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WEIGHT MINIMIZATION OF HONEYCOMB HEAT SHIELDS

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FOREWORD

This report was prepared by the Structures Division of the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command at Wright-Patterson Air Force Base, Ohio. The work was conducted under Project 1368, "Structural Design Concepts," Task 136804, "Re-entry and Hyperthermantic Structures." This effort was performed March — April 1965 by Mr. Robert T. Achard. The manuscript of this report was released by the author May 1965 for publication as an RTD Technical Report.

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ABSTRACT

A procedure for the design of minimum weight honeycomb structures for use as radiative heat shields is presented. Formulations are applicable to panels critical to intracell buckling and face compressive yield. Minimum weight is shown to occur at the condition where design variables produce a critical buckling stress equal to the compressive yield stress.

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SYMBOLS

C Core thickness (distance between facing mid-points)

D Density of core to face joint (bond, braze, weld); expressed in weight per unit area

E Young's modulus

E_T Tangent modulus

E' Effective modulus

F Stress in honeycomb face

F_y Face material compressive yield strength

K Empirical constant

M Panel bending moment; per panel width

n Empirical exponent

R $C \div S$

S Core cell size

t Thickness

T Effective core thickness

u Definition; $(m \div KE')\ \exp\left(\frac{1}{n+1}\right)$

w Panel weight per surface area

W Panel specific weight, $(w-D) \div \rho_f$; expressed in units of length

ρ Density

μ Poisson's ratio

Subscripts

b Intracell buckling

by Both intracell buckling and yield are critical

c Expanded core

f Face

m Minimum

o Overall minimum

s Function of cell size (S)

sc Core material (solid)

T See symbol E_T

y Compressive yield

INTRODUCTION

Aerospace application of sandwich panels involves numerous interdependent considerations; for example, heat transfer requirements, available materials and fabrication techniques, mechanical loading characteristics or histories, flutter criteria, extraterrestrial particle impingement, and thermal stresses or deflections. Design must account for both general and local structural instabilities, while usually being oriented towards providing minimum weight structure. This report presents a weight minimization study of honeycomb sandwich heat shields, where the sandwich facings are assumed to be isotropic, the panels polytropic in planes parallel to the facings, and where failure is by intracell buckling or yield of a panel face.

A honeycomb sandwich is subject to the following failure modes:

a. General buckling, where the panel buckles as a unit. This mode is produced by edge compression, mechanically or thermally induced.

b. Core shear, where crimping or shear buckling through a cell wall occurs. This mode can be produced by shear or edge loading. It often follows initial general buckling.

c. Core compressive failure by forces normal to the faces.

d. Face wrinkling of one or both faces, where the face separates from the core by tensile failure in the face/core joint and possibly tearing of the core, or where the face buckles inward due to a compressive core failure (usually buckling). This mode is often produced from edge stresses, where failure of one or both faces may occur. Differential thermal expansion of panel faces or shear bending moments may produce this type of failure in the face under compression.

e. Intracell buckling or dimpling, where failure of one or both faces occurs within the confines of the honeycomb cells. Loads similar to those described for face wrinkling may produce this failure, especially for overly thin faces.

f. Tensile or compressive plastic deformation of the faces, by edge or bending loads.

Considering these failure types, typical heat shields will not experience excessive edge, shear, or normal face loading. These forces are small due to deflection allowance at expansion gaps, adequate core shear strengths, and low normal aerodynamic pressures (1 to 4 psi at critical design temperature). Reference 1 shows a low temperature gradient and low differential thermal expansion through a honeycomb panel at steady thermal state since the panel thermal resistance is much less than that of the backup insulation. It should, however, be noted that rapid transient or non-uniform (across the face) heating may create adverse bowing or warpage with associated permanent deformation or failure. This study is for the case where transient or non-uniform heating is not critical. In addition, it is assumed that core gages and face-to-core-joint strength are adequate to preclude face wrinkling under stress. Therefore, this study is for a honeycomb panel designed for intracell buckling by the action of bending stresses.

The work was initiated to verify and amplify a design procedure utilized during the conduction of the technical program of Reference 1. The procedure included the formulation of minimum weights for a honeycomb panel at the unique design case where the face stress was critical both for intracell buckling and compressive yield, and where other modes of panel failure were not applicable. At this particular situation panel structural design for minimum weight could be accomplished using either buckling or yield analysis. Intuitively, both analyses might seem to give the same weights. This study, to the contrary, shows that the two different analyses produce minimum weights that are unequal, by a small percentage difference. However, in a practical design situation the difference may be neglected, as was done in Reference 1.

This study presents an overall view or geometrical model of the structural mechanics and weight analysis for a honeycomb panel subject to compressive failure of one face due to panel bending moments. In addition, the analysis techniques and qualitative characteristics described are applicable to weight minimization for other modes of panel loading and failure.

The study has been conducted with core cell size (S) core depth (C) and facing thickness (t_f) as variables. Design constraints for minimum weight make t_f a dependent variable as a function of C and S. Design bending moment, core wall thickness, and material properties are taken as given constants. The design moment (M) and core thickness could also be treated as variables, the former as a function of panel length. For this study, panel and core wall size are assumed to have been determined by other factors; for example, thermal deflection, flutter, panel attachments, minimum core gages, fabrication requirements, and so forth.

