APPLICATIONS EXPERIENCE WITH THE FORMAT COMPUTER PROGRAM

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Experience in using FORMAT in the design of current commercial aircraft structures is summarized. Experience to date ranges from the gross problems of major aircraft components such as a wing-fuselage intersection, to highly localized problems such as stress distributions associated with propagation of fracture. Applications have been primarily in support of the design of the DC-10 and include basic analyses of internal forces and deflections, expansions of basic solutions to take advantage of structural symmetry, joining of complex substructures, and investigations of alternate designs by the method of subsequent modifications. An additional special case is the analysis of the pier extension of the La Guardia Airport runway.

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SECTION I

INTRODUCTION

FORMAT (Fortran Matrix Abstraction Technique) is a Fortran computer program system designed for direct and simple implementation of the extensive structural technology published in matrix notation. In its current form, the system consists of three phases: (I) automatic generation of matrices required for basic, joining, symmetry expansion, stability, and vibration analyses by the force and displacement methods, (II) matrix and pseudo-matrix operations in user-specified sequences, including the structure-cutter for automatic selection of redundants in the matrix force method, and (III) printing and plotting of results directly on report pages. For classic applications, the three phases are executed in order and the user specified sequence of matrix operations is a straightforward solution of the matrix equations of the force or displacement method. More general applications involve intricate utilization of the three phases including multiple steps in phase 2 to perform symmetry expansions and multiple joining of substructures.

The development of FORMAT is a continuing effort. The initial study was an investigation of feasibility. Fifteen existing matrix methods in five different technologies (structures, dynamics, aerodynamics, thermodynamics, and flight testing) were identified as suitable applications, and it was concluded that the fully developed FORMAT system would provide the means for effective integration of all the technologies fundamental to the development of aerospace structures (Reference 1). The second stage of development produced a basic linear analysis capability consisting of an operational matrix abstraction program (References 2 and 3) and two auxiliary user-coded subroutines for automatic generation of matrices required for basic analysis by the force and displacement methods (References 4 and 5, respectively). The third (and current) stage of development has provided a highly automated system for analysis of internal forces and deflections, critical loads and buckling modes, and resonant frequencies and vibration modes. The loading can be mechanical or thermal, and both force and displacement methods are fully implemented. The project report includes a complete user guide for the three-phase system (Reference 6), and program documentation for the additional capability (References 7 through 9).

References 2 and 6 include example applications to demonstrate the validity of the system and its use, however, the example applications are necessarily restricted in size and scope. This paper describes some of the major applications of FORMAT during its first year of production use by the McDonnell Douglas Corporation. In general, the structures were idealized by lumped parameters (bars and shear panels) and were solved by the force method. The computer program was essentially as described in References 3 and 6 through 9, and both the IBM 7094 and GE 635 electronic data processing machines were used. For completeness, a brief summary of the FORMAT system precedes the description of the applications.

SECTION II

THE FORMAT SYSTEM

The FORMAT system consists of three independent digital computer programs as follows:

Phase I - Case and Matrix Generation
Phase II - Matrix Abstraction
Phase III - Special Output

Figure 1 shows the data flow in the FORMAT system for conventional applications. Tabulations of joint coordinates, load/reaction vectors, and elements (e.g., bars) of the idealized structure are the input data for Phase I. The Phase I program (a) merges these data onto a case data set with various editing options, and (b) generates the corresponding matrices required for a matrix solution to the specified problem, e.g., calculation of stresses and deflections, and vibration modes and frequencies by the matrix displacement method. The matrices generated in Phase I are saved on matrix data sets (e.g., magnetic tapes) for immediate or subsequent use in Phase II.

Using the matrix data from Phase I, the Phase II program executes the matrix abstraction sequence specified by the user in terms of built-in matrix and pseudo-matrix operations such as matrix addition, subtraction and multiplication, extraction of eigenvalues and eigenvectors, the structure-cutter, and saving of specified matrices on matrix data sets.

The Phase III program uses the matrices computed and saved in Phase II, and the corresponding case data from Phase I. Computed results are printed in tabular form with self-explanatory labels for direct use in reports. Two types of pictorials are made: structural geometry from the case data set, and graphical display of values of matrix elements.

It is important that the three programs are independent and can be operated individually or in any logical sequence, for example, Phase II might be used independently to solve a matrix problem involving data generated externally, or Phase III might be used to plot structural geometry from an edited case data set prior to generation of matrices.

PHASE I

The Phase I program provides for generation of case and matrix data sets. A case data set is a file of structural input tables stored in order by case on a logical data set (e.g., magnetic tape). A case of structural input tables includes joint coordinates, load/reaction vectors, and structural elements. The basic activity in case data set generation is to edit existing case data sets by deleting a case, adding a case or replacing a case. A new case can be specified on punched cards or generated wholly, or partially, internally. Three techniques are used for internal generation: (1) duplication of data entries or tables on existing case data sets, (2) automatic generation of joint coordinates for a regular gridwork on several standard surfaces (line, triangle, warped quadrilateral, etc.), and (3) symmetric transformation of an existing case to provide a mirror image case.

A matrix data set is a file of matrix data stored in order by matrix on a logical data set (e.g., magnetic tape). The primary function of the Phase I program is to generate matrices as specified by the user from tables on a generated case data set. Matrices are generated for analysis by the force or displacement method, or for simulation of a continuous load distribution by a system of discrete loads. The force and displacement generators both provide for thermal and mechanical loading, and generate matrices required for (1) basic static analysis, (2) joining of complex substructures, (3) systematic disconnecting of symmetric and antisymmetric reactions to take advantage of structural symmetry, and (4) vibration and stability analyses.

The basic force method generator is based on the Redundant Force Method (Reference 10). It provides for idealizations consisting of general bars (axial load, torsion, and bending in the two principal planes), panels that carry shear only, and triangular and parallelogram membrane elements. The basic generator produces matrices describing the equilibrium of the structure, the element flexibilities, and various auxiliary transformation matrices. As a part of the Redundant Force Method, these matrices are used in Phase II in three basic steps: (1) "solution" of the equilibrium equations using the structure-cutter, (2) formation and solution of the continuity equations to determine the redundants, and (3) computation of internal forces and deflections. For other cases, matrices are generated by procedures described in References 11 through 13.

The basic displacement method generator is based on the Direct Stiffness Method (Reference 14). It provides for general bars, symmetric trapezoidal panels that carry shear only, triangular and rectangular membrane elements, and triangular and quadrilateral plate bending elements. The basic generator produces a singular stiffness matrix, the boundary condition transformation to reduce it to a nonsingular form, and various auxiliary transformation matrices. As a part of the Direct Stiffness Method, these matrices are used in Phase II in two basic steps: (1) reduction of the stiffness matrix to a nonsingular form and solution to determine displacements, and (2) computation of stresses and deflections. For other cases, matrices are generated by procedures described in References 15 and 16.

PHASE II

The Phase II program is an expansion of the basic matrix abstraction capability described in References 2 and 3. The addition is a capability to extract eigenvalues and eigenvectors from a real matrix of order less than 2000. The current operational capability is summarized in Table 1. Phase II executes in two sequential steps: (1) input data is organized and checked, and an executable program is planned for the second step, and (2) the specified operations are performed. The salient features of this program are as follows.

- The user controls the calculations via simple matrix abstraction statements without concern for internal storage allocations. Internal storage allocations are automatically generated in step 1 for optimum utilization of available data storage devices.

- Matrices can be large, up to 2000 order on an IBM 7094 with 32K core storage unit. (Exception: EIGEN is core limited). The program is easily modified to take advantage of machines with larger working storage.

TABLE I

MATRIX OPERATIONS AVAILABLE IN FORMAT - PHASE II

Code	Operation
PRINT	Print matrices
SAVE	Save matrices
IF	Conditional transfer
ADD	Matrix add
SUBT	Matrix subtract
MULT	Matrix multiply
TMULT	Matrix transpose-multiply
SMULT	Scalar-matrix multiply
EMULT	Element-by-element multiply
TRANSP	Matrix transpose
POWER	Matrix elements raised to constant power
INVERS	Matrix inversion
SEQEL	Solution of equations-elimination
SEQIT	Solution of equations-iteration
STRCUT	Structure-cutter
ADJOIN	Matrix adjoin
EIGEN	Eigenvalue/eigenvector extraction-real symmetric matrix (core limited)
EIGEN2	Eigenvalue/eigenvector extraction-real matrix
ENVROW	Maximum and minimum elements in rows of a matrix
ENVCOL	Maximum and minimum elements in columns of a matrix
DIAGON	Diagonalize a column matrix
RENAME	Rename a matrix
USERXX	Execute the operation in the user-coded subroutine XX

- Up to nine user-coded subroutines can be loaded with the data and executed in line with built-in operational capability.

- Data is stored in compressed mode (i.e., with zero values deleted) when the relative density of nonzero values is less than 50 percent on an individual column basis. As practical, it is processed directly in compressed mode.

PHASE III

The Phase III program provides for special display of data in tables and figures. The report form printing significantly improves usability of the results. The data is printed in tables with self-explanatory headers and identification data taken directly from the Phase I input data tables. It is significantly easier to use than the utility matrix printing because an auxiliary coding of correspondence is not required. Two separate printing modules are provided to accommodate the differences in the force and displacement methods. Tables are printed on standard 11 x 8-1/2 paper with suitable margins and sequenced page numbers. Standard page header data may be specified by the user.

