ELECTROMAGNETIC VIBRATION DAMPERS*

by

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ABSTRACT

A non-magnetic metal moving through a region of non-uniform magnetic field experiences a drag force. For some simple, one-dimensional or axi-symmetric cases, it is possible to obtain an exact analytical solution. For more complex geometries, finite element (FE) methods are the most practical means of calculating the force between a configuration of magnets and a moving conductor. This paper describes how FE calculations can be performed and shows that good agreement can be obtained between FE calculations and the measured response. When a conducting plate, bar or rod is constrained to move near certain configurations of high energy density, permanent magnets, a large drag force proportional to the relative velocity is produced. This drag force can be used to damp mechanical motion. This paper presents several candidate magnet-conductor configurations that could be used as vibration damper assemblies. The next step is to design damper assemblies for particular modes of a specific structure and then to compare the calculated with the measured performance of these dampers.

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1.0 INTRODUCTION

The reduction or elimination of unwanted structural motion is an ever present problem in mechanical structures. Many very clever and effective solutions have been developed to address vibration damping under a wide variety of circumstances. This paper shows that electromagnetic damping as described herein should become one of the candidate technologies that is routinely considered for adding passive damping to structures. Several modifications of the passive damping approaches discussed in this paper are also candidates for combined active and passive dampers but these are not discussed here.

2.0 GENERAL BACKGROUND THEORY

Currents are induced to flow in any conductor moving through a region of localized magnetic field; these currents and fields obey Maxwell's equations

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad (1)$$

and

$$\nabla \wedge \underline{H} = \underline{J} \qquad (2)$$

For non-magnetic metals such as aluminum, the appropriate constitutive equations for the moving conductor are

$$\underline{J} = \sigma \underline{E} + \sigma \underline{v} \wedge \underline{B} \qquad (3)$$

and

$$\underline{B} = \mu \underline{H} \qquad (4)$$

where $\underline{v}(\underline{r},t)$ is the velocity of the conductor relative to the magnetic field $\underline{B}(\underline{r},t)$, σ is the electrical conductivity and μ is the magnetic permeability. Following standard convention, solutions are developed in terms of a vector and scalar potential such that

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} + \nabla\phi \qquad (5)$$

$$\underline{B} = \nabla \wedge \underline{A} \qquad (6)$$

Substituting Equations (5) and (6) into Equation (3) gives

$$\underline{J} = \sigma\left(-\frac{\partial \underline{A}}{\partial t} + \underline{v} \wedge \nabla \wedge \underline{A}\right) - \sigma\nabla\phi \qquad (7)$$

Under most conditions at low frequencies, the time derivative of $\underline{A}$ will be much smaller than the velocity term and one can write

$$\underline{J} = \sigma \underline{v} \wedge \nabla \wedge \underline{A} - \sigma\nabla\phi \qquad (8)$$

With no loss of generality for 2D current flow, one can take $\underline{A} = (0,0,A)$ and $(\partial A/\partial z) = 0$. Consequently,

$$B_x = \frac{\partial A}{\partial y} \; ; \quad B_y = -\frac{\partial A}{\partial x} \; ; \quad B_z = 0 \qquad (9)$$

One is free to chose the gauge such that $\nabla \cdot \underline{A} = 0$. Let us consider the special case of a conducting plate moving in the y-direction (therefore $\underline{v} = (0,V,0)$) with the magnetic field confined to the x-y plane as required by Equation (9). Combining Equations (2), (4), (6) and (8) gives

$$-\frac{1}{\mu}\frac{\partial^2 A}{\partial x^2} - \frac{1}{\mu}\frac{\partial^2 A}{\partial y^2} + \sigma V \frac{\partial A}{\partial y} - \sigma \nabla \phi = 0 \qquad (10)$$

Solving Equation (10) gives the magnetic field and its gradients (and hence the current density induced in the conductor).

The total power dissipated by the moving conductor is given by

$$P = \frac{1}{\sigma} \int_{conductor} \underline{J} \cdot \underline{J}^* \, dx\, dy\, dz \qquad (11)$$

The equations developed above neglect any skin depth effects. If conditions are such that motion causes a significant screening of the inside of the conductor, then the term in $\partial A/\partial t$ in Equation (7) must be included. The solution is straightforward but considerably more complex than the outlined given above.