BASIC DESIGN PROCEDURE

The weight of a honeycomb panel (Figure 1) with equal face thicknesses of the same material is expressed:

$$w = \rho_c C + 2\rho_f t_f + D \tag{1}$$

From Reference 2,

$$\rho_c = (2 t_c \rho_{sc}) \div S \tag{2}$$

or

$$\rho_c = (2 t_c \rho_f) \div S \tag{3}$$

if both core and facing are of the same material. For a general case where different materials may be used for core and facing (and where the weight of the braze or bond material attaching face to core is included) a parameter, T, can be defined and substituted into Equation 2 to allow the general expression of core weight to be in terms of facing density. To accomplish this let:

$$\rho_{sc} t_c = \rho_f T \tag{4}$$

Then Equation 2 becomes:

$$\rho_c = \frac{2T\rho_f}{S} \tag{5}$$

In determining minimum weights only those terms in Equation 1 that have variables (t_f, S, C) are effectively utilized in the minimization analyses. Equation 1 can then be written as:

$$W = \frac{2TC}{S} + 2t_f \tag{6}$$

where

$$W \equiv (w - D) \div \rho_f \tag{7}$$

For the panel under bending loading:

$$F = \frac{M}{t_f C} \tag{8}$$

where F will be critical either for intracellular buckling or compressive yield. Reference 2 recommends an empirical formula for the former failure mode that takes the following form:

$$F_b = KE' \left(\frac{t_{f,b}}{S}\right)^n \tag{9}$$

where dimensionless K and n reflect a best fit to available data and:

$$E' = \frac{2 E E_T}{E + E_T} \tag{10}$$

From Equations 8 and 9:

$$t_{f,b} = \left(\frac{F_{f,b}}{KE'}\right)^{1/n} \cdot S = \left(\frac{M}{CKE'}\right)^{\frac{1}{n+1}} \cdot S^{\frac{n}{n+1}} \tag{11}$$

for buckling design.

For compressive yield, Equation 8 is rearranged:

$$t_{f,y} = \frac{M}{F_y C} \qquad (12)$$

Equation 6 can now be written for the constrained cases where stresses are at the critical level:

$$W = \frac{2TC}{S} + \frac{2M}{F_y C} \qquad (13)$$

and

$$W_b = 2T\frac{C}{S} + 2\left(\frac{M}{KE'}\right)^{\frac{1}{n+1}}\left(\frac{S^{\frac{n}{n+1}}}{C^{\frac{1}{n+1}}}\right) \qquad (14)$$

for compressive yield and interface buckling, respectively.

The following symbols are substituted into Equations 12 and 13:

$$R \equiv \frac{C}{S} \quad \text{and} \quad u \equiv \left(\frac{M}{KE'}\right)^{\frac{1}{n+1}}$$

yielding

$$W_y = 2TR + 2\frac{M}{F_y SR} \qquad (15)$$

and

$$W_b = 2TR + 2uR^{-\left(\frac{1}{n+1}\right)} S^{\frac{n-1}{n+1}} \qquad (16)$$

Reference 2 lists:

$$n = 1.5$$
$$K = 0.764$$

Later revisions (Reference 3) of the referenced military handbook and Reference 4, which uses the newer formulations, indicate data to be best represented by:

$$n = 2$$
$$K = \frac{2}{1-\mu^2} \approx 2.2 \quad \text{for } \mu \approx 0.3$$

A numerical value for K will not effect weight minimization analyses. K is, therefore, left algebraic in this study. Specific values of n are reflected in the minimization analyses for buckling and yield in a quantitative, but not qualitative manner, for the range of applicable n. The quantities, n and K, are involved in the yield analysis only where $F_y = F_b$.

Weight minimization of Equations 13 through 16 can be accomplished with respect to C or R, both giving identical results, but with somewhat different insight. The former method will give C as a function of S for minimum weight. It is directly useful where core sizes (S) are limited to a few specific values, due either to available gages or other design considerations. A specific value of C can thereby be determined for the given S, dictating panel design. In addition, the values of C and S corresponding to minimum weight conditions can be used to portray design curves in the C-S plane as later derived and shown in Figures 2 through 5.

Use of R to analyze weight limits and determine minimums is of interest since it can be readily used to describe a three dimensional picture of W with respect to C and S. This would involve plotting W along constant R planes. Limiting values of W are easily visualized when expressed along R planes.

COMPRESSIVE YIELD ANALYSIS

Setting $\frac{\partial W}{\partial C}$ or $\frac{\partial W}{\partial R}$ equal to zero in Equations 13 and 15, values of C and R giving minimum weights for this design condition can be determined:

$$C_{m,y,s} = \left(\frac{M\ S}{F_y\ T}\right)^{1/2} \quad (17)$$

$$R_{m,y,s} = \left(\frac{M}{F_y\ ST}\right)^{1/2} \quad (18)$$

$$W_{m,y,s} = 4T\ R_{m,y,s} = 4T\ \frac{C_{m,y,s}}{S} \quad (19)$$

General weight versus C, S, and R relations are presented in Figure 2. Limiting values of W are listed in Table I. Figure 4 sketches Equation 17. $W_{m,y,s}$ and $R_{m,y,s}$ both are functions of S and decrease as S increases, forming a single curving trough.

