THE DIELECTRIC TENSOR OF ALTERNATING-LAYER MATERIAL WITH APPLICATION TO HONEYCOMB AND BROADBAND RADOMES

Jack Kotik

TECHNICAL RESEARCH GROUP

§1. Dielectric honeycomb material has attracted radome fabricators because of its high strength-weight ratios (in sandwich form). Rational design of such radomes requires that the effect of honeycomb on microwaves be calculable. If the ratio of cell size to wavelength is small enough, the honeycomb will act as a homogeneous anisotropic material. We shall calculate its constitutive tensors $\tilde{\mu}$, $\tilde{\epsilon}$.

We first calculate the constitutive tensors of alternating-layer medium, which is shown in Fig. 1.

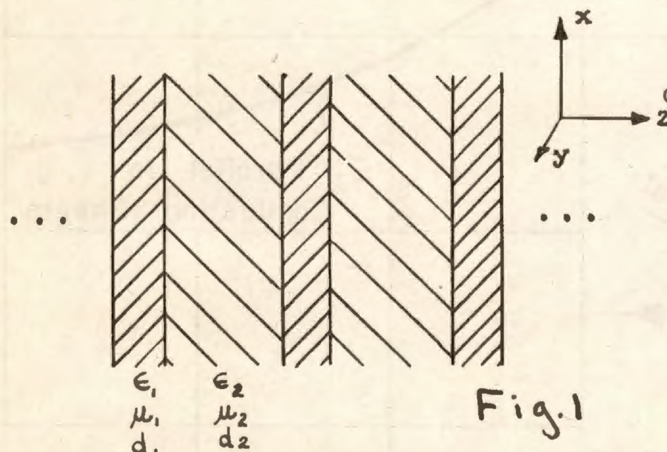


Fig. 1

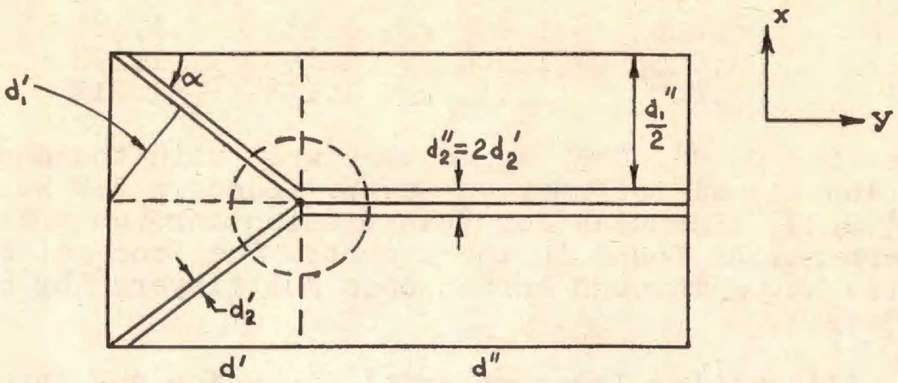
Static (long-wavelength) considerations indicate that

$$\epsilon_\pi = f_1 \epsilon_1 + f_2 \epsilon_2 \qquad 1.1$$

$$\epsilon_\sigma^{-1} = f_1 \epsilon_1^{-1} + f_2 \epsilon_2^{-1} \qquad 1.2$$

where $f_1 = \frac{d_1}{d_1 + d_2}$, ϵ_π is the dielectric constant for $\vec{E}$ parallel to the z-axis and ϵ_σ is the dielectric constant for $\vec{E}$ normal to the z-axis. The direction of propagation is quite immaterial, so that in the coordinate system of Figure 1, the elements of $\tilde{\epsilon}$ are $\epsilon_{xx} = \epsilon_{yy} = \epsilon_\pi$, $\epsilon_{zz} = \epsilon_\sigma$, and the other elements vanish. The results for $\tilde{\mu}$ are completely similar. Formulas 1.1, 1.2 are essentially the formulas for condensers in parallel and in series. This rapid derivation gives results which can be obtained in a more rigorous manner, as in QSR No. 1 on AF33(616)-2973.

§2. Honeycomb can be analyzed by dissection. Invoking symmetry we may take as a unit cell the following:



Subcells $'$, $''$ will be in parallel (series) for $\vec{E}$ parallel to x,z (y). Cell $''$ is alternating-layer material, and hence ϵ_x, ϵ_y, ϵ_z are given in terms of 1.1, 1.2. Observing that the $'$ cell may itself be bisected, and that the result is almost a unit cell (although not a natural one) of alternating-layer material, we have