3.0 FINITE ELEMENT CALCULATIONS

The standard starting point for electromagnetic finite element (FE) calculations is Equation (10) with the velocity dependent term equal to zero. It is well known that the solution of a partial differential equation (PDE) containing a term like ($v\, \partial A/\partial y$) such as in Equation (10) is difficult to solve using numerical procedures because there is a tendency to generate oscillatory solutions.

Variational calculus shows that, if a functional F' satisfies the equation

$$\frac{\partial}{\partial x}\left[\frac{\partial F'}{\partial(\partial A/\partial x)}\right] + \frac{\partial}{\partial y}\left[\frac{\partial F'}{\partial(\partial A/\partial y)}\right] - \frac{\partial F'}{\partial A} = 0 \qquad (12)$$

then F' is a solution to the PDE given by Equation (10). With some considerable efforts, we have shown that

$$F' = \exp(-\mu\sigma y V)\left[\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2 + 2A\sigma\nabla\phi\right] \qquad (13)$$

reproduces Equation (10) and hence can be used in the Ritz method for obtaining a FE solution to Equation (10).

Using the functional given by Equation (13), we have developed a FE solution to Equation (10). One particular case is shown in Figure 1 where an aluminum plate is moving with a velocity of 1 m/sec between the poles of a magnet that produces a maximum field of about 1 T in the gap region. It is clear that the magnetic field lines within the plate are altered substantially by current induced with the plate when it is moving.

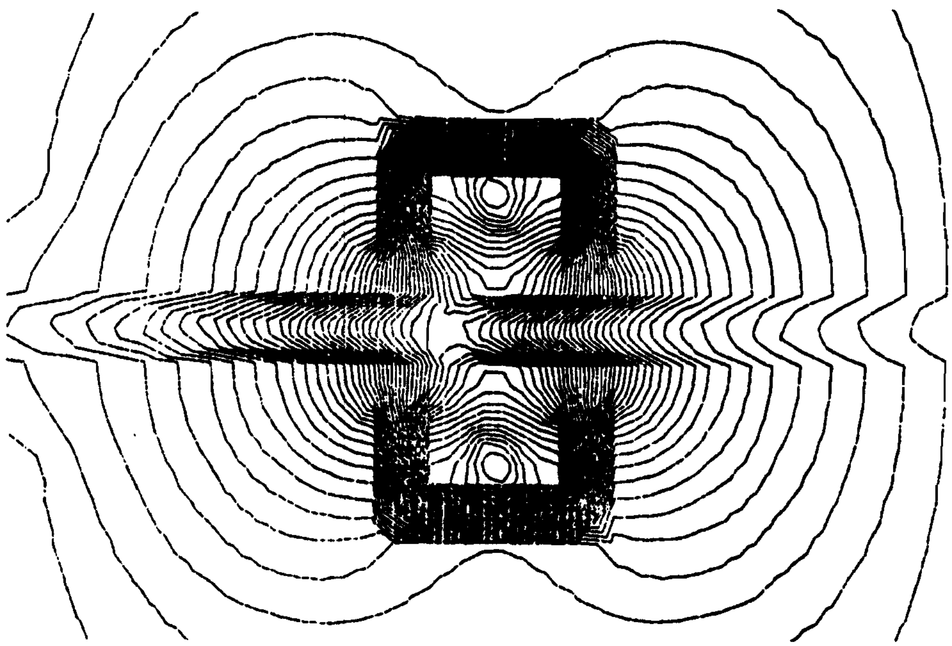

Figure 1 A finite element calculation showing the magnetic field configuration due to a non-magnetic conductor moving in a magnetic field.