Traveling away from the origin, or W axis, along the $W_{m,y,s}$ trough, increasing values of core spacing (S) reach a limit where intracell buckling becomes the dominant failure mode. As derived in the Appendix, the limit of S for compressive yielding is:

$$S_{m,y} = T^{1/3} M^{1/3} (KE')^{\frac{2}{3n}} F_y^{-\left(\frac{n+2}{3n}\right)} \quad (20)$$

and design weight would be:

$$W_{m,y} = 4T^{1/3} M^{1/3} (KE')^{-\frac{1}{3n}} F_y^{\frac{1-n}{3n}} \quad (21)$$

INTRACELL BUCKLING ANALYSIS

Similar to the procedure used for compressive yield:

$$C_{m,b,s} = \left[\frac{u}{(n+1)T}\right]^{\frac{n+1}{n+2}} \cdot S^{\frac{2n+1}{n+2}} \quad (22)$$

$$R_{m,b,s} = \left[\frac{u}{(n+1)\ T}\right]^{\frac{n+1}{n+2}} \cdot S^{\frac{n-1}{n+2}} \quad (23)$$

$$W_{m,b,s} = (4+2n)\ T\ R_{m,b,s} = (4+2n)T\ \frac{C_{m,b,s}}{S} \quad (24)$$

For n = 3/2, Equations 22 through 24 become:

$$C_{m,b,s} = \left(\frac{2}{5}\ \frac{u}{T}\right)^{5/7} S^{8/7} \quad (25)$$

$$R_{m,b,s} = \left(\frac{2}{5}\ \frac{u}{T}\right)^{5/7} S^{1/7} \quad (26)$$

$$W_{m,b,s} = 7T\ R_{m,b,s} = 7T\ \frac{C_{m,b,s}}{S} \quad (27)$$

These relations are sketched and limits tabularized in Figure 3 and Table I, respectively. Thus, the minimum weight is a function of the variable, S, and weight is lowered by minimizing S. As S is reduced along the varying $R_{m,b,s}$ plane or $W_{m,b,s}$ trough, a value of S is reached where the failure mode changes from buckling to compressive yield. Therefore, the analysis has a practical limit at this point of mode change.

The Appendix derives minimum values for $S_{m,b}$ and $W_{m,b}$ along the $W_{m,b,s}$ trough. These are expressed with n written algebraically and for the two appropriate numerical values, 1.5 and 2. For the general case:

$$S_{m,b} = (n+1)^{1/3} T^{1/3} M^{1/3} (KE')^{\frac{2}{3n}} F_y^{-\left(\frac{n+2}{3n}\right)} \quad (28)$$

$$W_{m,b} = (4+2n)(n+1)^{-2/3} T^{1/3} M^{1/3} (KE')^{-\frac{1}{3n}} F_y^{\left(\frac{1-n}{3n}\right)} \quad (29)$$

For n equal to 1.5 the minimums are expressed:

$$S_{m,b} = \left(\frac{5}{2}\right)^{1/3} T^{1/3} M^{1/3} (KE')^{4/9} F_y^{-7/9} \quad (30)$$

$$W_{m,b} = 7\left(\frac{2}{5}\right)^{2/3} T^{1/3} M^{1/3} (KE')^{-2/9} F_y^{-1/9} \quad (31)$$

DUAL YIELD/BUCKLING ANALYSIS

As previously shown and derived in the Appendix, panel weights using yield or buckling analyses reach respective minimums on the W_m troughs where $F_y = F_b$. Each analysis produces its own minimum and the two are unidentical both for weight and design dimension (C and S). However, at any point where $F_y = F_b$, W_y must equal W_b (Equations 15 or 16). Using the $W_{m,b,s}$ trough as an example, this relation can be clarified: At point 1 (Figure 4), $W_{m,y} = W_b \neq W_{m,b}$. Thus W_b at point 1 is not a minimum for the S associated with this point. $W_{m,b}$ would occur at point 2, were buckling analysis valid at this point, which it is not.

Following this train of thought, Figure 4 shows that between points 1 and 3 neither yield nor buckling (separately) weight minimization analyses are valid. Between these points an analysis for both failure modes must be developed. This is readily accomplished by equating Equations 15 and 16. Thus, for $W_b = W_y$:

$$R = \frac{M\,(KE')^{1/n}}{S^2 \quad F_y^{\frac{n+1}{n}}} \qquad (32)$$

or

$$C = \frac{M\,(KE')^{1/n}}{F_y^{\frac{n+1}{n}}}\; S^{-1} \qquad (33)$$

For this condition Equations 15 and 16 yield:

$$W_{by} = \frac{2T\;M\,(KE')^{1/n}}{F_y^{\frac{n+1}{n}}\; S^2} + 2S\left(\frac{F_y}{KE'}\right)^{1/n} \qquad (34)$$

Minimizing W_{by} with respect to S:

$$S_o = (2)^{1/3}\, T^{1/3}\, M^{1/3}\, (KE')^{\frac{2}{3n}}\quad F_y^{-\left(\frac{n+2}{3n}\right)} \qquad (35)$$

$$W_o = 3S_o\left(\frac{F_y}{KE'}\right)^{1/n}$$

$$= 3(2)^{1/3}\, T^{1/3}\, M^{1/3}\, (KE')^{-\frac{1}{3n}}\quad F_y^{\frac{1-n}{3n}} \qquad (36)$$