The graphic displays employ the standard software available for the Stromberg-Carlson 4020 cathode ray tube (CRT) device. The geometry of a structure on a case data set can be displayed in a series of full views or traces. The figures are formed by tracing the boundaries of each structural element listed in the case data. Load and reaction vectors may be included. This capability is useful in checking for errors in coordinates of joints and definition of element boundaries.

A utility matrix plotting capability is also included. Graphs are made with matrix element values (a_{ij}) plotted as ordinates versus row number (i) for specified ranges of row number and specified columns (j) in one or more matrices. Any available matrix data stored on a matrix data set can be plotted. Scales are selected automatically so that the data fills the CRT screen, and labels for the coordinate axes may be specified by the user.

SECTION III

APPLICATIONS

FORMAT has been used extensively as a production tool by the Douglas Aircraft Division since FORMAT II was made operational in early 1967. The applications include the Douglas commercial and military product lines, and various research and development activities. This section summarizes some of these applications, primarily component development for the DC-10.

The DC-10 is a three-engine jet transport designed for 250 to 350 passengers on ranges up to transcontinental. The general structural arrangement is shown in Figure 2. The structural design is particularly complex in the wing-fuselage intersection and the aft fuselage/empennage/pylon zone, and therefore these structures were analyzed in significant detail using FORMAT during preliminary design. Virtually the entire aircraft will be analyzed in detail during design.

To clarify the terms "preliminary design" and "design" a representative design schedule summary is shown in Figure 3. The basic project is bracketed by the milestones "decision to build" and "type certification." The DC-10 project is approximately 50 percent through "design" and the analyses reported herein were performed prior to decision to build (preliminary design) or as part of the design. Additional detail will be incorporated during substantiation and redesign.

AFT FUSELAGE/EMPENNAGE/PYLON

The idealization for preliminary design of the aft fuselage/empennage/pylon structure is shown in Figure 4. The structure is symmetric and therefore only one-half of the structure was modelled. The root of the vertical stabilizer is complicated by the engine inlet duct and the loads from the pylon for the rear engine. The aft fuselage shell is complicated by cutouts for a passenger door, access to an auxiliary power unit, and the moving center section of the horizontal stabilizer.

During preliminary design, three versions of this structure were analyzed.

1. A bifurcated duct with three spars.
2. A straight-through duct with three spars (shown in Figure 4).
3. A straight-through duct with four spars.