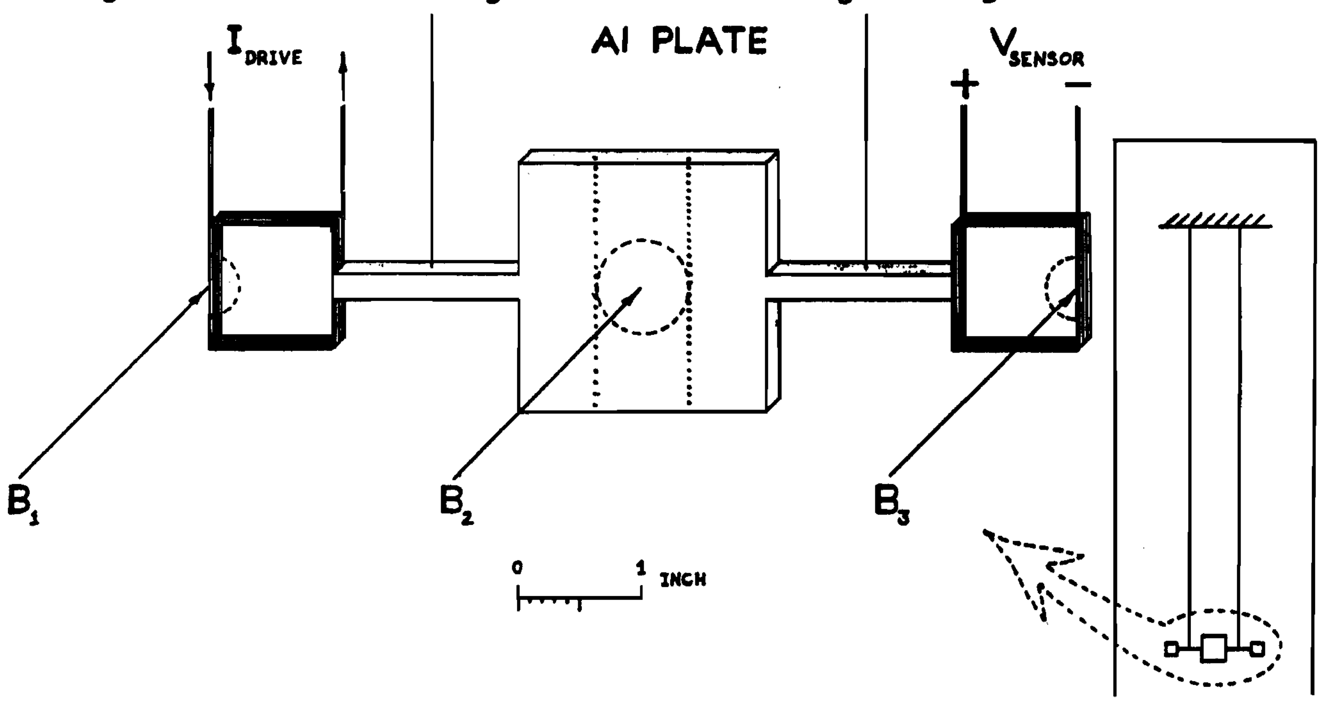

Figure 2 A pictorial illustration of the experimental arrangement used to determine the viscous drag coefficient of a conductor (Al block) moving within a reasonably localized field region, B_2. A current I (drive) through a coil that passes through a region, B_1, of reasonably constant field produces a well defined driving force on the rigid system shown in the figure. The frequency of the drive current is changed in order to map out response curves. The velocity amplitude of the response is determined by the voltage induced in a pickup coil moving in region, B_3. This coil is positioned in the field so that the pickup voltage is proportional to the horizontal velocity of the rigid system. Naturally, this voltage is also proportional to the frequency.

4.0 COMPARISON WITH EXPERIMENTS

One of the most widely studied and easily understood mechanical systems is the damped forced oscillator. This system, shown pictorially in Figure 2, was chosen for a quantitative evaluation of passive electromagnetic damping. Aluminum plates up to 6 mm thick were placed as shown at the end of a long string to form a pendulum. For the case described here, this pendulum had a frequency of 1.06 Hz. The Al plate could be driven by a linear motor shown pictorially as B_1 on the left hand side in Figure 2. The horizontal velocity produced by this driving force was measured using a calibrated electromagnetic velocity sensor shown pictorially on the right hand side in Figure 2.

This geometry does not satisfy all of the constraints imposed on the FE solution, namely the magnetic field in the z-direction (vertical direction in Figure 2) is non-zero in some regions. We handled this by first calculating the damping per unit volume assuming the plate to be infinite in extent and the magnetic field to be constant within the rectangular region defined by the dotted lines in Figure 2. The actual damping was calculated by using the calculated damping per unit volume and the actual volume of conductor over which there existed a magnetic field greater than 0.7 of the maximum gap field.

