

INTERACTIVE CHARACTERIZATION AND DATABASE STORAGE OF COMPLEX MODULUS DATA

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ABSTRACT

Test data for viscoelastic damping materials often do not provide thorough coverage of temperature and frequency. A methodology for processing damping material complex modulus data and presenting it in a manner that is meaningful to the damping designer is discussed. The use of a computer program for characterization of complex modulus data is described. A database program for storage and retrieval of characterized complex modulus data is also described.

1. Introduction

Successful design of passive damping treatments using viscoelastic materials (VEM's) such as elastomers depends upon several factors. One important factor is accurate knowledge of the sensitivity of VEM properties to variations in temperature and frequency. Since it is impossible to test a viscoelastic material at every combination of temperature and frequency, the material is tested at discrete temperatures and frequencies and a mathematical relationship is developed that characterizes the material at all other combinations of temperature and frequency. This process is referred to as characterization.

The equations used in characterization are all of a parametric nature, often easily represented on computers. The hard part of characterization is to correctly choose the equation parameters so that they accurately represent the VEM's. Interactive computer graphics have greatly improved the process of choosing and adjusting the correct parametric values.

As the body of characterized viscoelastic material data grows, the need for a centralized VEM database becomes increasingly important. Computers may again be used to allow damping designers and fabricators access to a database of VEM information based on the characterization parameters and material properties.

This paper provides a methodology for obtaining initial parameter values to represent analytically the complex modulus of VEM's and describes computer programs used for VEM characterization and for database storage and retrieval.

2. Complex Modulus Theory

For infinitesimal strain and rate of strain, the time-dependent stress-strain relations for a viscoelastic material can be described by linear differential equations with constant coefficients. This linear behavior requires

$$\beta \equiv \left. \frac{d \log G_M}{d \log f_R} \right|_{\text{transition}} = \left. \frac{d \log G_R}{d \log f_R} \right|_{\text{transition}} = \left. \frac{d \log G_I}{d \log f_R} \right|_{\text{transition}} \quad (1)$$

and

$$\eta_{\max} \cong \tan \frac{\pi \beta}{2} \quad (2)$$

where

- G_M = magnitude of the complex modulus
- G_R = real (storage) part of the complex modulus
- G_I = imaginary (loss) part of the complex modulus
- η = G_I/G_R (also known as the loss factor or $\tan \delta$)
- f_R = reduced frequency = $f_i \alpha_T(T_i)$

$$\begin{aligned}
 f_i &= \text{experimental frequency} \\
 T_i &= \text{experimental temperature} \\
 \alpha_T &= \text{temperature shift function}
 \end{aligned}$$

It has been shown by Rogers that a solution to this requirement is given by the fractional complex modulus equation¹

$$G^*(f_R) = \frac{G_e + G_g z^\beta}{1 + z^\beta} \quad (3)$$

where

$$z = jf_R/f_{R_0}$$

The parameters, G_e , G_g , and f_{R_0} , as well as parameters for α_T must be found such that the curve described by Eq. 3 fits the data within the error bounds of the material test. Initial values for the parameters are first determined graphically and then are iterated and regressed for the best mathematical fit.

3. Initial Parameters

Values for G_e and G_g may be obtained directly by drawing a plot of η versus G_M , as shown in Figure 1. Note, this plot is a useful indicator of data quality in that resonance and other qualitative errors such as broken fixturing will appear as data points that do not follow the overall inverted "U" shape from G_e to G_g (Figure 2). To evaluate β , the equation

$$\eta_{max} = \frac{\left(1 - \frac{1}{A}\right) \tan \frac{\pi\beta}{2}}{1 + \frac{1}{A} + \sqrt{\frac{2}{A} \cos \frac{\pi\beta}{2}}}, \quad \text{where } A = \frac{G_g}{G_e} \quad (4)$$

is derived from Eq. 3. A value for η_{max} is obtained from the plot in Figure 1. β is then calculated iteratively. The transition region is defined by choosing an η_{cutoff} value from the plot in Figure 1. The use of η_{cutoff} to define the transition region is shown in Figure 3.