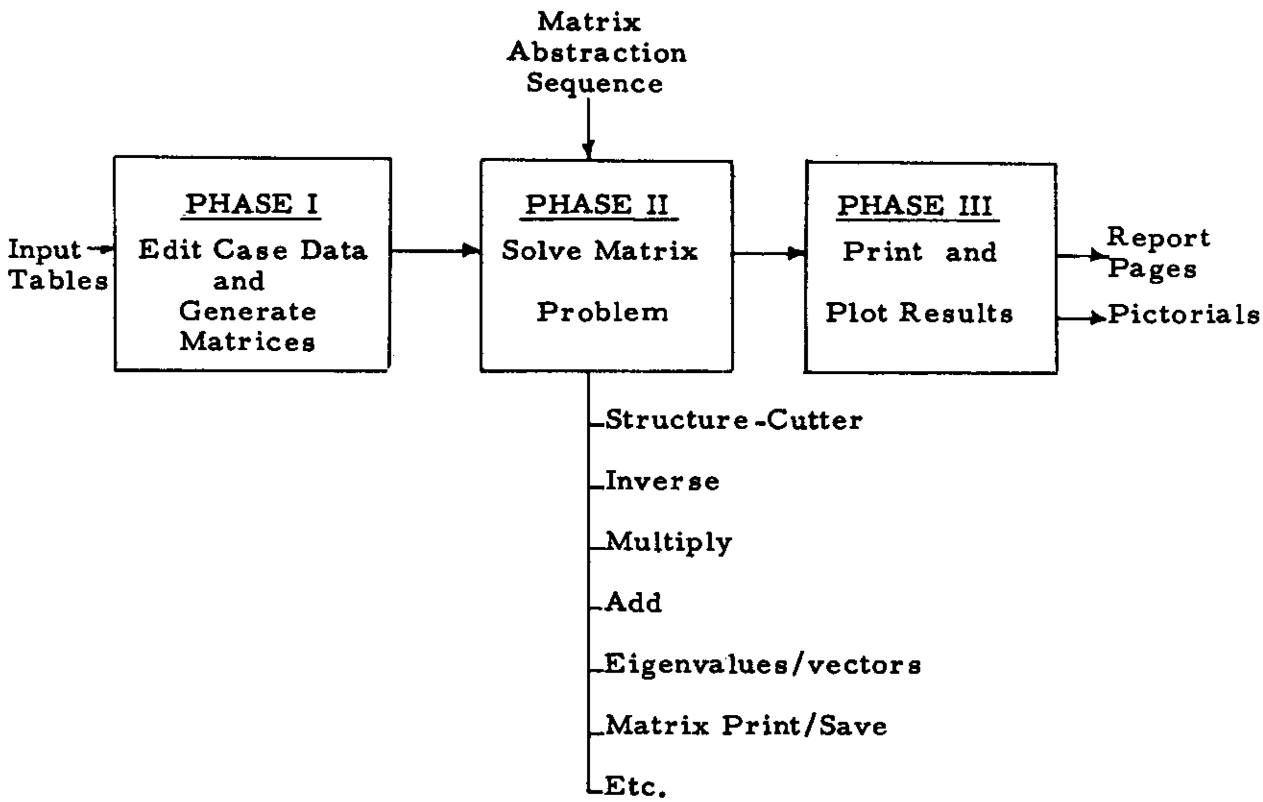


Figure 1. Operation of the FORMAT System

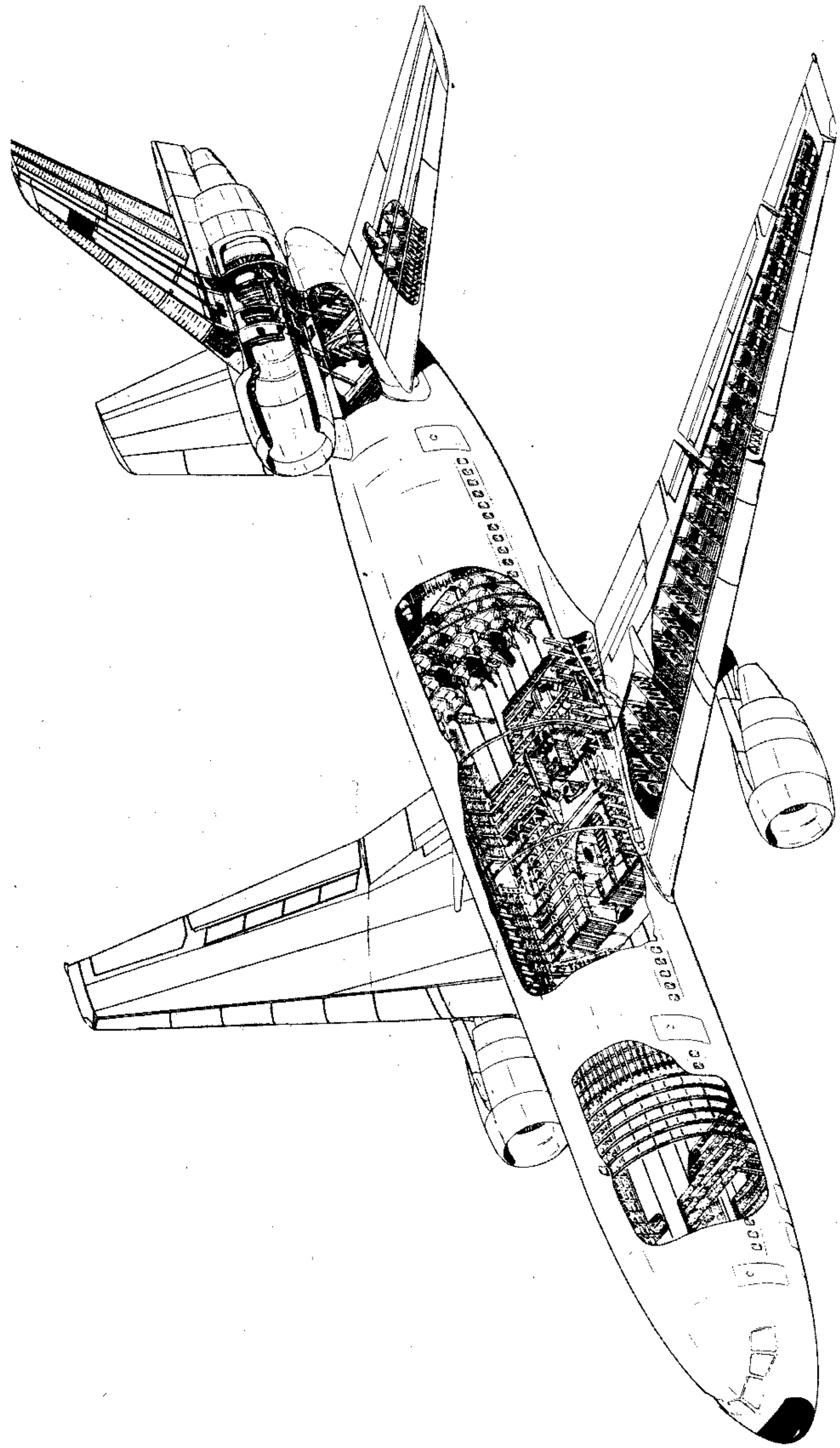


Figure 2. DC-10 Structural Arrangement

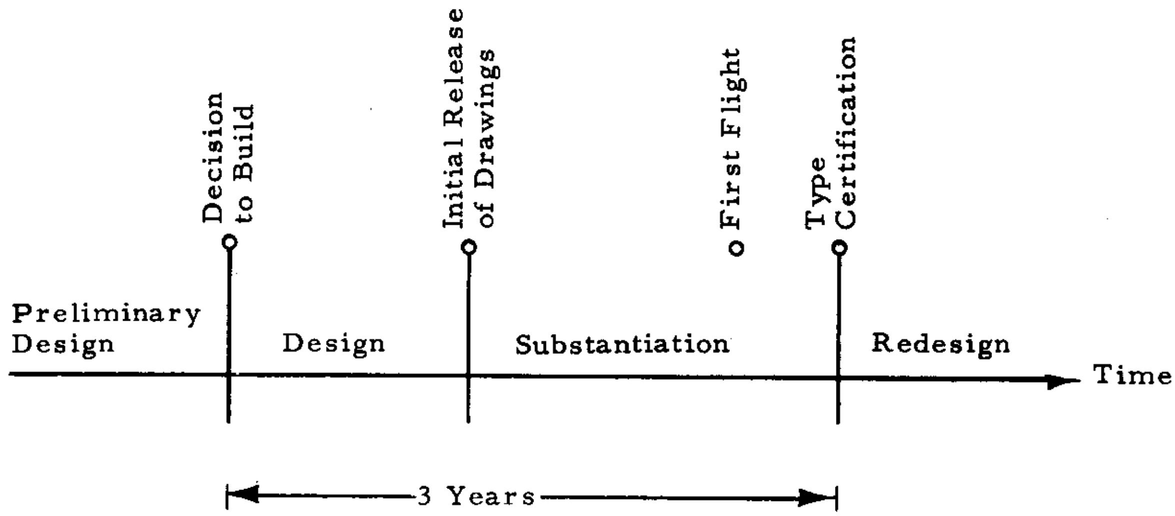


Figure 3. Design Schedule

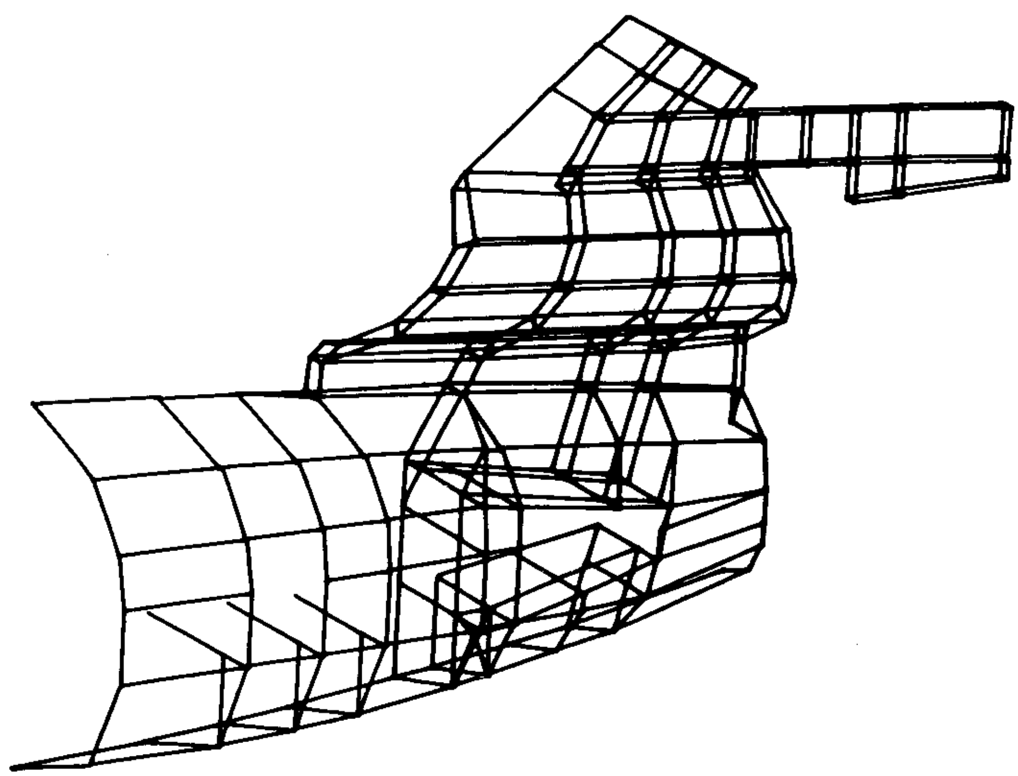


Figure 4. Aft Fuselage/Empennage/Pylon Idealization - Preliminary

The results from version 1 and 2 were used in a major design decision: (1) straight spars for the vertical stabilizer with inlet air for the rear engine ducted around the primary stabilizer root structure, or (2) a straight-through duct with ring shaped frames in the stabilizer spars to carry the loads around the duct. On the basis of overall considerations, including structural weight and engine performance, version 2 was selected. The results from versions 2 and 3 were used in the second major design decision: (1) three spars with simpler construction, or (2) four spars with improved static and dynamic fail-safe behavior. Version 3 was selected.

The three versions were analyzed as three separate FORMAT problems of nominal size (1400 unknown internal forces). The method of subsequent modifications was used to modify the results for solutions for versions 2 and 3 to determine internal forces and deflections for conditions of failed spars, individually, in turn. These modified solutions faithfully simulate the conditions of failed members and were the basis of the selection of version 3.

The idealization for design is significantly more detailed (20,500 unknown internal forces). This idealization consists of 16 substructures (Figure 5), including seven which are symmetric about the buttock plane. Joining of the substructures is a cascade process: (1) substructures 1 through 8 (SS1, SS2, . . . , SS8) are joined to form substructure A (SSA), (2) substructures 9 through 16 are joined to form substructure B (SSB), and (3) substructures A and B are joined to provide a unit solution and real load solutions for the total structure. The complete process of analysis of substructures and symmetric halves (e.g., 1/2 - SS1), and subsequent joining is shown in Figure 6. Horizontal position in the chart is an index of elapsed time, e.g., the analyses of substructures 1 through 8 were done as an overlapping series of tasks. It is important that the entire process is a time-phased plan. Each step provides data that is used directly in design, and data for later use such as joining.

WING-FUSELAGE INTERSECTION

The preliminary design idealization of the wing-fuselage intersection is shown in Figure 7. The detail structural arrangement includes a special extended rib structure at the root to distribute the rear spar loads over multiple fuselage frames, large fuselage cut-outs for storage of the main gear, and flat pressure panels to isolate the wing center section and wheel wells from cabin pressurization.