5.0 A DRIVEN DAMPED HARMONIC SYSTEM

A driven, damped, harmonic system is described by the equation

$$M\ddot{x} + 2b\dot{x} + \omega_o^2 x = P\sin(\omega t) \tag{14}$$

where M is the mass of the moving system, P is the peak driving force, b is the damping or drag coefficient, ω_o is the system resonant frequency. The steady state solution is given by

$$x_o = \frac{P/M}{\left[(\omega_o^2 - \omega^2)^2 + 4b^2\omega^2\right]^{1/2}} \tag{15}$$

The experimental setup shown in Figure 2 gives directly the peak velocity. The damping coefficient, b, can be obtained directly from these measurements. To do this, let us rewrite Equation (15) as

$$\frac{1}{(\omega x_o)^2} = \left(\frac{M}{P}\right)^2 \omega^2 \left[1 - \left(\frac{\omega_o}{\omega}\right)^2\right]^2 + 4\left(\frac{M}{P}\right)^2 b^2 \tag{16}$$

Plotting $(\omega x_o)^{-2}$ against $\omega^2[1-(\omega_o/\omega)^2]^2$, one obtains a straight line with slope $(M/P)^2$ and intercept of $(2Mb/P)^2$ from which one obtains b. It is also customary to define a damping constant k = 2Mb.

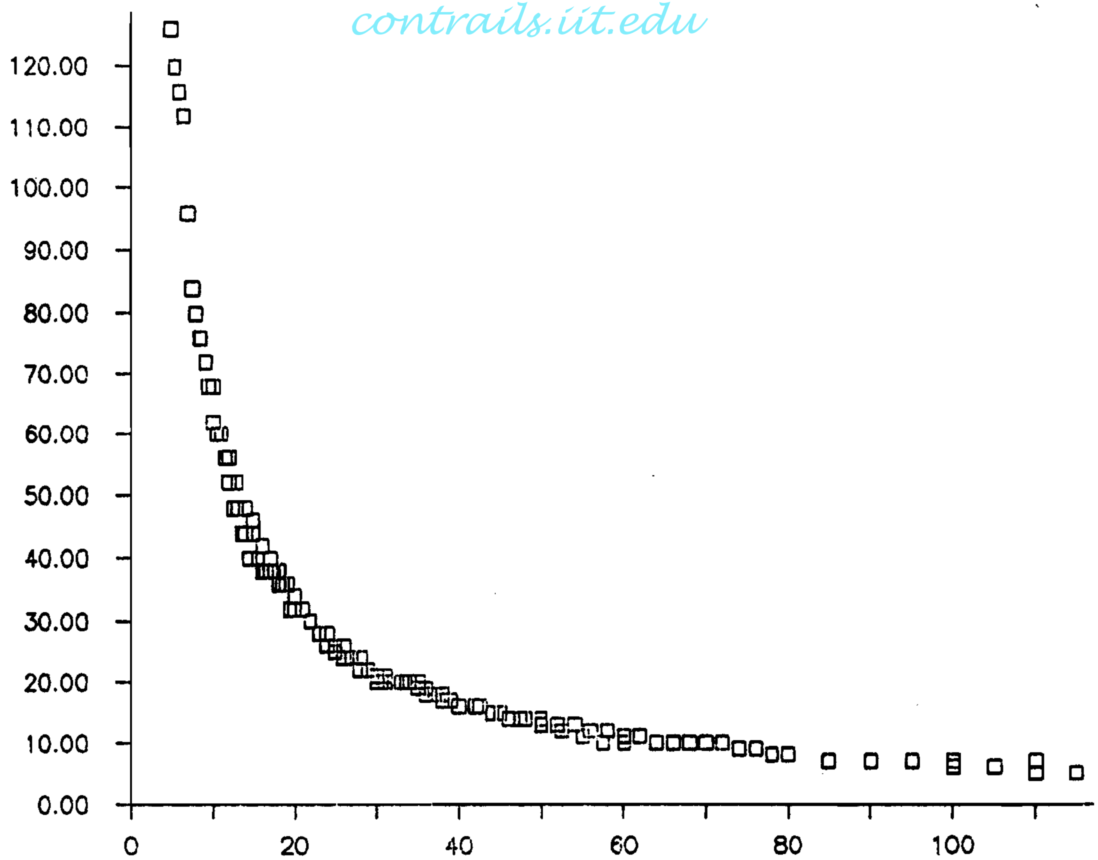

Figure 3 The velocity amplitude as a function of frequency obtained using the experimental setup shown in Figure 2.