4. Temperature Shift Function

Historically, the WLF equation has been used to define α_T .² This has not been able to shift all viscoelastic material data correctly outside the transition, however. A new approach is to use a spline fit of the slopes of α_T for a relatively small number

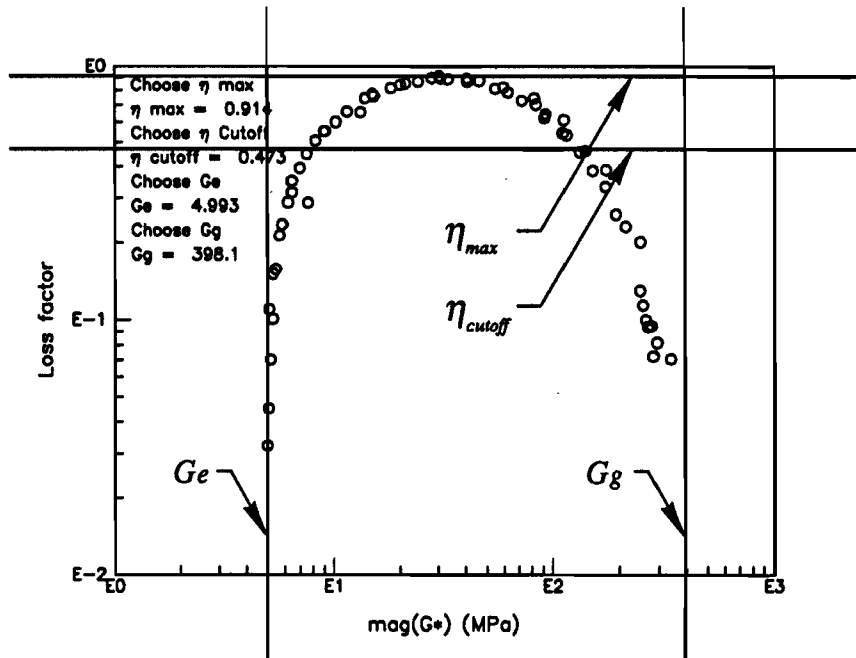


Figure 1. Obtain η_{max} , η_{cutoff} , G_e , and G_g

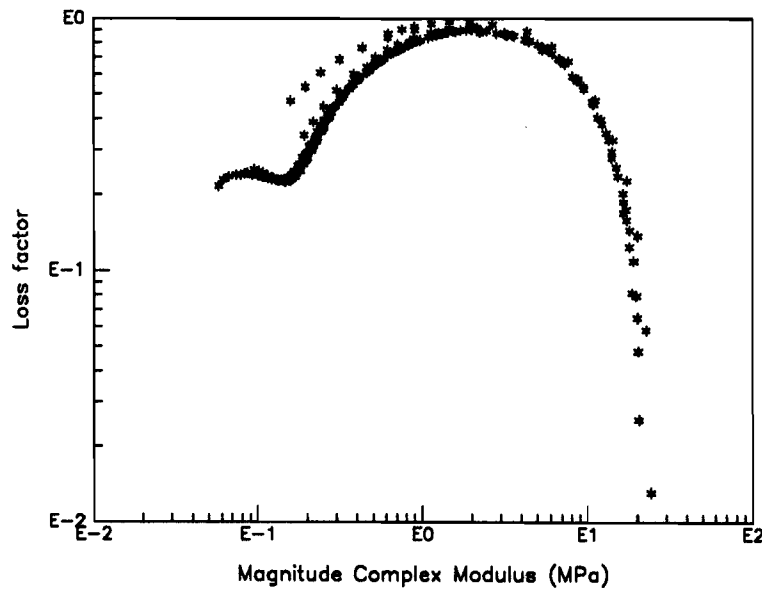


Figure 2. Qualitative error

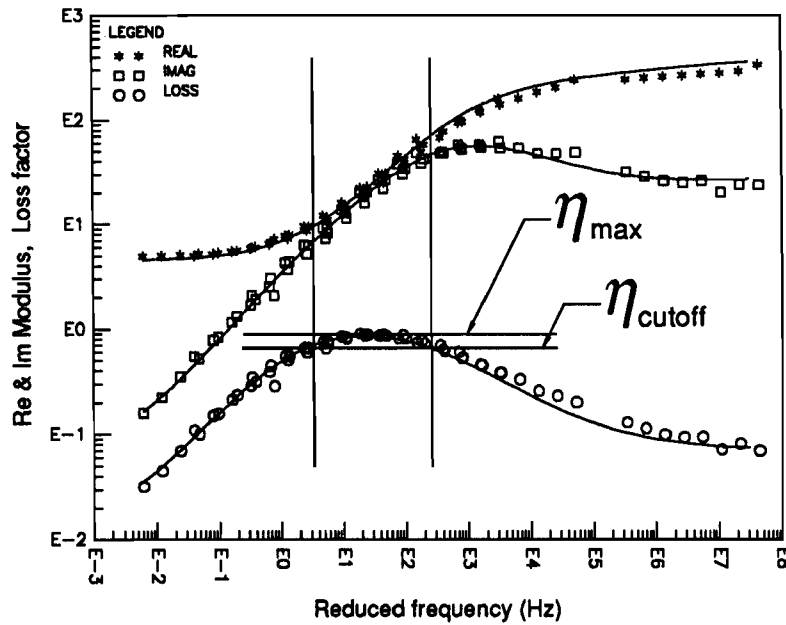


Figure 3. Transition region defined

of equally-spaced temperature points (e.g., 5 points) to define α_T . The reference temperature, T_Z , is obtained by fitting a quadratic function through the data points of $\log \eta$ versus T , solving for zero slope, and rounding to the nearest evenly-spaced temperature point.

Initial values for the reference slope, S_{AZ} , and f_{R0} are obtained by solving Eq. 3 for α_T/f_{R0}

$$\frac{\alpha_T(T_i)}{f_{R0}} = \frac{1}{j f_i} \left[\frac{G_i^* - G_e}{G_g - G_i^*} \right]^{\frac{1}{\beta}} \tag{5}$$

Since Eq. 5 is valid in the transition region, a quadratic is fit through the data points defined within the transition by $\eta \geq \eta_{cutoff}$ for α_T/f_{R0} as a function of temperature. Using $\alpha_T \equiv 1.0$ at T_Z , f_{R0} is obtained from the reciprocal of the quadratic at T_Z . S_{AZ} is obtained as the slope of the quadratic at T_Z multiplied by the initial f_{R0} .

A modified version of the WLF equation is then used

$$\log \alpha_T = -\frac{S_{AZ} (T - T_Z) (T_Z - T_\infty)}{(T - T_\infty)} \tag{6}$$

with T_∞ set equal to 10.0 to generate initial values of slope at all the other temperature-slope points.