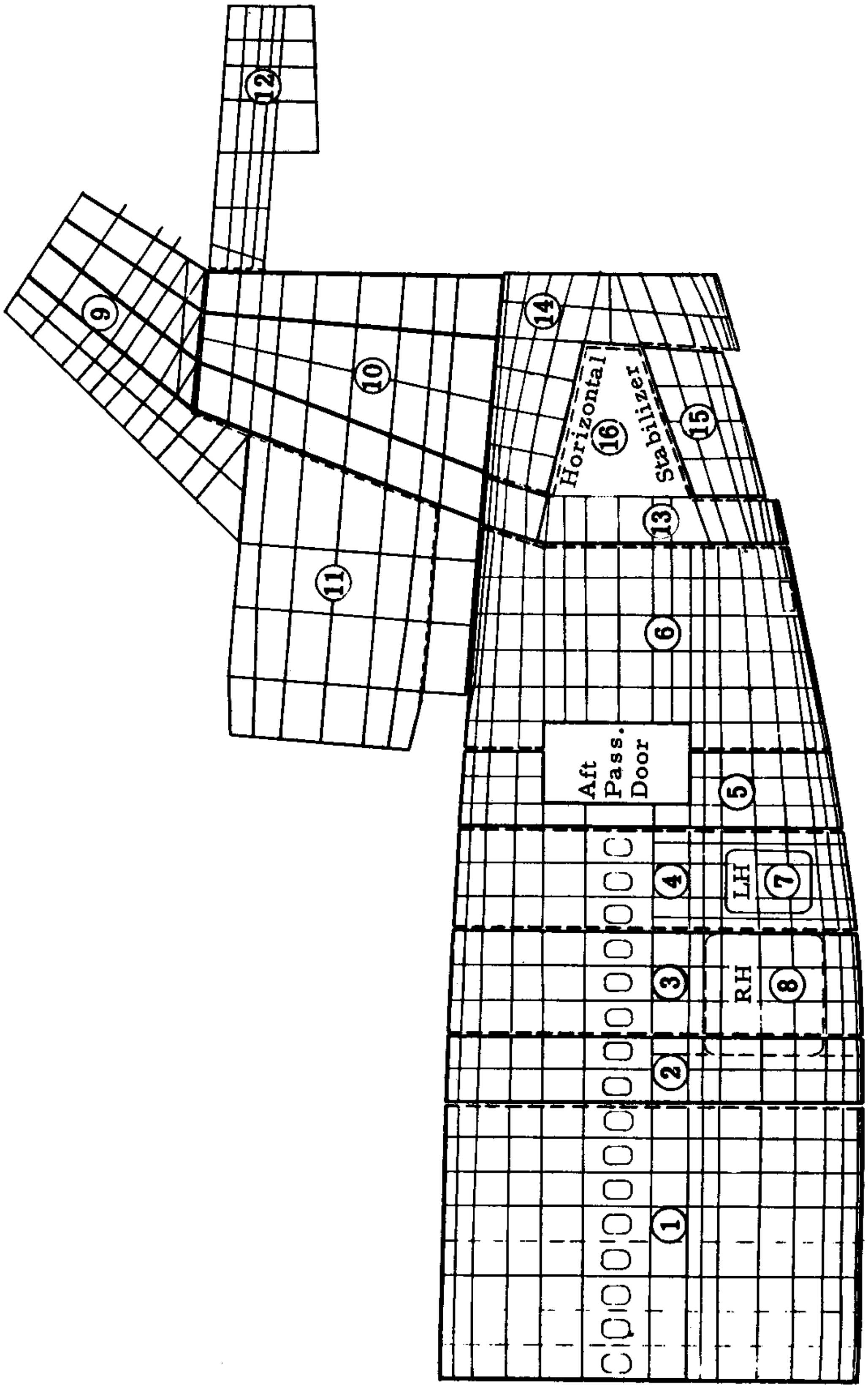


Figure 5. Aft Fuselage/Empennage/Pylon Idealization — Design

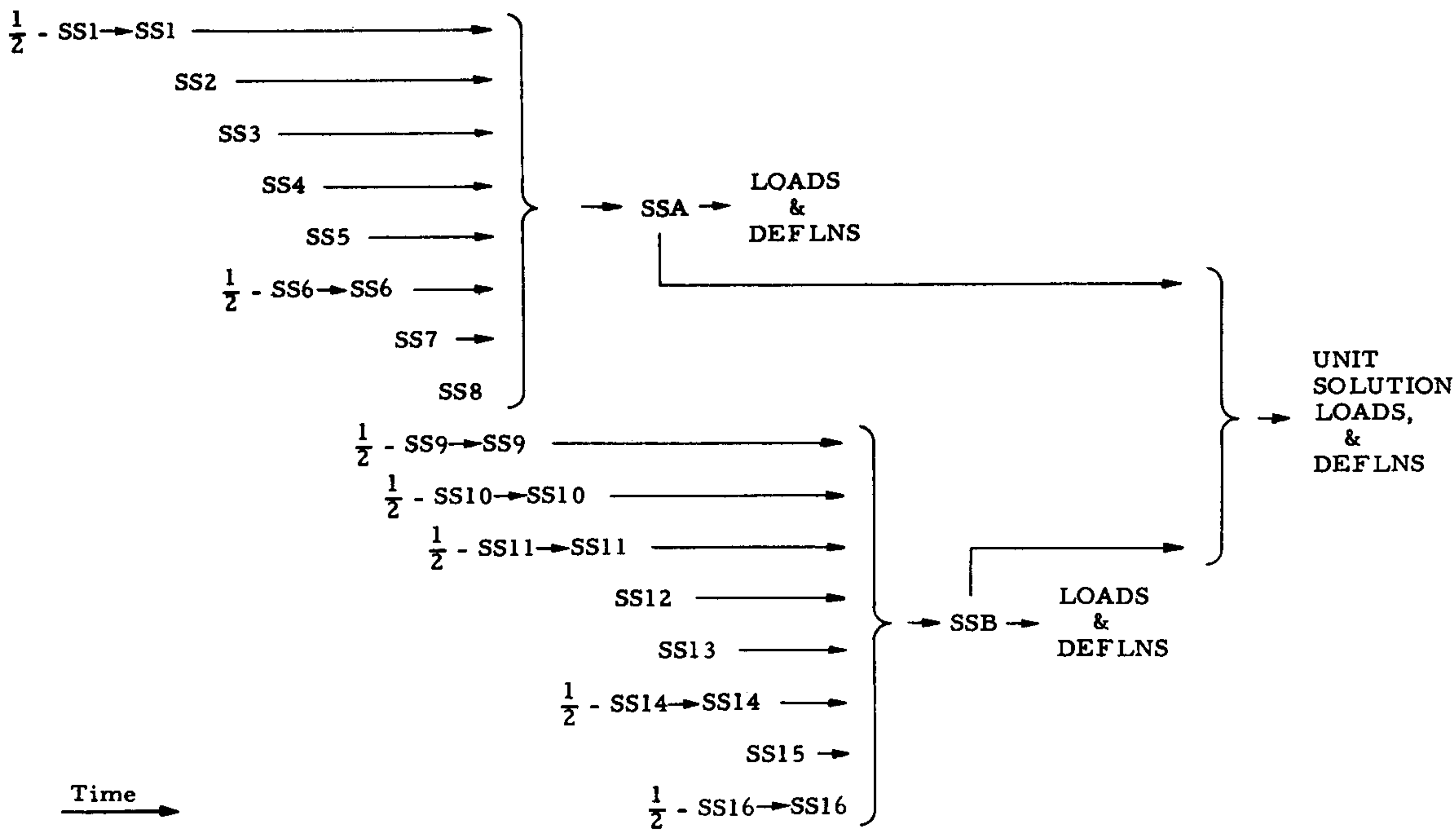


Figure 6. Aft Fuselage/Empennage/Pylon - Design Analysis Flow Chart

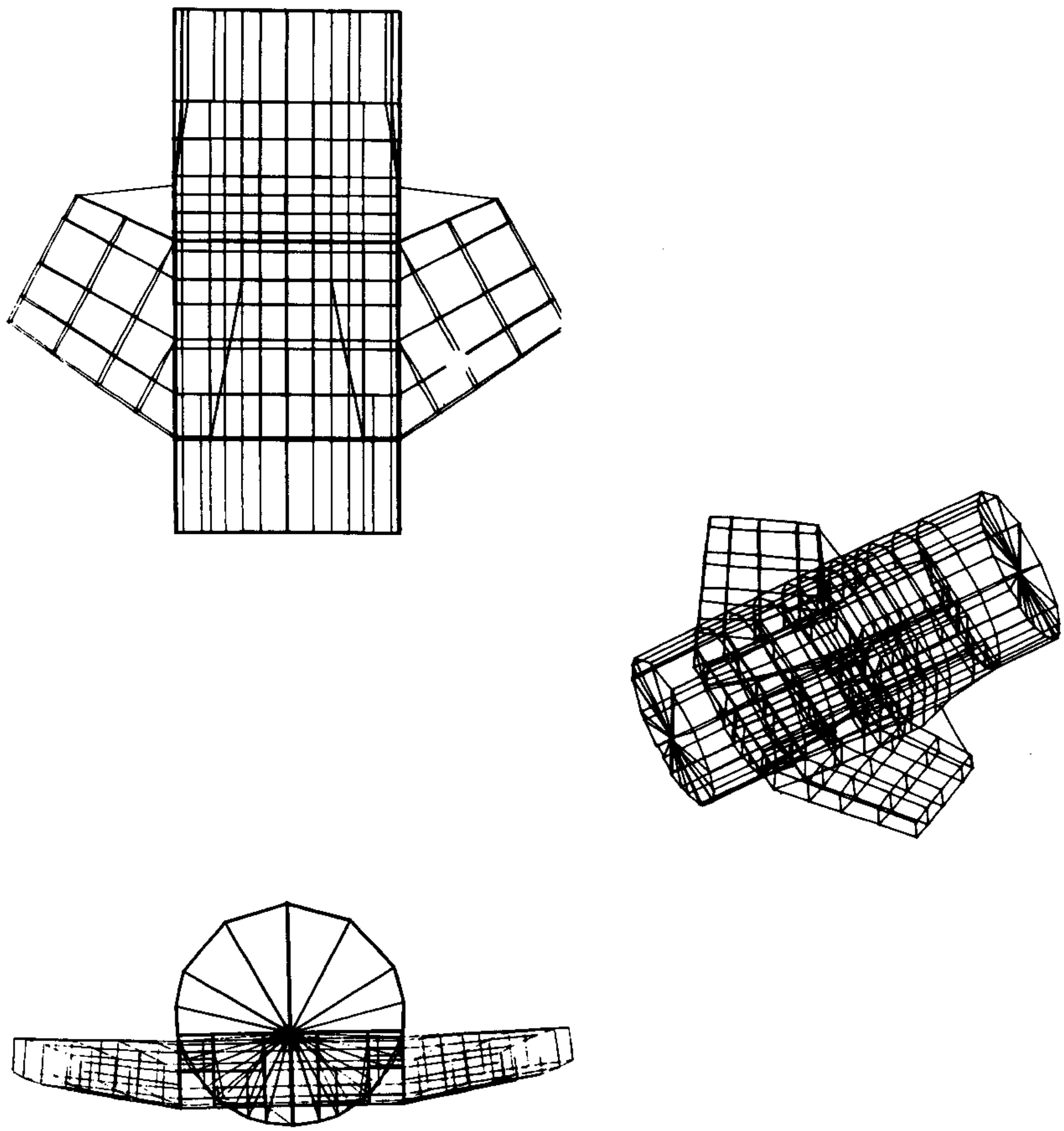


Figure 7. Wing - Fuselage Intersection Idealization — Preliminary

A symmetric half of the model was analyzed as a single structure of capacity size (approximately 2000 unknown internal forces). This solution was expanded for symmetry and 26 variations of the expanded model were analyzed using the method of subsequent modifications. The radial bars at the ends of the fuselage section are part of a pair of "rigid" end fixtures included to simplify end loading and to prevent warping of the cross-sections.