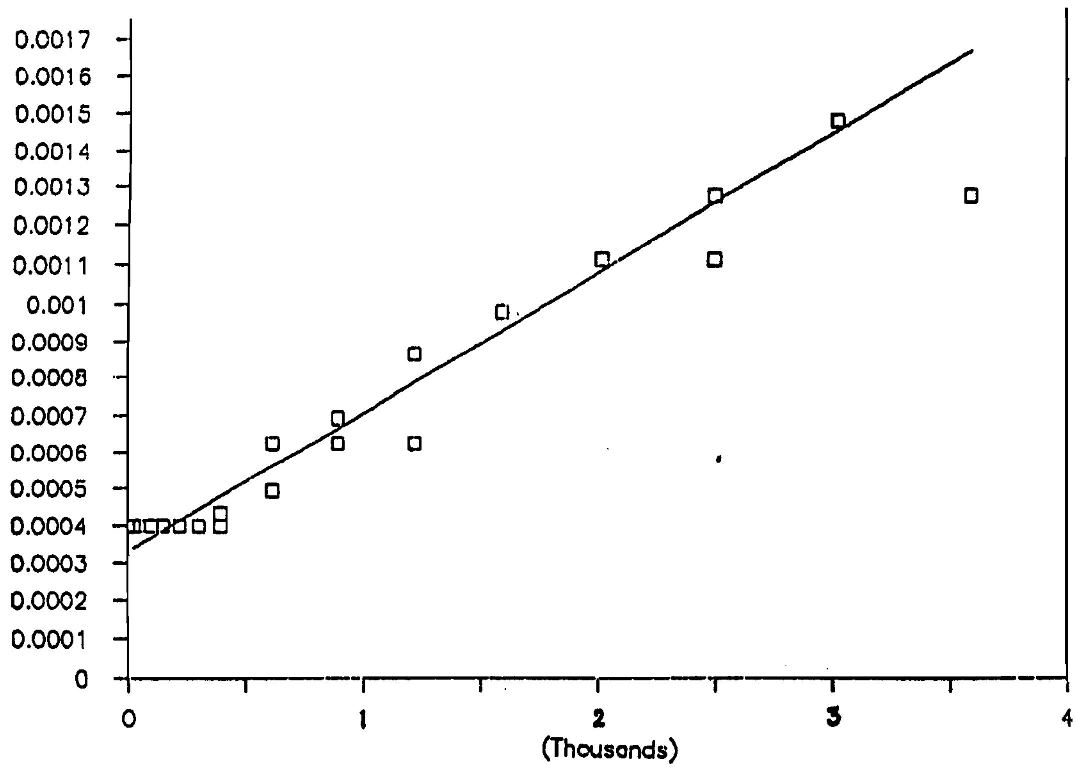

Figure 4 This shows experimental data plotted as described in the text; from the slope and vertical intercept, one obtains the experimental damping constant. The central field, B_z, for a 0.5 inch square pole is about 1.5 T.

Figure 3 shows the velocity amplitude as a function of frequency, $f = \omega/2\pi$ for a particular value of magnetic field. When plotted as Equation (16), one obtains the graph shown in Figure 4. From this and many similar plots, one finds that, as expected, the power dissipated by electromagnetic damping is quadratic in both velocity and magnetic field. At the highest field of 1.5 T where we have the greatest accuracy in our measurements, the damping factors are

$$b(EXP) = (101 \pm 6)\ /sec; \quad k(EXP) = (21 \pm 1)\ kg/sec$$

A FE calculation performed as described above for this same case yields

$$b(FE) = 72\ /sec \qquad ; \ k(FE) = 15\ kg/sec$$

We regard this as good quantitative agreement. Of course, better agreement could be obtained using a 3D FE code but this would be a great deal more time consuming to develop. A single point calibration that normalized the calculated magnetic field to the measured value in the gap would also reduce the difference between calculated and measured values for the damping.

6.0 POTENTIAL DAMPER CONFIGURATIONS

Although our example of a pendulum is an excellent case for demonstrating that there is good quantitative agreement between FE calculations and the measured behavior of a damped harmonic system, the magnet and conductor configuration that was used is not very practical. For many applications, we expect that it will be most practical to have magnets near only one surface; that is, it will not generally be practical to place the moving conductor within the gap of a permanent magnet. Figure 5 shows one magnet configuration that provides good damping. An array of rectangular permanent magnets is placed with alternating magnet poles adjacent to each other as shown in Figure 5. This magnet stack is attached rigidly to some portion of the structure that will move relative to the conductor that is adjacent to the magnet assembly. Damping results when the magnet assembly moves relative to the conductor. The dimension of the magnet pole height shown in Figure 5 determines the magnetic field liftoff coefficient or how rapidly the magnetic field decreases with distance form the pole face. This, in turn, determines the thickness and closeness of conducting material that should be used in the damper. In general, a damping constant of about 20000 $kg/sec/m^2$ of pole area can be obtained for each 1 mm in thickness of Al conducting material. Clearly, for the greatest damping, such a damper should be placed between two points on a structure having the largest relative velocity.