Finally, α_T is calculated as the integral of the spline of the slopes where the constant of integration is given by the $\alpha_T \equiv 1.0$ at T_Z relationship.

The accuracy of the α_T parameters is checked by looking at a plot of the complex modulus data versus the reduced frequency. The parameters must be adjusted iteratively to remove any isotherm "shingles."

5. Complex Modulus

Improvements to Eq. 3 which add adjustment factors to account for non-linearity have been offered by Bagley, Rogers, Nashif, and others.^{3 4 5} All equations work for some damping materials. None can adequately fit all sets of VEM data. Present methods use a piecemeal approach. An equation that has successfully fit the type of material of interest in the past or that is the most general is used and parameters are adjusted using regression and trial-and-error to get the best fit. If the best fit is not adequate, a different equation is tried. This approach has been implemented on a computer with nine different complex modulus equations available.⁶

Initial estimates of parameters vary for each model. For example, if the series fractional Maxwell equation, given by

$$G^* = G_e + \sum_{k=1}^n \frac{G_k}{1 + z_k^{-\beta_k} + \Delta_k z_k^{-\rho_k}}, \quad z_k = j \left(\frac{f_R}{f_{R0}} \right)_k \quad (7)$$

where

$$\begin{aligned} G_e < G_k < G_g & \quad (\text{stepping logarithmically}) \\ \beta_k & = \text{slope of storage modulus corresponding to } G_k \\ \Delta_k & = \text{pole multiplier} \\ \rho_k & = \text{pole exponent} \end{aligned}$$

is chosen, β_k is set equal to the previously calculated β , Δ_k is the slope of the glassy intercept with the abscissa on a Cole-Cole plot (i.e. G_I versus G_R), and ρ_k is set to 0.1 for all k .⁷