One of the design decisions was to determine the frame-wing flexible attachment concept for the frames in the region of the rear spar (Figure 8). Many concepts were evaluated, including: (1) fully continuous frame with reduced depth at the attachment to the sweep break bulkhead, (2) one "pin connection" above the floor (Alternate A), and (3) two "pin connections" below the floor (Alternate B). On the basis of extensive studies of relative stiffness of related components and consideration of structural weight, producibility and passenger comfort, the fully continuous frame concept was selected. A more detailed model is being analyzed for the design cycle.

FORWARD FUSELAGE

The forward fuselage idealization for design is shown in Figure 9. The central figure is a manually prepared composite side view of the nine substructures and the nine satellite figures are computer generated views of the individual substructures.

The major complications in this structure are the irregularities of the flight compartment structure including the windshield, the nose gear support structure, and the nine gross cut-outs for the various doors. The passenger windows are simulated by equivalent structure. A stress summary for this structure for one load condition is shown in Figure 10. Direct stresses are plotted in the upper set of diagrams. The deviation from simple bending (My/I) is significant. Numerical values on the idealization sketch are panel shear flows (lb/in.) and lumped parameter bar forces (kips). Seven of the nine substructures are symmetric. Solutions for these seven structures were expanded and then included in a single joining step to form the complete solution. The total number of unknown forces is 15916.

The preliminary design idealization of this structure was a two-dimensional "flat plate" model representing the projection of the real structure on the buttock plane. This "deep beam" approach gave good preliminary estimates of side wall shear flows and also served to check the design model data.

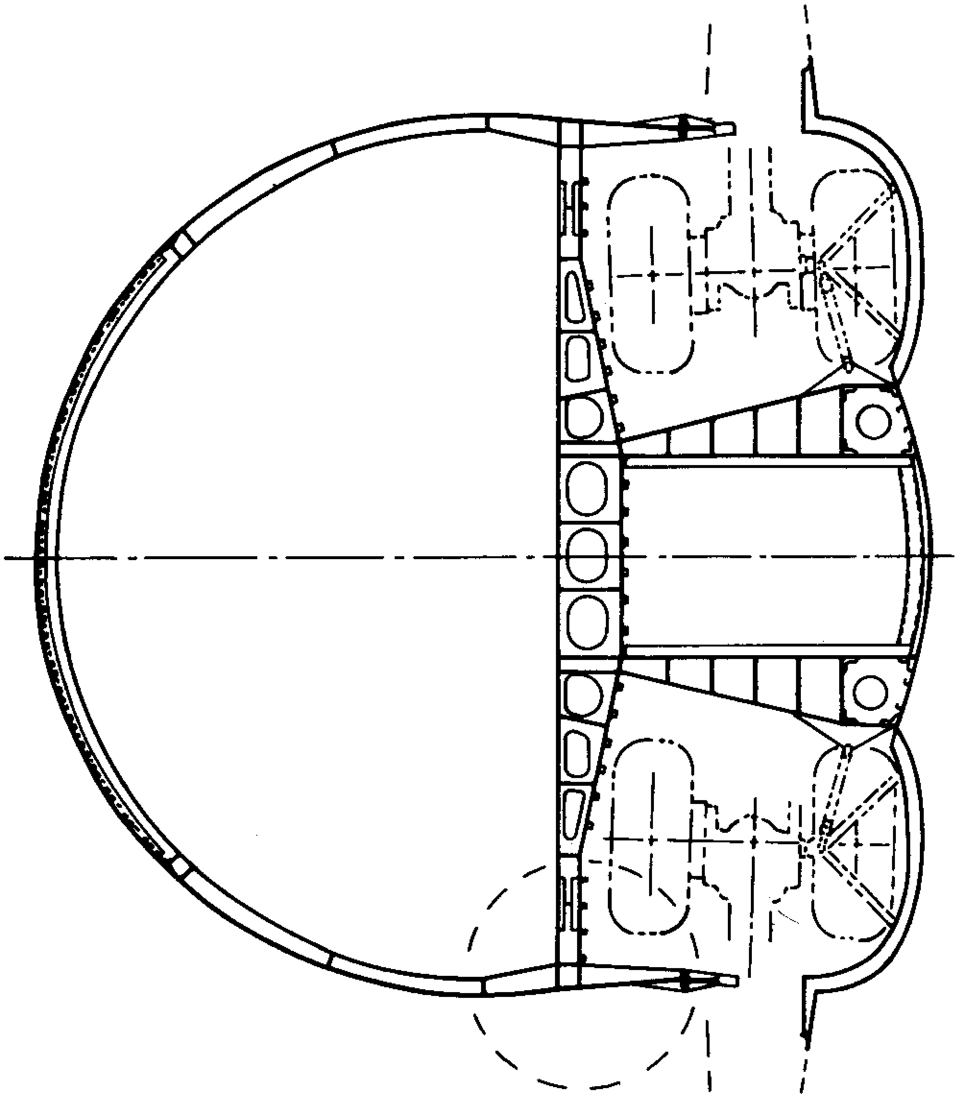
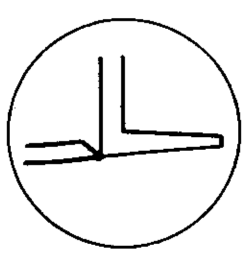
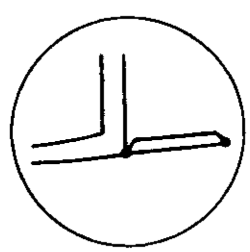


