayer near the ground, the radial diffusion of momentum and energy should dominate the axial diffusion. Accordingly, we assume  $M_2, K_2 = 0$  and consider solution of the nearly parabolic boundary value problem prescribed by Eqs. (4) and the following boundary conditions.

At the symmetry axis and at the ground, the boundary conditions are

$$u = \frac{\partial T}{\partial r} = 0 \quad \text{on } r = 0, \quad (7a)$$

$$v = 0 \quad \text{on } y = 0. \quad (7b)$$

The asymptotic conditions to be used above many flame heights should reflect the restructuring of a high-velocity, high-temperature, radial flow to a slower-moving, weakly buoyant, nearly vertical flow characteristic of the free-convection column. Based on a formal matching of asymptotic expansions (3) (in the limit  $\epsilon \rightarrow 0$ ) for the separate strongly-buoyant and convection column flows, it is found that the necessary condition is

$$P + Ay \rightarrow 0 \quad \text{as } y \rightarrow \infty, \quad r \leq 1. \quad (7c)$$

It can be shown that Eq. (7c) also implies  $u \rightarrow 0, T \rightarrow 1$  as  $y \rightarrow \infty$ .

At the fire/column periphery ( $r \approx 1$ ), large gradients in temperature, pressure (7), and the level of turbulence are expected. Jump conditions at  $r = 1$  are used to analyze this local behavior. Writing Eqs. (4) in conservation form and integrating from  $r = 1^-$  to  $r = 1^+$  yields the following jumps

in mass, momentum, and energy at the periphery:

$$\begin{aligned} [\rho u] &= 0 \quad , \\ [\rho u^2] &= [P] + \left[ M_1 \frac{\partial u}{\partial r} \right] \quad , \\ [\rho u T] &= \left[ K_1 \frac{\partial T}{\partial r} \right] \quad , \end{aligned}$$

where  $[W] = W^+ - W^-$ . Since the leading-order ambient density and temperature are  $\rho^+ = T^+ = 1$ , integration of Eq. (4c) yields  $P^+ = -Ay$  on  $r = 1^+$ . Expanding the jump conditions, using the leading-order thermodynamic properties and assuming  $M_1^+, K_1^+ \ll M_1^-, K_1^-$ , the boundary conditions applicable at  $r = 1$  are:

$$\begin{aligned} \frac{\partial u}{\partial r} &= \frac{1}{M_1} \{P + Ay + \rho u^2(1 - \rho)\} \quad , \\ \frac{\partial T}{\partial r} &= \frac{1}{K_1} u(1 - \rho) \quad . \end{aligned} \tag{7d}$$

The boundary value problem defined by Eqs. (4) (with  $M_2, K_2 = 0$ ) and Eqs. (7) is independent of both the far-field and the free-convection-column flows. Such an uncoupling implies that the mechanics of the source region is controlled principally by the heat release and the resulting pressure gradients produced by the strong buoyancy.

## RESULTS

For the special case of weak heating ( $q$  small), relatively small temperature changes and velocities are expected, and a leading-order description of those perturbations is provided by a linearization of Eqs. (4) and (7) about the ambient, no-flow state (1). The resulting, simplified equations are decoupled and may be solved in succession for the perturbation temperature ( $T_1$ ), density ( $\rho_1$ ), pressure ( $P_1$ ), radial velocity ( $u_1$ ) and vertical velocity ( $v_1$ ) (1). This solution provides a concise description of the basic interchanges of energy and momentum in and around the burning zone as well as illustrating the structure of the solution.

For example, with  $v \ll 1$ ,  $q(r, y) = v$  in the burning zone and  $q(r, y) = 0$  elsewhere, solution of the simplified energy equation (see Ref. (1)) yields a temperature increase

$$T_1 = v/4\sigma \quad . \tag{8a}$$

Sequential solution of the linearized state, momentum, and continuity equations yields

$$\rho_1 = -T_1 = -v/4\sigma \quad , \tag{8b}$$

$$P_1 = -A(1 - y)/4\sigma \quad , \tag{8c}$$

$$u_1 = -vP_1 r/M_1 = -vA(1-y)r/4\sigma M_1, \quad (8d)$$

$$v_1 = vA(y - y^2/2)/2\sigma M_1. \quad (8e)$$

Solving each equation in turn suggests the following physical interpretation. The heat release increases the temperature ( $T_1$ ) and thus the buoyancy (decrease in density). The buoyancy produces a pressure gradient (Eq. (8c)) which induces the fire-wind inflow  $u_1$ . Finally, the inflow is kinematically turned upward ( $v_1$ ) to form the initial part of the convection column.