6. Graphical Presentation

Jones and, more recently, Jones and Rao have developed methods to present complex modulus data graphically.^{8 9} These are the reduced-temperature nomogram (also known as the international plot) and inverted "U" plot respectively (Figures 4 and 5).

The international plot consists of the real and imaginary moduli displayed logarithmically in megaPascals (MPa) on the left vertical axis along with the dimensionless loss factor. The horizontal scale is the reduced frequency defined in Eq. 3.

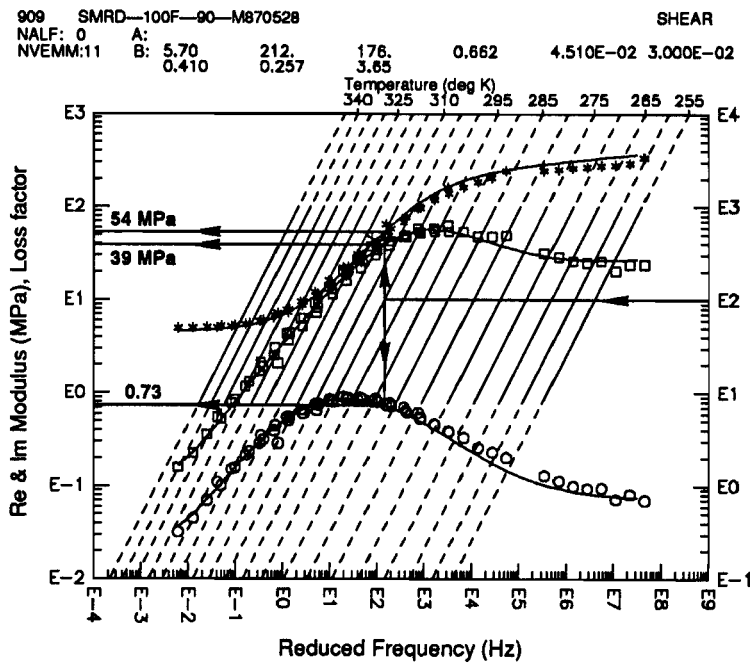


Figure 4. Reduced-temperature nomogram (international plot)

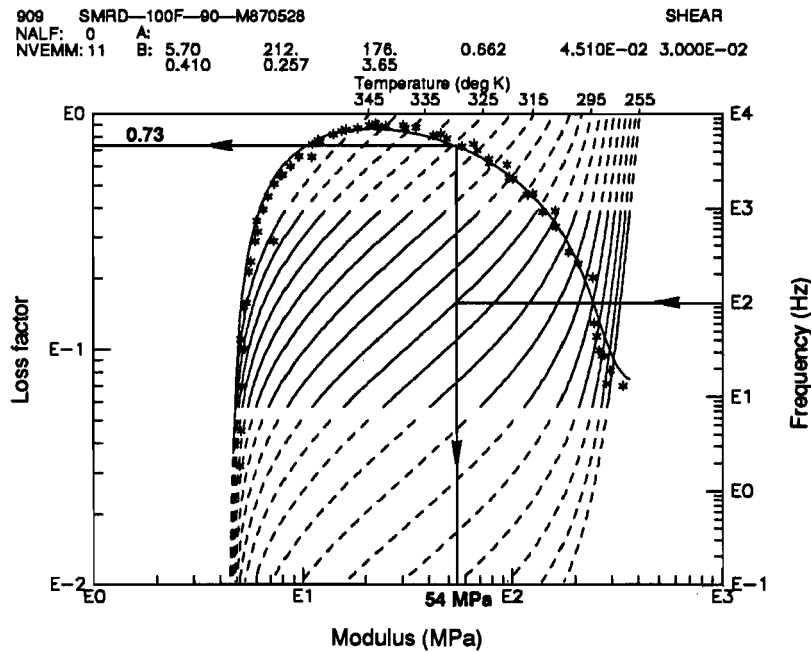


Figure 5. Inverted "U" plot

The right vertical axis is cyclic frequency displayed logarithmically in hertz (Hz). Lines of constant temperature are superimposed on the plot from the relationship