Figure 8. Wing-Fuselage Intersection; Frame-Wing Attachment



Alternate A



Alternate B

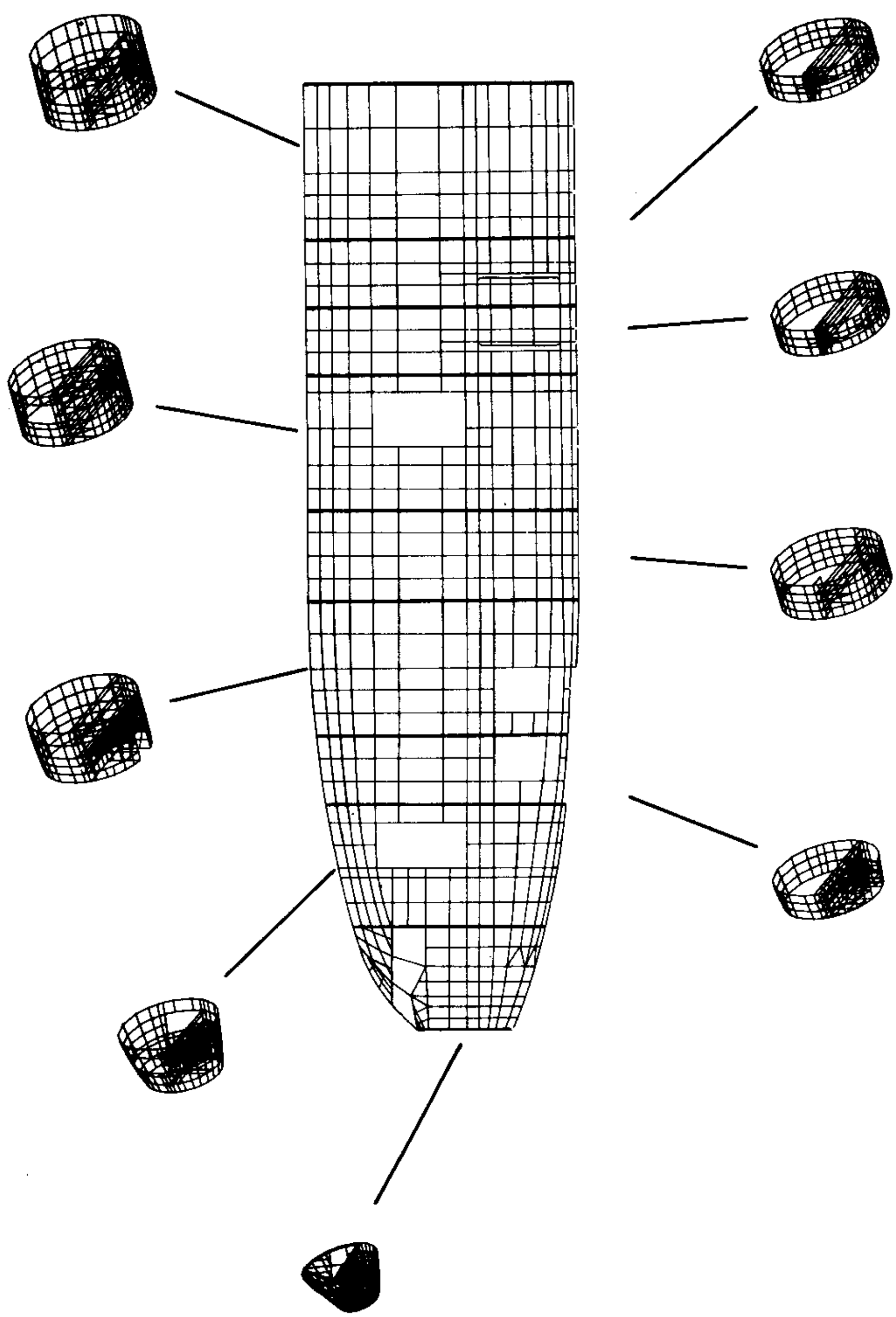


Figure 9. Forward Fuselage Idealization — Design

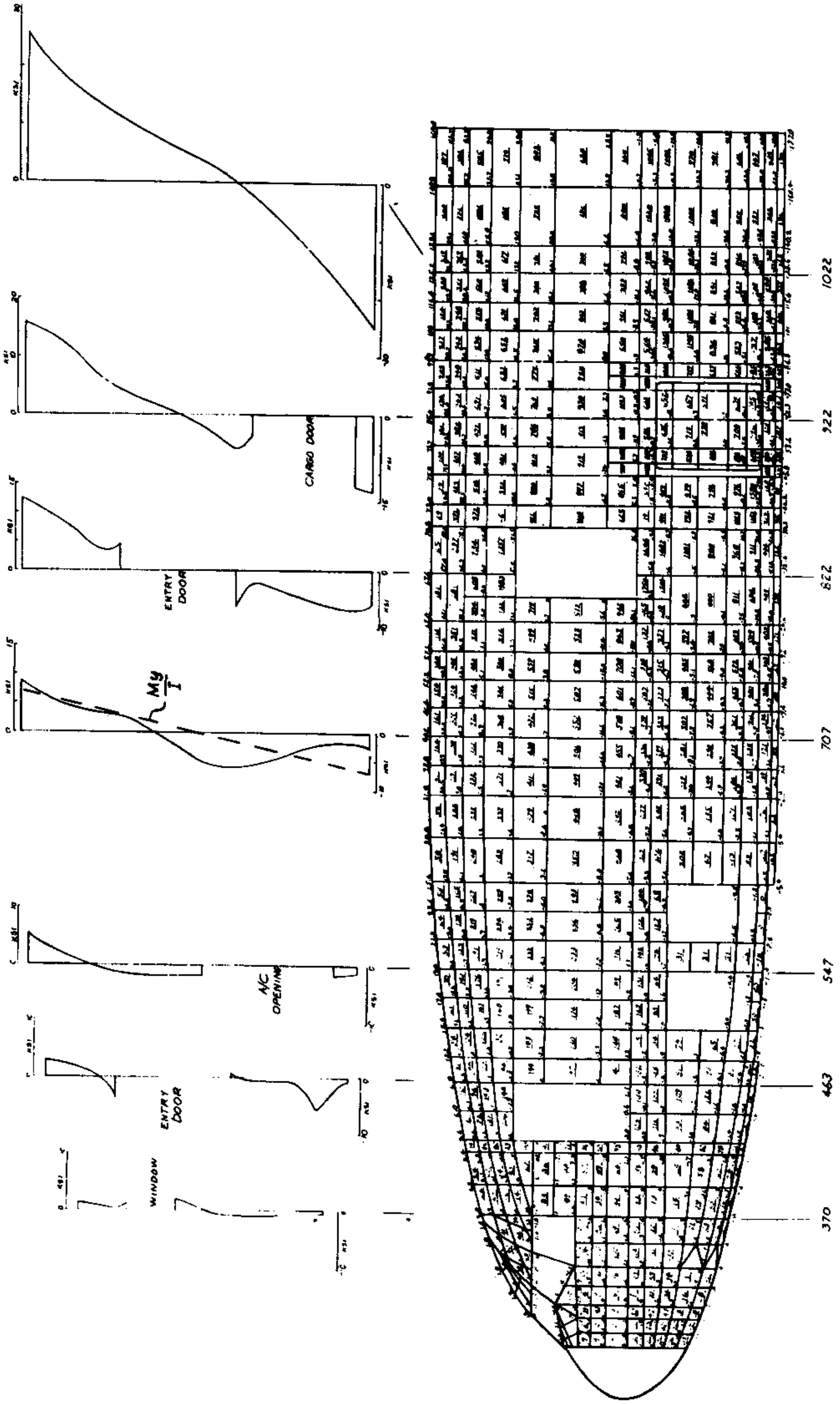


Figure 10. Forward Fuselage — Stress Summary

OTHER DC-10 COMPONENTS

Numerous smaller scale structural components of the DC-10 have been analyzed using FORMAT. Two examples are shown in Figures 11 and 12. Figure 11 is a computer generated view of an assembly of four substructures. It is a view looking down and forward at a segment of the main wing box and the primary flap structure. The flap is shown in the 50-degree down position. The simpler beam-like segment represents the flap. It is supported by two remote hinges (interior) and by rollers on radiused tracks at the ends. The flap is idealized as a separate substructure to facilitate multiple position joining simulating representative deflected positions. The analysis also includes a special joining technique to represent conditions of single failure in the flap positioning mechanism.

Figure 12 is an idealization of a fitting with an I-shaped cross-section and discrete angle changes. The "pyramid" at the upper end simulates a solid loading point. This relatively simple example represents a completely different class of profitable application. Other examples include local behavior of fuselage frames, wing bulkheads, small cut-outs, and landing gear structure assemblies.

NOTCHED PLATE

The notched plate shown in Figure 13 has been analyzed as part of a continuing program to improve the basic technology for fail-safe design. The idealization of the plate was a three-dimensional gridwork of bars with included shear panels. The gridwork was dense in the region of the notch to accommodate the severe stress gradients. Only one-eighth of the plate was modelled (accounting for symmetry) and bar flexibilities were coupled to account for Poisson's ratio.

Test data were generated for comparison (Figures 14 and 15). The strain gage installation was intricate. Gages were installed on the surface along the transverse (x-axis) centerline and along the base of the notch as indicated by the test points in the figures. Gages mounted in the base of notch were 1/64-inch gages and were mounted as close as 0.025 inch center-to-center. The test-theory comparison for the variation of y-direction strains (ϵ_y) along the transverse centerline is very good (Figure 14), and the comparison through the thickness is good (Figure 15).