In general, Eqs. (4) and (7) are solved (1) by a numerical procedure that involves repeated iteration to find a pressure distribution consistent with the asymptotic condition Eq. (7c). Figures 1-5 show typical results obtained for the 1943 Hamburg firestorm (8). For that case, the fire dimensions and heat release were (approximately)  $R = 1500$  m,  $H = 60$  m, and  $QH = 57$  kcal/m<sup>2</sup>-sec (8, 9). The radiation mean free path was taken as 20 m and  $M_1, K_1$  as 2.0. For those values, the velocity scale  $U$  is 16.8 m/sec,  $A$  and  $\sigma$  are 2.08 and 0.066, respectively.

The temperature rise and subsequent pressure drop in and around the fire due to the combustion heating are shown in Figs. 1 and 2. The changes are maximal in the center of the fire, where the high-speed radial inflow stagnates, but decay rapidly with increasing height. Above several flame heights, a state of weak buoyancy is attained. The induced radial inflow and the resulting vertical upflow are shown in Figs. 3 and 4. The turning of the strong inflow in the source region to form a low velocity, free-convection column is shown in Fig. 5.

## DISCUSSION

The model developed here describes the velocity and thermodynamic fields generated by a large urban fire. The analysis focuses on the turning region, which includes the burning zone and the region below the established free-convection column. Such an approach allows estimates to be made of the conditions necessary for shelter design and of the environment facing survivors and rescue workers.

A finite-volume heat source is used to model the combustion processes, and large changes in temperature and density are allowed. A one-parameter eddy-viscosity model is used to describe the turbulent stresses, and a gray-body approximation employed to model radiative losses. Jump conditions are derived to describe rapid changes in physical quantities at the fire periphery. Those conditions effect model problem closure, allowing the induced fire winds to be computed directly, without extensive far-field calculations.

Sample results illustrate the generation of high-speed fire winds by the heat release and buoyancy production, and simulate the velocity and thermodynamic fields created in the Hamburg firestorm. Extensions of the theoretical treatment could include predictions of specie concentrations (1) as well as the extent beyond the fire region of the high-velocity radial inflow.

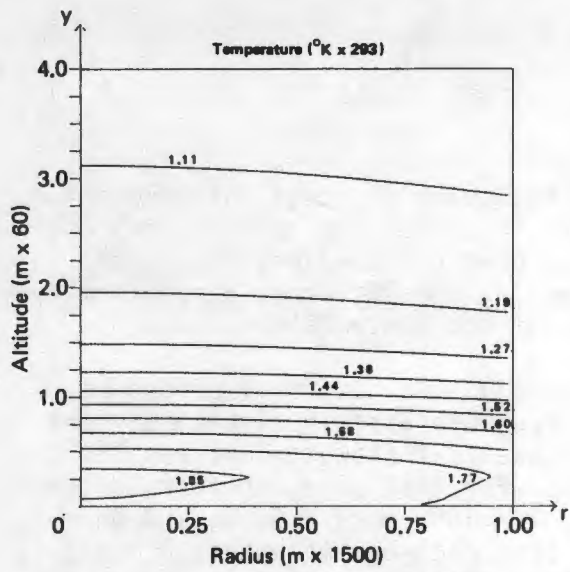


Fig. 1. Temperature contours Hamburg firestorm.

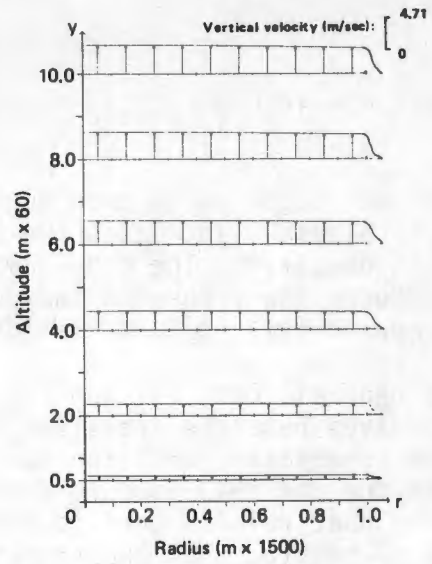


Fig. 4. Vertical velocity profiles, Hamburg firestorm.