2.1 $$\epsilon'_x = \epsilon'_\sigma \cos^2\alpha + \epsilon'_\pi \sin^2\alpha$$

2.2 $$\epsilon'_y = \epsilon'_\sigma \sin^2\alpha + \epsilon'_\pi \cos^2\alpha$$

2.3 $$\epsilon'_z = \epsilon'_\pi$$

where ϵ'_π , ϵ'_σ are determined from 1.1, 1.2 using the geometry of the preceeding figure. The quantities α , d_1' , d', d'' depend on the details of the circled region and details of the cell boundaries, but for ordinary honeycomb in which $d_2'' \ll d_1''$ the uncertainty is small. The subcells are combined to give

2.4 $$\epsilon_x = f'\epsilon'_x + f''\epsilon''_x$$

2.5 $$\epsilon_y^{-1} = f'\epsilon'^{-1}_y + f''\epsilon''^{-1}_y$$

2.6 $$\epsilon_z = f'\epsilon'_z + f''\epsilon''_z, \text{ where}$$

$$f^{','' } = \frac{d^{','' }}{d' + d''}$$

This analysis is rough but may be expected to be rather accurate for $d_2'' \ll d_1''$; naturally it is most accurate when $\lambda \gg d_1''$.

Measurements at WADC on a sample having $\epsilon_1 = 1$, $\epsilon_2 = 4\ (1 + .014j\ )$ yielded $\epsilon_x = 1.10$, $\epsilon_y = 1.15$, $\epsilon_z = 1.20$. There is generally some uncertainty regarding d_2' , d_2'' because of the way in which the fiberglass is impregnated with resin. However, the value of d_2' was believed to lie in the range .005" -

.008". Theory yields the following values:

d_2'	ϵ_x	ϵ_y	ϵ_z
.006"	1.097	1.135	1.184
.0065"	1.104	1.147	1.198
.007"	1.111	1.158	1.213

The values for $d_2' = .0065"$ agree very well with the measured values. Additional information may be found in QSR No. 1 on AF33(616)-2973. Formulas for transmission through anisotropic multilayers may be found in the report, "The Propagation of Electromagnetic Waves Through Anisotropic Multilayers" by S. Cutler, AF19(604)-1307.

Alternating-layer material, in which one layer is thin (say .010" - .020") and made of fiberglass and the other is thicker (say .050" - .100") and made of foam, has been used in the construction of wideband radomes. Our previous results can be used to obtain a simple analysis of such radomes. In the frequency band for which both layers are thin this radome wall is simply a slab of homogeneous anisotropic material with $\tilde{\mu} = 1$ and $\tilde{\epsilon}$ given by 1.1, 1.2. For instance, if $d_1 = .050"$, $\epsilon_1 = 1.07$, $d_2 = .010"$, $\epsilon_2 = 4$ we have

$$\epsilon_x = \epsilon_y = \epsilon_T = 1.558$$

$\epsilon_z = \epsilon$[illegible] $= 1.219$

We see that the high transmission in the low-frequency band is due to low dielectric constant rather than any "tuning" effect associated with the multilayer, and that a transmission calculation based on $\tilde{\epsilon}$ is simpler than the conventional calculation.

Redheffer has also investigated a slab of alternating-layer material (with layers parallel to the slab boundaries), from a different point of view, and found that it is equivalent to a homogeneous isotropic slab having $\tilde{\mu} \neq 1$ and a different thickness. This result is surprising but correct and merely indicates that the "natural" equivalent of alternating-layer medium is anisotropic, as in Section 1. "Natural" means that thickness and the property $\mu = 1$ are preserved. The situation may be cogently formulated via the following equivalence:

An anisotropic slab with thickness d and

$$\tilde{\epsilon} = \left\| \begin{matrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{matrix} \right\| , \quad \tilde{\mu} = \left\| \begin{matrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{matrix} \right\|$$

where $\epsilon_x = \epsilon_y$, $\mu_x = \mu_y$, has the same (complex) transmission coefficient at $\perp$-pol. (all θ and ω) as the isotropic slab having

$$(\perp) \qquad \underline{\epsilon} = \gamma_{\perp}\epsilon_x , \quad \underline{\mu} = \gamma_{\perp}\mu_x , \quad \underline{d} = d/\gamma_{\perp}$$

and the same (complex) transmission coefficient at $\parallel$-pol. (all θ and ω) as the isotropic slab having

$$(\parallel) \qquad \underline{\epsilon} = \gamma_{\parallel}\epsilon_x , \quad \underline{\mu} = \gamma_{\parallel}\mu_x , \quad \underline{d} = d/\gamma_{\parallel}$$

where $\gamma_{\perp} = \sqrt{\dfrac{\mu_z}{\mu_x}}$, $\gamma_{\parallel} = \sqrt{\dfrac{\epsilon_z}{\epsilon_x}}$. Hence a polarization-independent equivalence is possible only for those special media having $\gamma_{\perp} = \gamma_{\parallel}$ or $\tilde{\epsilon} = \underline{a}\tilde{\mu}$, where $\underline{a}$ is a scalar.