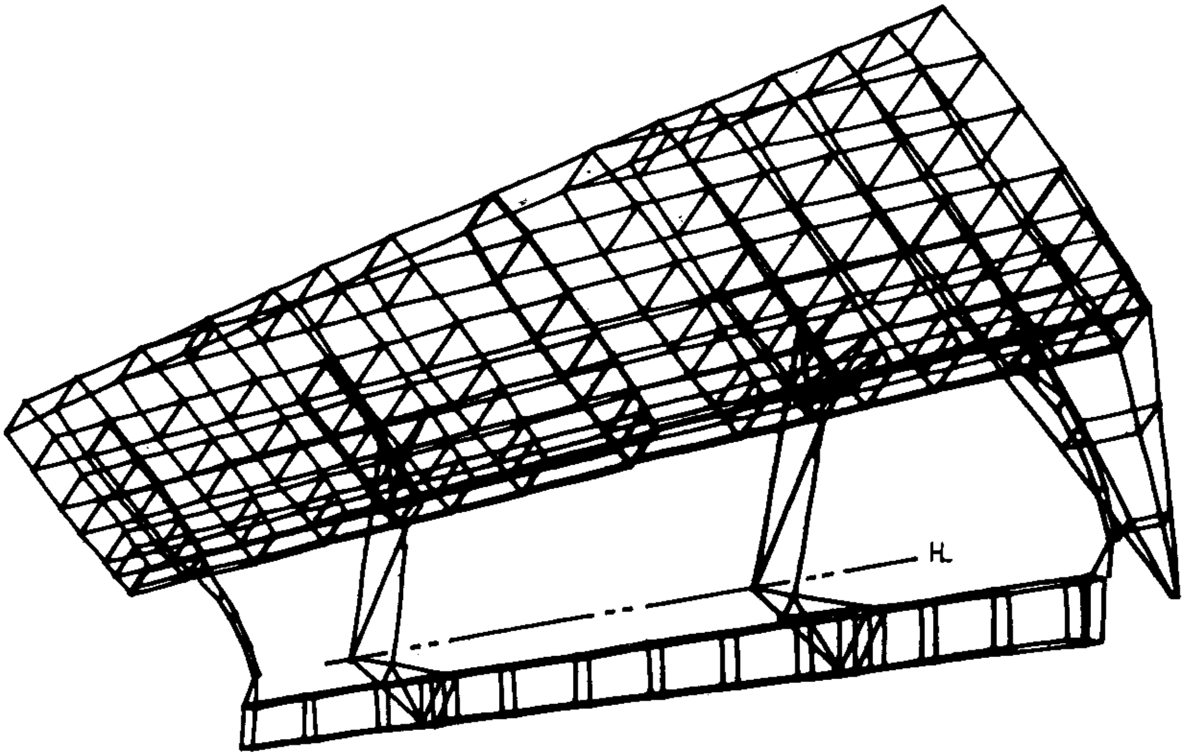


Figure 11. Wing/Outboard Flap Idealization - Design

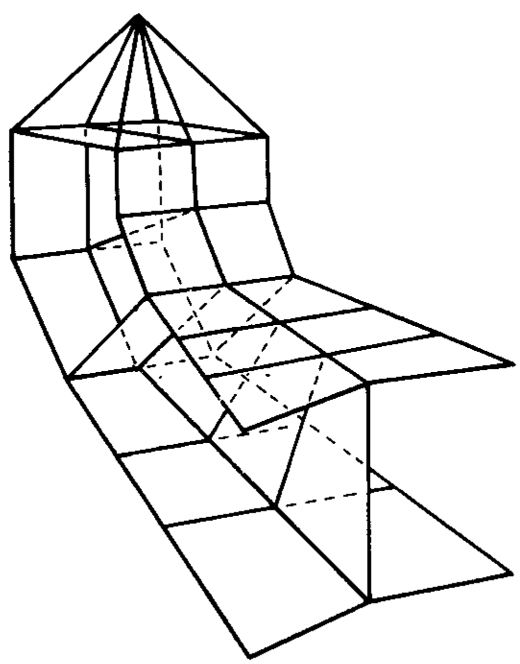


Figure 12. Main Gear Trunnion Fitting Idealization - Design

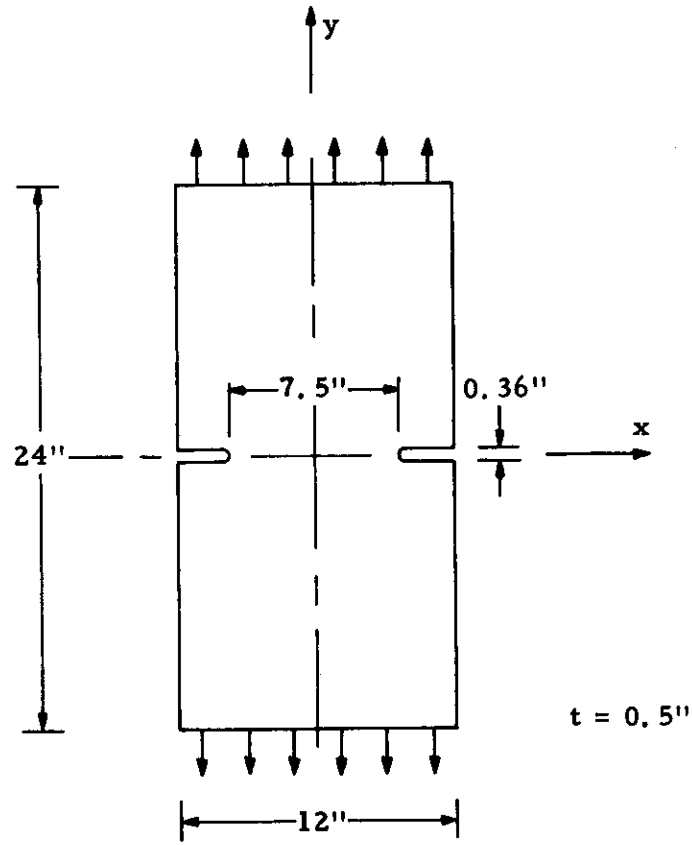


Figure 13. Notched Plate

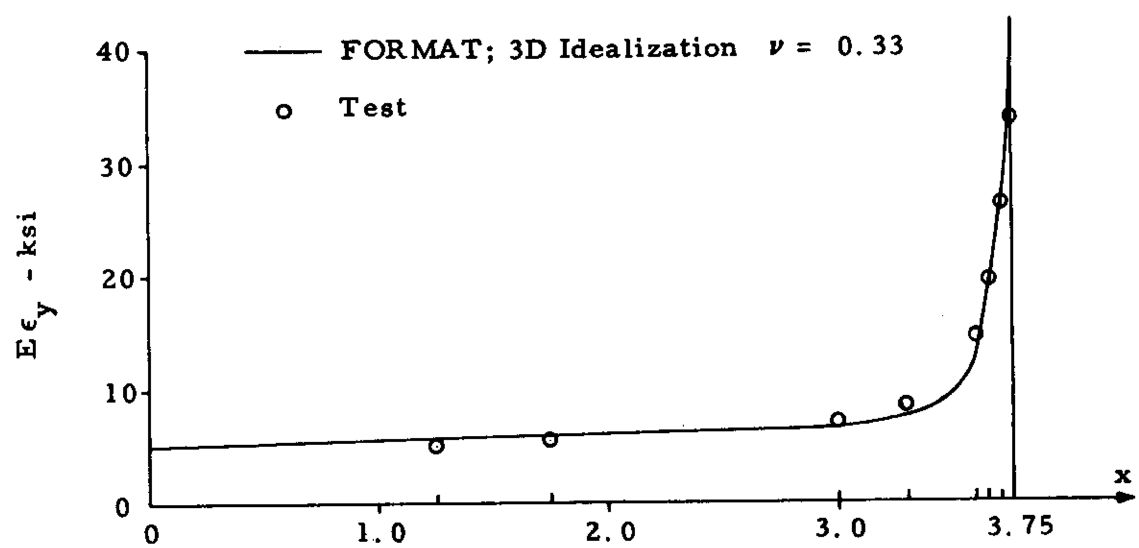


Figure 14. Test-Theory Strain Comparison on Surface Along Transverse Centerline

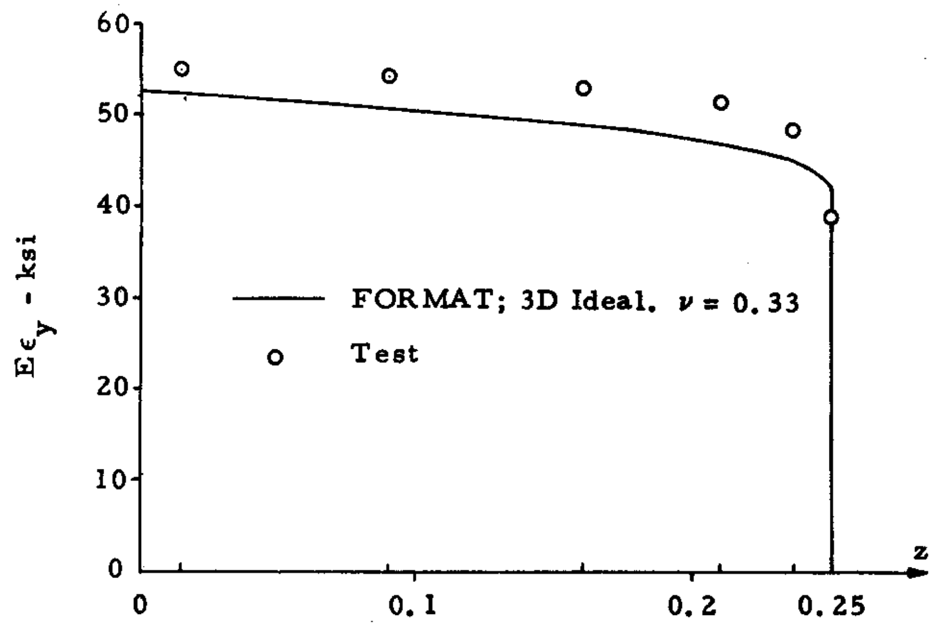


Figure 15. Test-Theory Strain Comparison Through Thickness at Base of Notch

This elastic solution is the basis for an analysis accounting for plasticity that also will be compared to the test data. It is hoped that the comparison will establish a firm foundation for analyzing the growth of a crack in a thick plate accounting for plasticity. Such a solution would be very valuable to the understanding of crack propagation and fail-safe design.

LAGUARDIA PIER RUNWAY EXTENSION

The runway of the LaGuardia Airport has been extended by construction of a pier. The strength of the pier is an important factor in the design of the landing gear of the DC-10, and accordingly, static and dynamic analyses have been performed by the McDonnell Douglas Corporation to gain an understanding of the behavior of the pier.

One structural unit of the runway extension is shown in Figure 16. The large prestressed concrete slab is supported by seven reinforced concrete girders, each of which is in turn supported by 17 reinforced concrete piles. Two general static analyses were performed to determine stresses and loads in the slab and supporting structure: (1) an "interior analysis" for the case of gear loads and pile settlement three or more bays lengthwise from an end

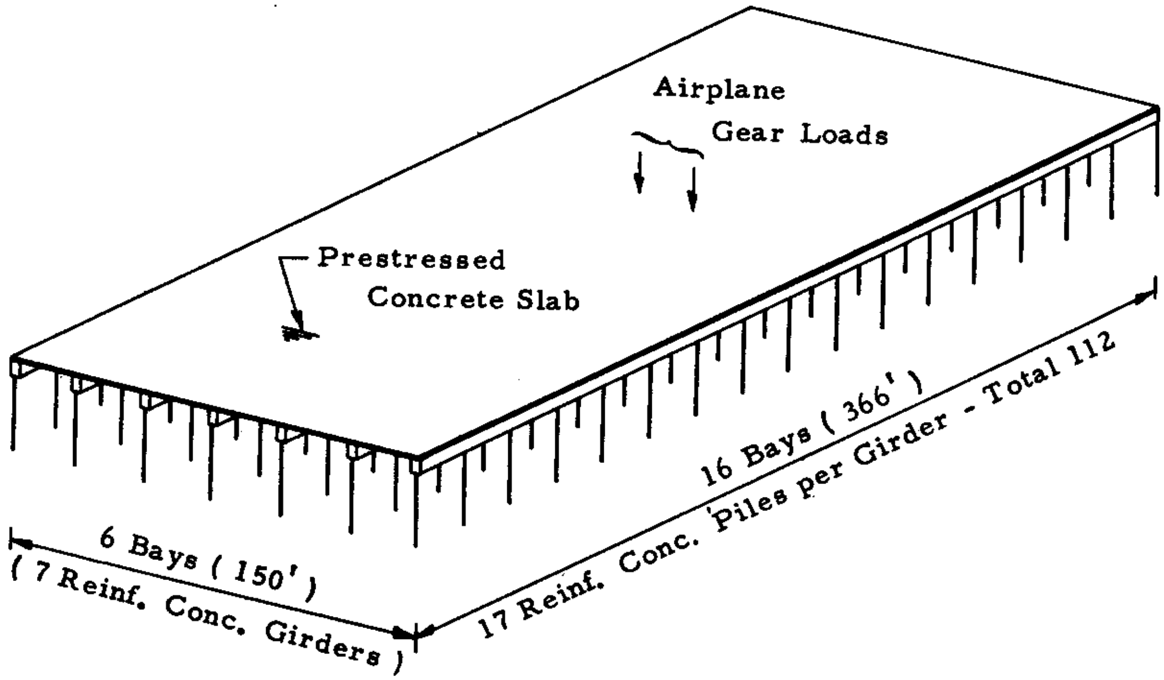


Figure 16. La Guardia Runway Pier Extension

(150 ft. edge), and (2) an "end analysis" which accounts for the short bay length and edge beam at each end. The model for the interior analysis is shown in Figure 17. The analysis approach was as follows.