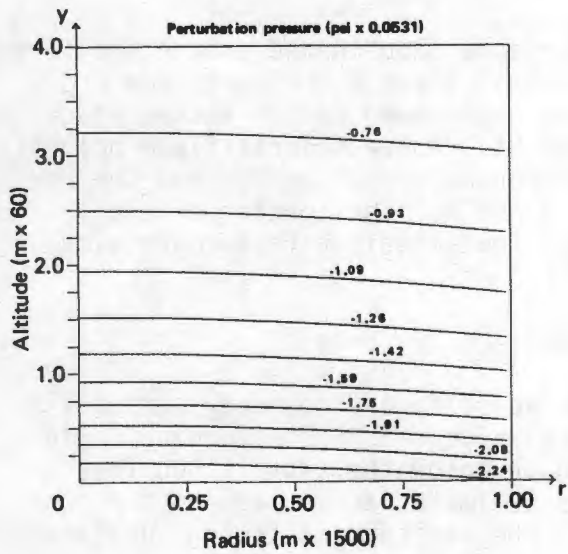


Fig. 2. Pressure contours, Hamburg firestorm.

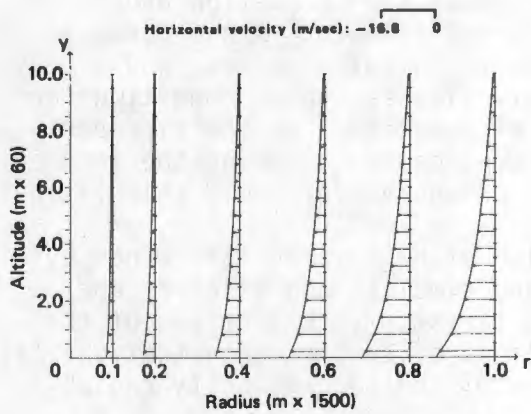


Fig. 3. Radial velocity profiles, Hamburg firestorm.

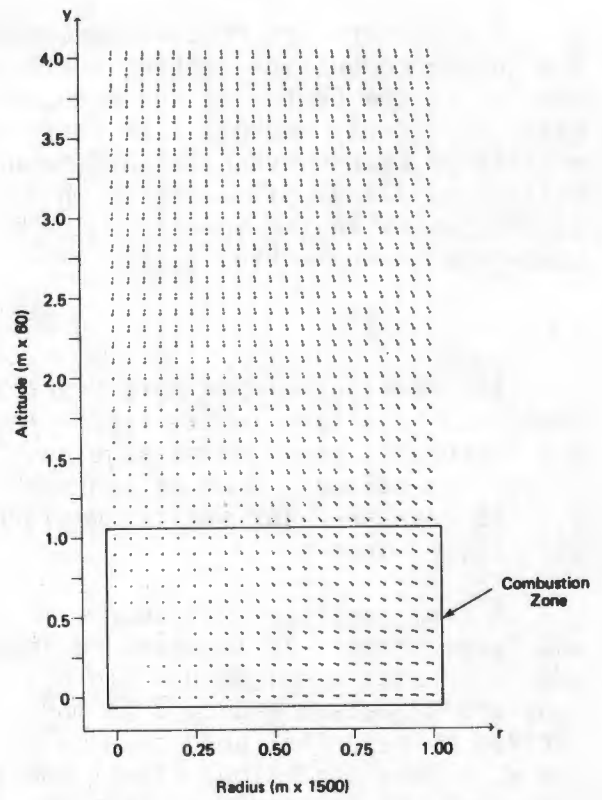


Fig. 5. Velocity field, Hamburg firestorm.

## ACKNOWLEDGEMENT

This research was sponsored by the Federal Emergency Management Agency, Office of National Preparedness, monitored by Dr. David Bensen and the Defense Nuclear Agency, Dr. Michael J. Frankel, Project Monitor.

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