For n = 1.5

$$S_o = (2)^{1/3}\, T^{1/3}\, M^{1/3}\, (KE')^{4/9}\quad F_y^{-7/9} \qquad (37)$$

$$W_o = 3(2)^{1/3}\, T^{1/3}\, M^{1/3}\, (KE')^{-2/9}\quad F_y^{-1/9} \qquad (38)$$

ANALYSIS COMPARISON

Three analysis methods were developed in this study. Each has a finite minimum weight where $F_y = F_b$. Comparing Equations 20, 21, 28, 29, 35, and 36, we note that the expressions for design factors T, M, K, E, and F_y are identical for similar equations. This naturally follows since the three points these equations describe fall along a common curve. T and M are independent of empirical constants (N and K), and weight will always be proportional to their cube root. Design factors are preceded by coefficients that are totally numerical, except for intracell buckling (Equations 28 and 29) where n is also included. Table II compares the coefficients and lists intersection points of the three equations formed by the analyses. These are shown in Figure 4 for n = 1.5. In this figure, the numerical coefficients for points 1 and 5 are independent of n.

The following relations can be expressed:

$$\frac{S_{m,b}}{S_o} = \left(\frac{n+1}{2}\right)^{1/3} \tag{39}$$

$$= 1.08 \text{ for } n = 1.5$$

$$= 1.14 \text{ for } n = 2$$

$$\frac{W_{m,b}}{W_o} = \left(\frac{2}{n+1}\right)^{2/3} \frac{2+n}{3} \tag{40}$$

$$= 1.01 \text{ for } n = 1.5$$

$$= 1.02 \text{ for } n = 2$$

$$\frac{S_{m,y}}{S_o} = (2)^{-1/3} = 0.79 \tag{41}$$

$$\frac{W_{m,y}}{W_o} = (0.75)(2)^{-1/3} = 1.06 \tag{42}$$

$$\frac{S_{m,y}}{S_{m,b}} = 0.693 \text{ for } n = 2 \tag{43}$$

Thus, for values of n given by References 2 or 3 the minimum weight from intracell buckling analysis (Equation 29) will be closer to the exact minimum (Equation 36) than will be the weight for panel design by yield. Panel designs by yield or buckling analysis will be appreciably different in dimension (Equation 43), but will only vary by approximately 5 percent in weight from each other. The choice of analysis method used is, therefore, quite insensitive to the numerical value of n, for its referenced range. The choice of analysis is not a function of K since this factor becomes algebraically identical in all analyses. The actual numerical values of K and n will effect panel weight. These quantities, however, cannot be arbitrarily varied to optimize weight since they are essentially fixed quantities, dependent upon empirical data or recommended values (References 2 and 3).

For typical panel dimensions and material properties the following analysis shows the effect of n and K on S_o and W_o:

Using columbium alloy C-103 at 2600°F where

$$E' \approx 5 \cdot 10^6 \text{ psi } (3.5 \cdot 10^2 \text{ Kg/mm}^2)$$

and

$$F_y \approx 8000 \text{ psi } (5.6 \text{ Kg/mm}^2)$$

$$\frac{S_o(n=1.5)}{S_o(n=2)} = \frac{(0.764\,E')^{4/9}}{(2.2\,E')^{1/3}} \cdot F_y^{(-7/9+2/3)} = 1.4 \tag{44}$$

$$\frac{W_p\ (n=1.5)}{W_o\ (n=2)} = 1.4\left(\frac{F_y}{0.764E'}\right)^{2/3} \div \left(\frac{F_y}{2.2E'}\right)^{1/2} = 0.85 \tag{45}$$

Units used do not effect these ratios, regardless of n. Although logical, this fact may not always seem apparent; especially in an expression like Equation 44 where different powers are involved.

Therefore, in the range given by References 2 and 3 the actual design dimensions are quite sensitive to the empirical values of K and n; minimum weights are moderately sensitive. The above ratios are produced by partial balances between the effects of K and effects of n on the ratio $(E' \div F_y)$. In Equations 7 and 8 the effect of K taken to appropriate powers of n is:

$$\frac{(0.764)^{4/9}}{(2.2)^{1/3}} = 0.68 \text{ for } S_o \text{ ratios} \tag{46}$$

$$\frac{(0.764)^{-2/9}}{(2.2)^{-1/6}} = 1.21 \text{ for } W_o \text{ ratios} \tag{47}$$

Similarly, the effect of E' and F_y to powers of n can be determined for S_o:

$$(E')^{(4/9-1/3)} \cdot F_y^{(7/9+2/3)} = \left(\frac{E'}{F_y}\right)^{1/9} \tag{48}$$

For the above typical values, this ratio becomes 2.05. In a like manner the effect on W_o is determined to be:

$$\left(\frac{F_y}{E'}\right)^{1/18} = 0.7 \qquad (49)$$

The effect of n is appreciable due to the generally large ratio of $E' \div F_y$. The smaller this ratio is, up to the limit where Equation 49 becomes the reciprocal of Equation 47, the more W_o is insensitive to n.

From Equation 19 and especially Equation 27, minimum weights as functions of S are only weakly dependent on this variable, that is, to powers appreciably less than one. Similarly, C is a fairly weak function of S for yield analysis. However, for buckling analysis C is strongly dependent on S, and R is approximately independent of S. In the intracell buckling regime, small adjustment of S, from that analytically determined, to conform to a standard honeycomb size will not appreciably effect panel weight as long as the proper C is utilized with S. Therefore, since $S_{m,b,}$ is approximately equal to S_o these small adjustments of S will not cause excessive divergence from optimum weight.