$$\log f_R = \log f_i + \log \alpha_T (T_i) \quad (8)$$

These isotherm lines are usually calculated for steps of five degrees Kelvin and range from T_L to T_H to preclude extrapolation of temperature for which viscoelastic materials are highly sensitive. The range of experimental frequency is indicated by the solid region of the isotherm lines. In the area of extrapolated frequency, the isotherms are dashed. The use of the international plot to read interpolated values of modulus and loss factor is demonstrated in Figure 4. To get modulus and loss factor values corresponding to 100 Hz and 300°K, one reads the 100 Hz frequency on the right-hand scale and proceeds horizontally to the 300°K temperature line. Then proceed vertically to intersect the curves along a line of reduced frequency. Finally, proceed horizontally from these intersections to the left-hand scale to read the values of 54 MPa for the real modulus, 39 MPa for the imaginary modulus, and 0.73 for the loss factor.

The inverted "U" plot utilizes similar methodology, but removes the reduced frequency scale and directly superimposes constant temperature lines onto a plot of loss factor versus the real part of the complex modulus. Cyclic frequency is still displayed on the right-hand axis. To follow the same example as above, start at the 100 Hz frequency value on the right-hand scale and move horizontally to the 300°K temperature line. Drop vertically downward to read 54 MPa off the horizontal axis for the real modulus, and proceed upward to the curve and then horizontally to read 0.73 off the left-hand vertical scale for the loss factor.

Other plots of interest include

1. $\log \alpha_T, d \log \alpha_T / dT$, and the apparent activation energy versus temperature
2. $\log f_i$ versus temperature
3. real and imaginary components of G^* , and η versus temperature

7. VEM Database

A VEM database program has been written to store and retrieve characterized viscoelastic material data.¹⁰ It uses the characterization equations and parameters to interpolate modulus and loss factor values to match design criteria. The focus of the first page, Search Specifiers (Figure 6), is toward finding materials using frequency, temperature, modulus range, and loss factor range. Up to three different search points using these property specifiers may be used. The search temperature may be in units of Fahrenheit, Rankine, Centigrade, or Kelvin. The search modulus

Page: **Search Specifiers**

Report Name: **SEARCH_1**
 Units: **English Fahrenheit** Points: **1**

	Point 1		Point 2		Point 3	
Frequency:	1.0		10.		1.00E+02	
Temp (deg F):	72.		72.		72.	
	Lower	Upper	Lower	Upper	Lower	Upper
Modulus (psi):	1.0	10.	10.	50.	1.00E+02	2.00E+02
Loss factor:	0.10	0.50	0.10	0.70	0.10	1.0

Search Reports

Report name: **SEARCH_1**
 Output to: **Screen**
 Units: **English Fahrenheit**
 Report type: **Tables & Plots**
 Tables: **Brief**
 Plots: **International** Markers: **On**
 Lower: **10.** Upper: **1.00E+03**

Search specifiers **Librarian** **Beam specifiers** **Setup**

Figure 6. Search Specifiers page

range values may be in units of pounds per square inch (psi) or megapascals. All units, however, must be in the same measurement system – English or SI. The program will also find materials based on their names, manufacturers, material types, and availability in the Librarian page.

Reports are generated on request when one or more materials in the database file fall within the range of at least one search point. All materials that meet that criteria are flagged for inclusion into a report. Once a report is generated, it may be sent to either the computer screen or a hardcopy device. Different fields may be set to define the form of the report. Output is in the form of text data and information, the international and inverted “U” plots, and constant temperature and constant frequency plots. Figures 7 and 8 are examples of the output from the database.

8. Observations

Most viscoelastic materials data are for engineering applications and justifiably do not provide scientific coverage of temperature and frequency. The challenge is to make the data useful to the damping designer and simultaneously indicate limitations.