Figure 6 shows an inertial damper that is a modified version of the damper in Figure 5. The non-magnetic springs keep the damper somewhat centered. When the structure to which this damper is attached is accelerated, the magnet assembly will move relative to the support Al tube. Energy will be dissipated as long as this relative motion exists.

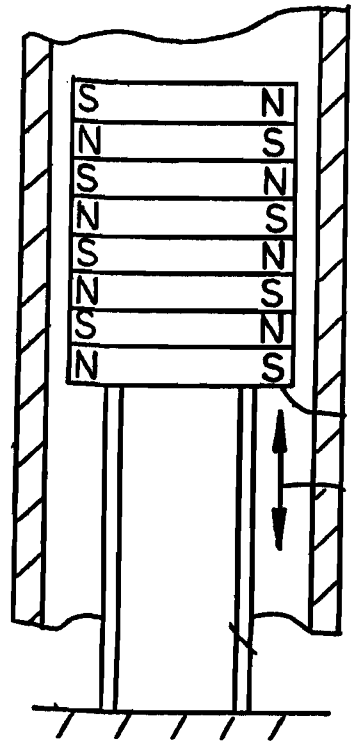

Figure 5 An electromagnetic damper that is analogous to a viscoelastic extensional shear damper. The magnet assembly is attached to one end of a tubular support strut by a very light, thin walled tube (it need only support the viscous drag or damping force between the magnets and the aluminum strut). Relative motion between the magnet assembly and aluminum strut results when the strut is lengthened or compressed due to an applied load.

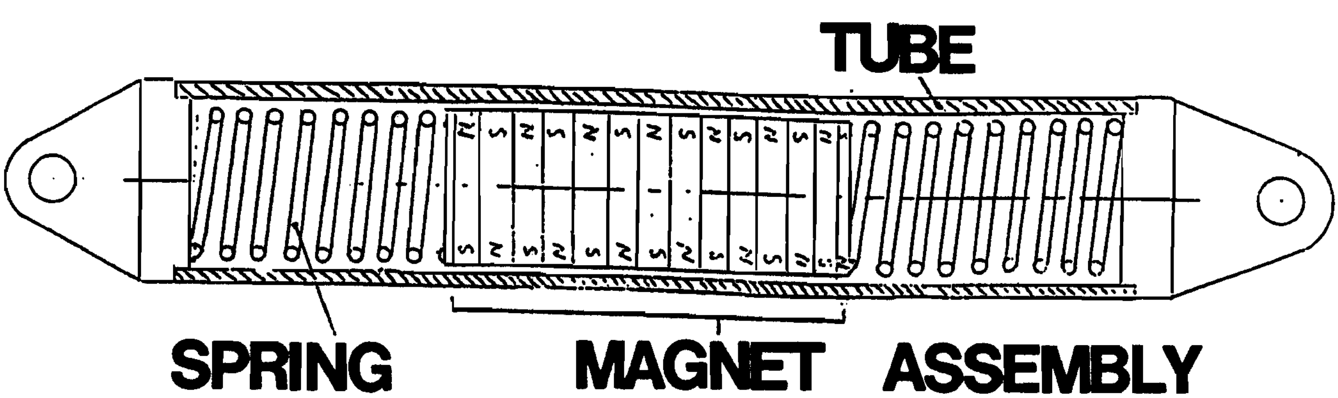

Figure 6 An inertial electromagnetic damper that operates by relative motion between the magnet assembly and the aluminum support tube. Spring constants are chosen so that the magnet assembly-spring resonant frequency is somewhat lower than the frequencies one wishes to damp. Under this condition, any acceleration of the structure (which is connected directly to the aluminum support tube) will produce relative motion between the magnet and aluminum tube. This damper is best substituted for a load bearing member although it can also be placed in parallel with structural members.

A damper assembly capable of withstanding very large loads and providing a large damping constant is shown in Figure 7.