1. The slab was treated as a substructure simply supported on four edges and subjected to force vectors corresponding to the airplane gear loads and the indicated 275 discrete joining vectors. Influence coefficients for forces and deflections were computed by evaluating the classic solution for plate bending.

2. A typical supporting structure unit consisting of a girder supported by eight piles was analyzed as a frame subjected to unit forces corresponding to the joining vectors (55), and unit support displacements. The solution to this typical unit was then expanded by direct duplication to form the solution for the second substructure.

3. The influence coefficients for the two substructures were used in a force method joining analysis to determine stresses in the slab, girder and piles for various conditions of airplane gear loading and pile settlement. Representative results are shown in Figure 18.

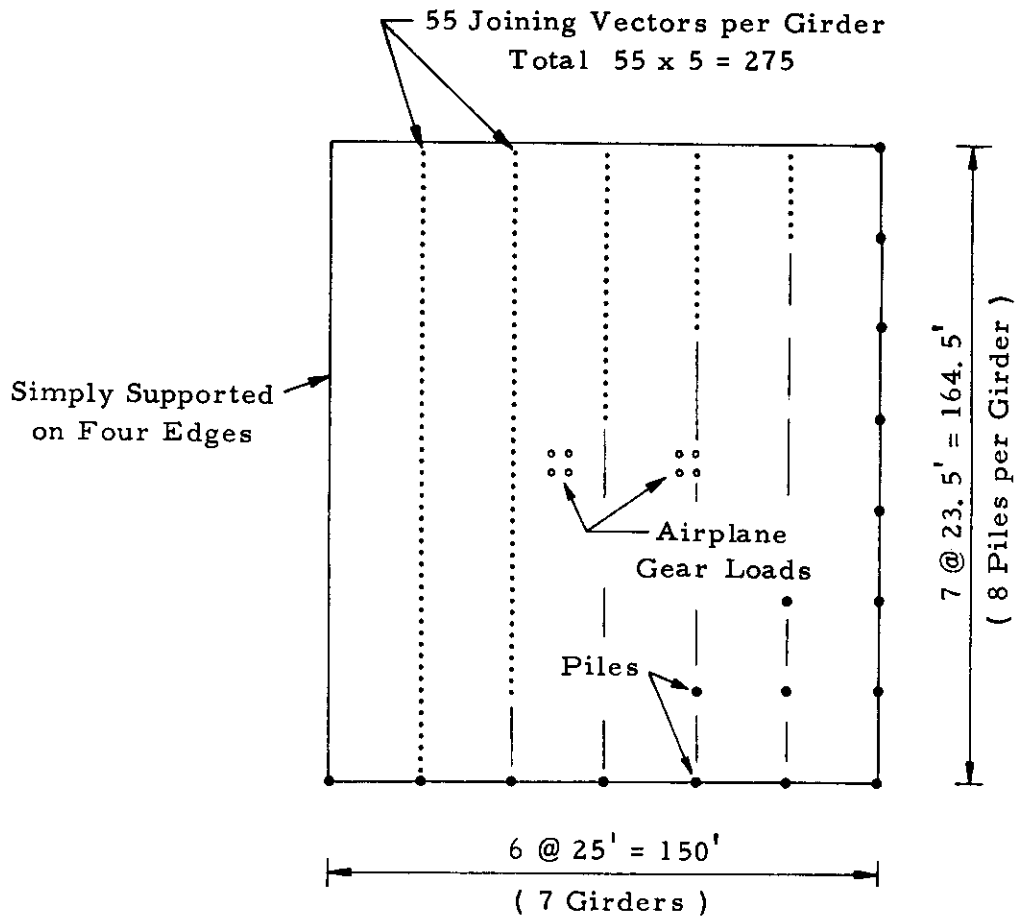


Figure 17. La Guardia Runway Pier Extension - Idealization for Interior Analysis

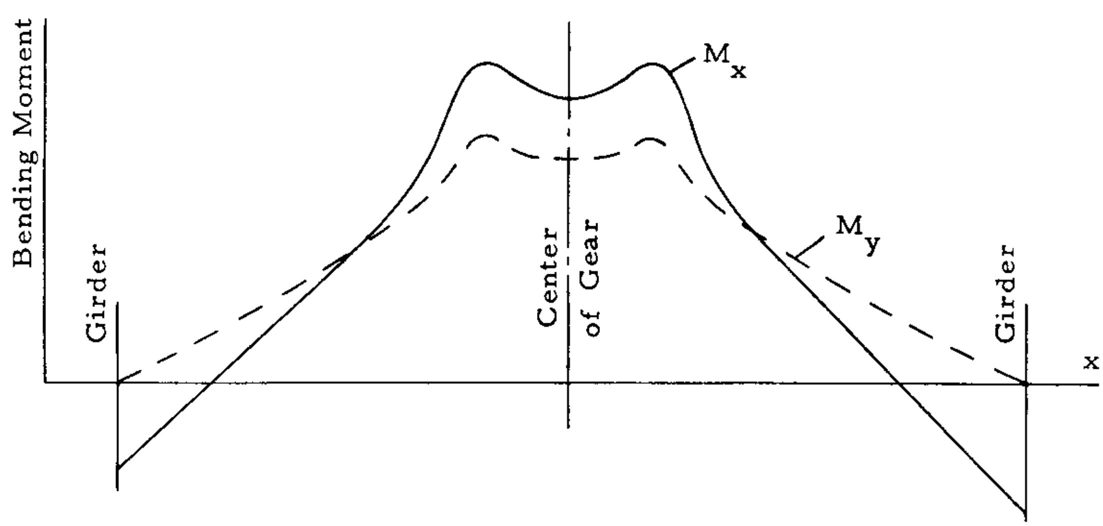


Figure 18. La Guardia Runway Pier Extension - Slab Bending Moments

SECTION IV
DISCUSSION

FORMAT has been readily adopted and extensively used by the Douglas Aircraft Company in the development of commercial systems. It also has been used extensively for military systems, including analyses of missile and space systems by other divisions of the McDonnell Douglas Corporation. One reason for its immediate acceptance, and the reason for dominance of the force method capability in FORMAT applications is the fact that the force method capability in FORMAT is a natural evolution of the Redundant Force Analysis computer program used by Douglas since 1959 (Reference 10). The other main reason is that many engineers have been formally instructed in the use of FORMAT and are encouraged to use it whenever economically profitable.

The formal instruction consisted of 17 two-hour classes on matrix structural analysis methods and their implementation via FORMAT. Seven senior users shared the task of preparing notes and giving lectures. Classes were held in-plant after normal working hours and average attendance was more than 90 percent. Sixty-two employees have completed the course. The students and the instructors have gained significantly from this activity.

The universality of FORMAT proved very beneficial during the DC-10 program. Due to a temporary shortage of in-plant computing capability, a significant amount of structural analysis was done using a GE 635 operated by a service bureau. Prior operational experience was limited to the IBM 7094 and the IBM 7094/7044 DCS, and yet the program was operational and the production work load was shifted with only a two-day delay for implementation. No other large program is known to be this universal.

The flexibility of FORMAT and the availability of approximately 60 qualified analysts is a significant advantage from the standpoint of job scheduling and manpower commitments. All major jobs are scheduled with consideration of time-phasing to best satisfy requirements for design data, and personnel are readily shifted to satisfy peak manpower requirements. This flexibility also stimulates creativity on the part of the analyst, for example, most applications involve idealizations and data processing that are customized for the particular case. In order to promote this creativity and yet avoid costly mistakes, an analysis review board has been established. The review board is composed of five senior engineers, each with 10 to 15 years experience and specializing in matrix structural analysis. The board reviews the proposed idealization, in concept and detail, and the computing plan for all major analyses. The board also is available for consultation for all other analyses, at the option of the cognizant analyst. The activities of the review board effectively focus many years of experience on current applications.

As is the case with all operational computer programs, errors have been found in FORMAT. In many cases, the difficulties have been circumvented by temporarily using alternate abstraction instruction sequences that avoid the error. As time permits, these errors are being isolated, corrected, and documented in the "FORMAT Update" system published and distributed by the Air Force Flight Dynamics Laboratory.

SUMMARY

The FORMAT system and some current applications have been summarized. These applications clearly demonstrate the broad scope of applications for which FORMAT can be effectively used.

ACKNOWLEDGMENT

The FORMAT computer program system was developed by the McDonnell Douglas Corporation under sponsorship of the Air Force Flight Dynamics Laboratory. The analysis of the notched plate, discussed herein, is part of a continuing investigation of structural reliability in the Independent Research and Development (IRAD) program of the McDonnell Douglas Corporation. All other applications described in this paper are part of the development program for the DC-10.

SECTION V

REFERENCES

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