Full directory of SEARCH_1

Material Name SMRD-100F-90 -M870528 Manufacturer MANUFACTURER Type UNSPECIFIED Available YES

This is the information field for SMRD-100F-90-M870528. This is where you would enter information about material handling, name and address of material suppliers, material properties such as outgassing, ease of use, etc.

SMRD-100F-90 -M870528 SHEAR

		ALPHA-T MODEL					
HALF	NA	A(1)	A(2)	A(3)	A(4)	A(5)	A(6)
0	0						
		COMPLEX MODULUS MODEL					
NUM	NR	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)
11	9	827.	212.	176.	0.662	4.510E-02	3.000E-02
		B(7)	B(8)	B(9)			
		0.410	0.257	3.65			

COMPLEX MODULUS DATA AS A FUNCTION OF TEMPERATURE AND FREQUENCY

Temp (Deg F)	Freq (Hz)	MReal (psi)	Eta	MImag (psi)	Alpha-t	
-2.440	7.600	3.5389E+04	0.1300	4601.	4.3215E+04	T
-2.440	15.60	3.6549E+04	0.1140	4167.	4.3215E+04	T
-2.440	31.20	3.7710E+04	9.9700E-02	3760.	4.3215E+04	T
-2.440	62.50	3.8580E+04	9.4100E-02	3630.	4.3215E+04	T
-2.440	125.0	3.9885E+04	9.4700E-02	3777.	4.3215E+04	T
-2.440	250.0	4.0755E+04	7.2100E-02	2938.	4.3215E+04	T
-2.440	500.0	4.2351E+04	8.1600E-02	3456.	4.3215E+04	T