7.0 ADDITIONAL CONSIDERATIONS

Like standard viscous damping, electromagnetic damping results from a force that is velocity dependent. This raises questions about the effectiveness of this damping at very low velocity. To evaluate the low velocity behavior in a qualitative manner, we constructed a simple loaded cantilever beam having an oscillation period of about 2 seconds. A stiff plate attached to the free end of the beam formed the moving plate of an electromagnetic damper assembly. This plate moved between the poles of an electromagnet having a pole area of 0.5 square inches and a gap field that could be as large as 1.8 T. A velocity sensor similar to the one shown in Figure 2 was used to measure the velocity of the free end of the beam. Figure 8 shows a sequence of velocity-time waveforms immediately after the beam was deflected 1 cm from its equilibrium position. Figure (8a) shows the behavior for zero applied field (about 0.05 T residual field). At a field of 0.67 T, Figure (8d) shows that one gets the most rapid return to equilibrium. Figure (8e) is very near the condition of critical damping while Figures (8f) and (8g) show that damping beyond critical damping can be achieved. Clearly, damping exists, as expected, down to the smallest measurable velocities.

8.0 SUMMARY AND CONCLUSIONS

In this paper, we have shown that the damping that results from a conducting, non-magnetic plate moving near the pole of a permanent magnet can be understood in a very quantitative manner. In addition, the expected quadratic dependence upon relative velocity (between the plate and magnet) and magnetic field has been demonstrated. Several magnet geometries that are adaptable to practical damper configurations have been suggested. To date, no quantitative measurements on any of these assemblies have been made.

Electromagnetic dampers have some advantages over other means that have been used to achieve damping. Since the energy is dissipated within an excellent thermal conductor, there is no problem in removing heat when large average powers are involved. Nearly all the temperature dependence arises from the electrical conductivity (see Equation (11)). This is a very mild temperature dependence compared to that encountered in using viscoelastic materials (VEMs). A single damper assembly could operate very well over a temperature range of several hundred Kelvin. Behavior of electromagnetic dampers (EDs) is extremely predictable under a wide variety of conditions. EDs can tolerate operating at elevated temperatures (in some cases, up to about 1000 K) and in very high radiation (neutron, gamma or X-ray) fluxes.

Although the detailed description of EDs given in this paper is only applicable at relatively low frequencies (say below 100 Hz), the basic

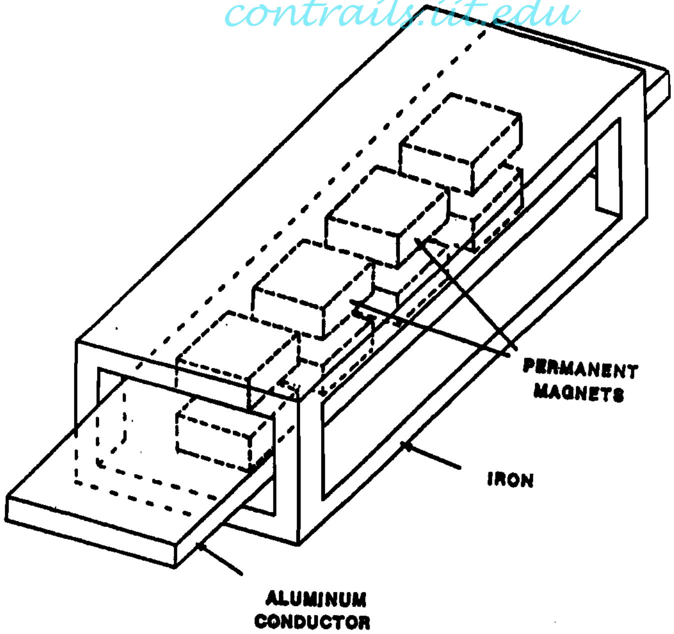

Figure 7 An electromagnetic damper that can produce large damping forces and handle large transient or steady-state loads.