APPLICATION

The previous discussions and derivations have determined that minimum weight design for a honeycomb sandwich panel critical for intracell buckling or face compressive yield will occur where the intracell buckling face stress equals the compressive yield allowable. Figure 4 shows that for values of cell size (S) between zero and dimension A, yield analysis is used for design. For S greater than that at dimension B, buckling analysis provides values of C and W for minimum weight. Between "A" and "B" the analysis is for both yield and buckling. Analyses (Sections 3 and 4) based upon each of the two distinct modes of failure, separately, were shown to give minimum weights at the finite points (1 and 3) where $F_y = F_b$: that is, the respective analysis' limit, where its failure mode ceased to be the only critical mode. A third analysis (Section 5), different from those solely for yield or buckling, actually determines the design point of <u>overall</u> minimum weight. Weight minimum determined by the three analyses are approximately equal, although design points (C and S) may vary appreciably.

In general, a three region, three dimensional design curve exists (Figure 4 plus a W coordinate) and gives a single overall design minimum. For the practical situation where the core is available in a select number of discrete sizes (S), the values of standard S that most closely surround that calculated by Equation 35 can be selected and judged for weight. Using the available S that gives the lower weight and determining the applicable failure mode (yield, buckling, or both) Equations 17 and 19, 22 and 24, or 33 and 34 can be used to calculate design dimension D and panel weight. Face design thickness, t_f, would subsequently be calculated from Equations 11, 12, or either. The practical minimum weight will probably occur for S in the buckling or dual buckling/yield regimes. This procedure will give a practical, minimum weight panel. Suffice to say, the panel design should also be verified against other possible failure modes and design should consider possible distortion stress from transient and/or uneven heating.

The particular value of n used in these analyses should reflect available data, where possible. In general, however, where the designer will use an available relationship, the formula of Reference 3 takes precedence over that of Reference 2. Therefore, the most recent formulation is recommended where empirical data is unavailable or insufficient to substantiate a different empirical relation. The equations of this study are applicable to any set of empirical constants.

CONCLUSIONS

A formulation for the design of honeycomb heat shields, subject to failure by intracell buckling and face compressive yield, was developed. This relation can be integrated into designs utilizing standard or available shield core sizes. A minimum weight honeycomb panel will result from the use of this analysis method. The ideal minimum panel weight will occur for a design condition where the panel face stress equals the compressive yield stress.

Three analysis methods are applicable to this problem, each giving approximately equal minimum weights for typical values of the empirical factors, n and K. One method gives the overall minimum. The other two (compressive yield and intracell buckling) would be used where available core sizes so warrant. Design by intracell buckling analysis will generally give a minimum weight closer to the ideal minimum than will yield analysis, regardless of the numerical value of n used. Design dimensions (C and S) will vary appreciably depending upon the analysis method used even though weights determined will be approximately equal. Panel weight will be sensitive to the numerical values of empirical design factors, n and K.

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APPENDIX

This Appendix determines values of S along each W_m trough where $F_y = F_b$ and the dominant mode of failure changes from compressive yield to intracell buckling or vice versa. Weight and design values for the two failure modes are compared.

1. For the $W_{m,b}$ trough:

$$F_b = KE' \left(\frac{t_{f,b}}{S_{m,b}} \right)^n = F_y$$

$$t_{f,b} = t_{f,y} = \frac{M}{F_b \, C_{m,b,s}} = \frac{M}{F_y \, C_{m,b,s}}$$

$$\therefore F_y = KE' \left(\frac{M}{F_y \, C_{m,b,s} \, S_{m,b}} \right)^n \qquad (50)$$

from Equation 22:

$$C_{m,b,s} = \left[\frac{u}{(n+1)\,T} \right]^{\frac{n+1}{n+2}} S_{m,b}^{\frac{2n+1}{n+2}}$$

where

$$u \equiv \left(\frac{M}{KE'} \right)^{\frac{1}{n+1}}$$

substituting into Equation 50:

$$F_y = \frac{KE'}{(F_y)^n} \cdot M^n \left[\frac{1}{(n+1)T} \right]^{\frac{-n(n+1)}{n+2}} \cdot \left(\frac{M}{KE'} \right)^{-\frac{n}{n+2}} S_{m,b}^{-3n\left(\frac{n+1}{n+2}\right)}$$

or

$$S_{m,b} = \left[(n+1)\,TM \right]^{1/3} (KE')^{\frac{2}{3n}} \, F_y^{-\left(\frac{n+2}{3n}\right)}$$

at this point Equation 24 gives:

$$W_{m,b} = (4+2n)(n+1)^{-2/3} T^{1/3} M^{1/3} (KE')^{-\frac{1}{3n}} F_y^{-\left(\frac{n-1}{3n}\right)}$$