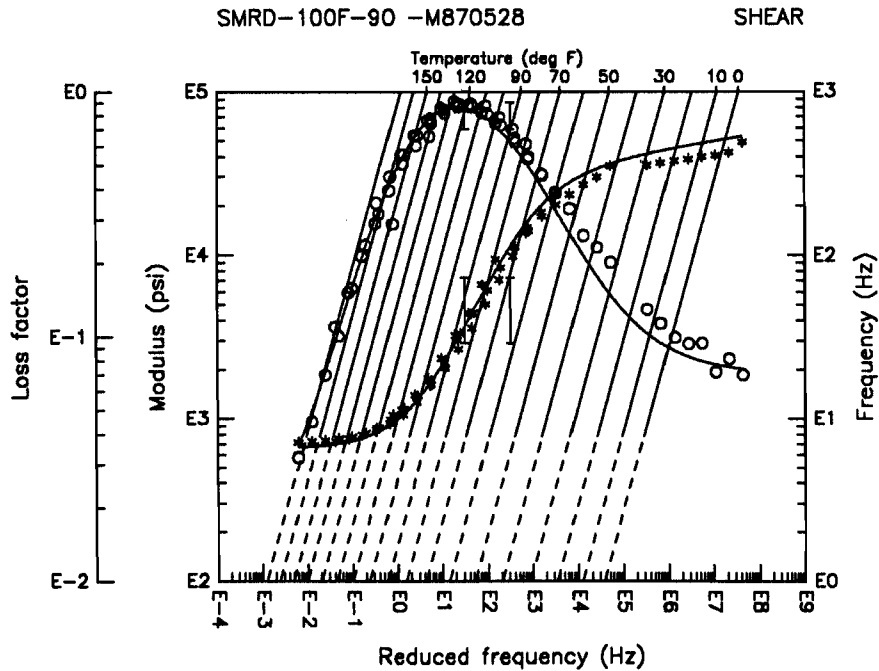


Figure 7. VEM database output
Text data and the International plot

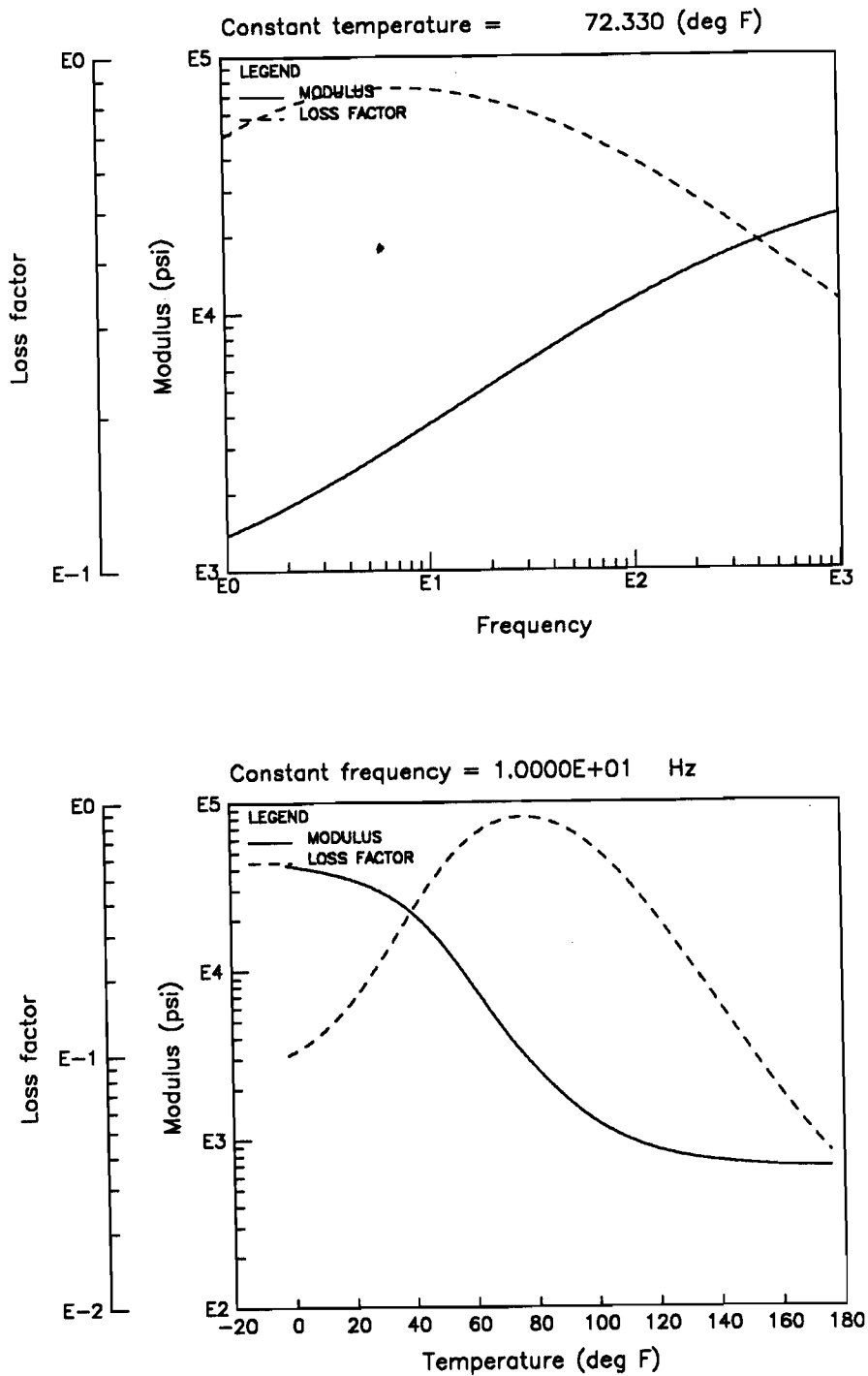


Figure 8. VEM database output (continued)
Constant temperature and constant frequency plots

Using the methodology and graphical presentation outlined in this paper, this challenge can be mostly met. Care must always be taken, however, when using the various accepted equations to insure that an appropriate representation of the complex modulus has been chosen. Errors of several orders of magnitude are still too easily introduced when the wrong parameters and/or the wrong equations are used.

The need for good characterization has always been present. With advances in damping design tools the need has become even more critical. As a design tool, the VEM database must have accurately characterized materials to be useful. Work must still be done to improve the equations used to characterize viscoelastic materials so that they will accurately characterize all damping materials.

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