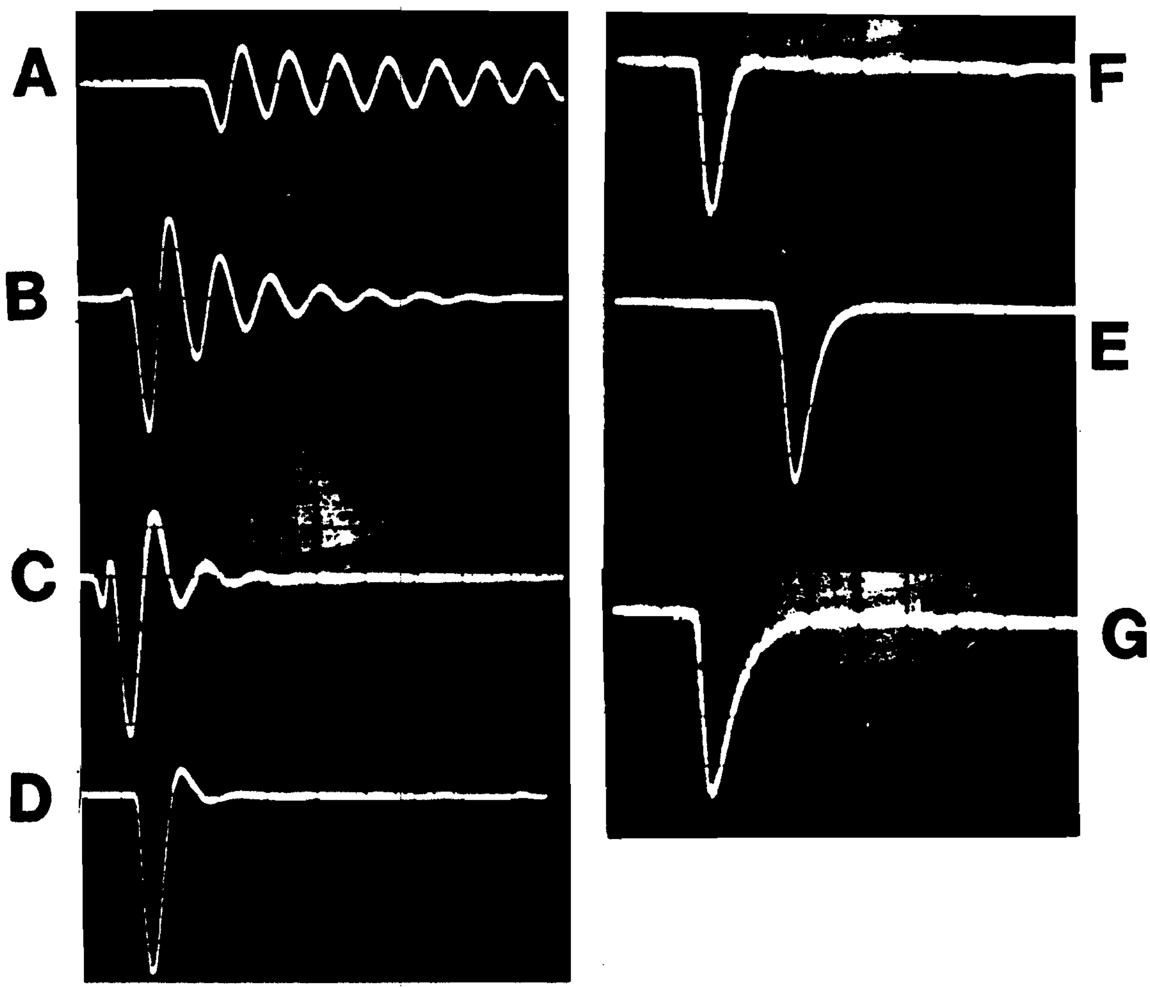

Figure 8 The output of a velocity sensor placed on the end of a vibrating, cantilever beam: (a) the damper moving in the residual field of the magnet, about 0.05 kG; (b) a damper magnetic field of 2.8 kG; (c) a damper field of 5.1 kG; (d) a damper field of 6.7 kG; (e) a damper field of 8.1 kG; (f) a damper field of 9.6 kG and (g) a damper field of 11.5 kG.

physics described by Equations (1) to (8) is valid up to several hundred megahertz. The primary effect of higher frequencies is to reduce the effective volume of conductor that is contributing to the damping. This can be overcome to some extent by using different conductor configurations. Basically, we see no problem in realizing damping up to many megahertz.

Another advantage of ED is that there is absolutely no hysteresis in either the amplitude or time behavior.

Varying the thickness of the conductor gives some degree of external control over the damper.

It should also be easy to couple the passive ED discussed in this paper with active control. For example, it is possible to embed current loops in (but insulated from) the conducting plate. Displacement or velocity sensors can be used in the conventional manner to feed current through these control loops to cancel unwanted motion. In fact, an inductive element attached to either the magnet or plate assembly can be used as the velocity sensor in this feedback loop because the time dependent fields that are produced external (or internal) to the conductor depend quadratically upon the plate velocity. These same current loops might also be used to extract small amounts of standby electrical power from the ambient mechanical noise. This standby power could be used to energize local field (velocity) sensors and thereby produce signals that could be used by the control system.

At low velocity, there can be very poor impedance matching in the sense that much more force is available for damping than is actually being used. When this is the case, ED will be improved by using a mechanical means of amplifying the displacement (velocity).