Substituting n = 3/2:

$$S_{m,b} = \left(\frac{5}{2} \right)^{1/3} T^{1/3} M^{1/3} (KE')^{4/9} F_y^{-7/9}$$

$$W_{m,b} = 7 \left(\frac{2}{5} \right)^{2/3} T^{1/3} M^{1/3} (KE')^{-2/9} F_y^{-1/9}$$

for n = 2:

$$S_{m,b} = (3TM\,KE')^{1/3} F_y^{-2/3}$$

$$W_{m,b} = 8\,(3)^{-2/3} (TM)^{1/3} (KE')^{-1/6} (F_y)^{-1/6}$$

2. For the $W_{m,y}$ trough Equation (50) becomes:

$$F_y = KE' \left(\frac{M}{F_y \, C_{m,y,s} \, S_{m,y}} \right)^n$$

Substituting from Equation 17:

$$F_y = KE' \left(\frac{M^{1/2} \, T^{1/2}}{F_y^{1/2} \, S_{m,y}^{3/2}} \right)^n$$

or

$$S_{m,y} = T^{1/3} M^{1/3} \, KE'^{\frac{2}{3n}} \, F_y^{-\left(\frac{n+2}{3n}\right)}$$

at this point Equation 19 gives:

$$W_{m,y} = 4\,T^{1/3} M^{1/3} (KE')^{-\frac{1}{3n}} F_y^{\frac{1-n}{3n}}$$

3. Comparing minimums for yield and buckling:

$$\frac{S_{m,b}}{S_{m,y}} = 1.36 \text{ for } n = 1.5 \text{ ; } = 1.44 \text{ for } n = 2$$

$$\frac{W_{m,b}}{W_{m,y}} = 0.95 \text{ for } n = 1.5 \text{ ; } = 0.96 \text{ for } n = 2$$

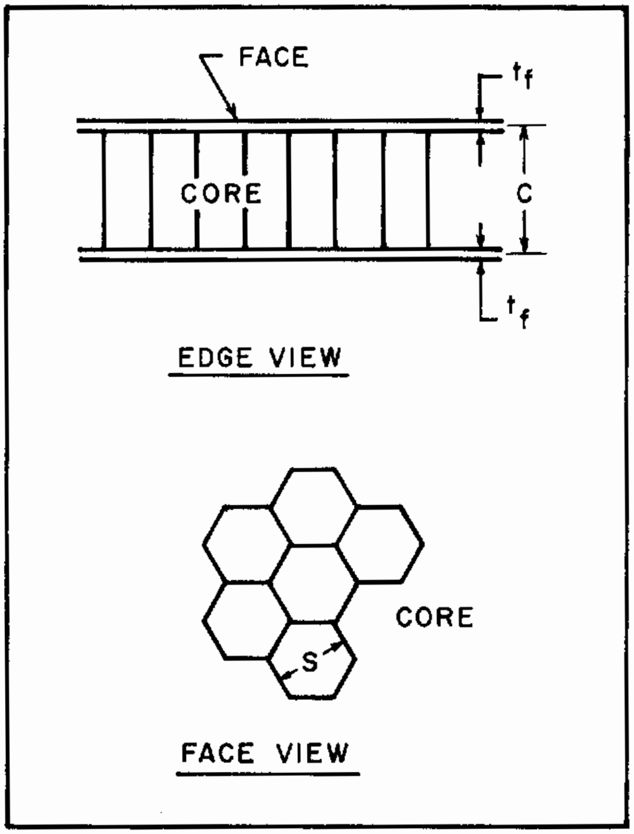


Figure 1. Honeycomb Sandwich Nomenclature

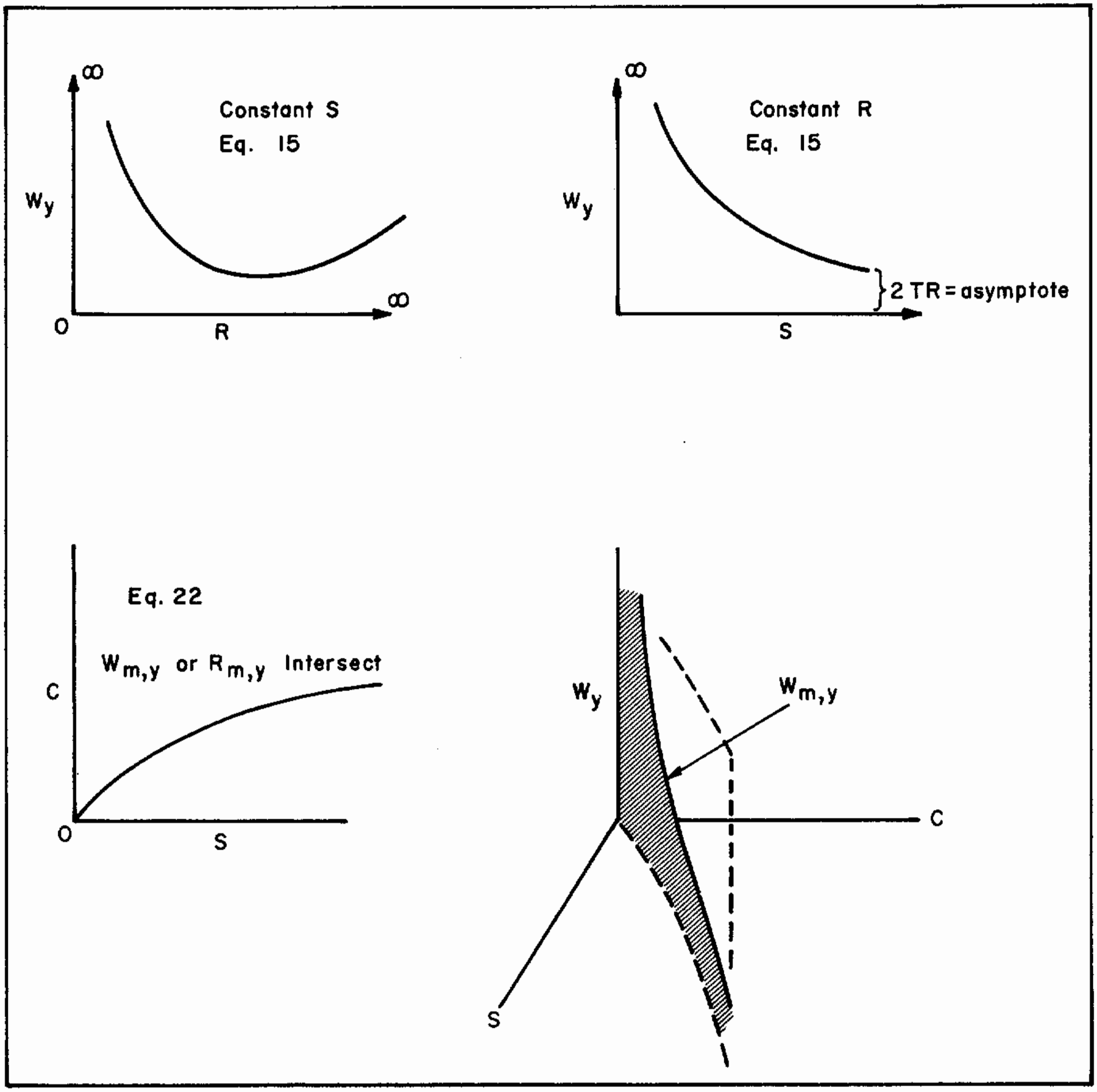


Figure 2. Panel Weight vs. Design Variables for Yield

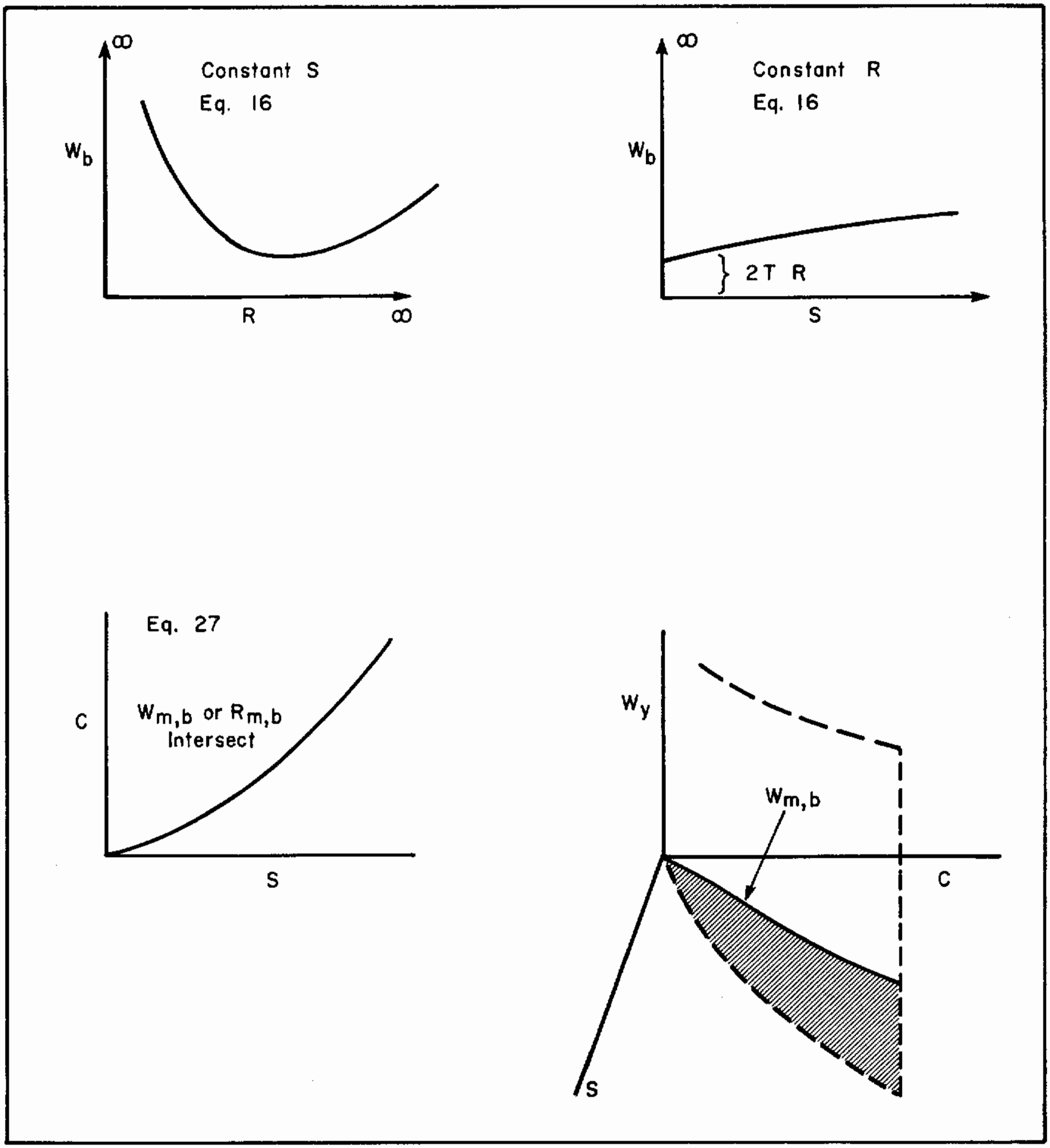
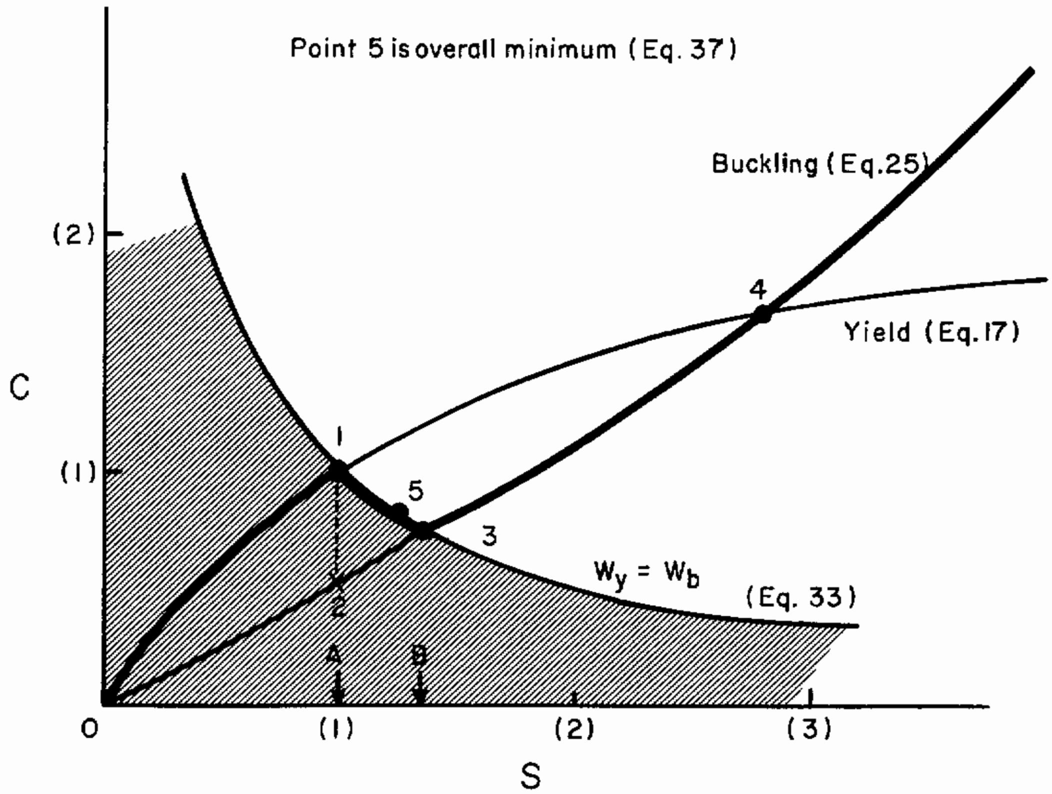


Figure 3. Panel Weight vs. Design Variables for Intracell Buckling



Notes: 1. Curves are sketched for n = 3/2.

2. Points 1,3,4,5 are exact for coordinates (in parentheses); remainder of curves are not exact for the coordinates.

3. See Table 3 for point values and coordinate dimension.

4. Yield is critical in shaded area.

5. Heavy line indicates minimum weight design curve.

Figure 4. Minimum Weight Curves on C-S Plane

TABLE I

Limits of W and W_m for Yield and Buckling Design

Compressive Yield Design	Limit of W_y (Equations 13,15)	Limit of $W_{m,y}$ (Equation 19)
$S \rightarrow 0$	∞	∞
** $S \rightarrow \infty$	$\frac{2m}{F_y C}$ or 2TR	0
$C \rightarrow 0$	∞	*
$C \rightarrow \infty$	∞	*
$R \rightarrow 0$	∞	*
$R \rightarrow \infty$	∞	*
Intracell Buckling Design	**Limit of W_b (Equations 14,16)**	**Limit of $W_{m,b}$ (Equation 27)**
** $S \rightarrow 0$	∞ or 2TR	0
$S \rightarrow \infty$	∞	∞
$C \rightarrow 0$	∞	*
$C \rightarrow \infty$	∞	*
$R \rightarrow 0$	∞	*
$R \rightarrow \infty$	∞	*

Notes:

* For W_m, C and R are considered functions of S; i.e., they are dependent.

** Values for this limit depend upon the other independent variable, C.

TABLE II

Numerical Values for Figure 4

Point	Equations	C	S	W_m or W_o
1	33 with 17 21	1 —	1 —	— 4
3	33 with 25 31	.738 —	1.35 —	— 3.81
4	17 with 25	1.67	2.78	—
5	37 38	.749 —	1.26 —	— 3.78

Notes: 1. Numerical values for points 3 and 4 are for n = 1.5; other coefficient values are general and independent of n.

2 The above values are coefficients for the following algebraic factors, with n = 1.5:

$$C: \quad T^{-1/3} \quad M^{2/3} \quad (KE')^{2/9} \quad F_y^{8/9}$$

$$S: \quad T^{1/3} \quad M^{1/3} \quad (KE')^{4/9} \quad F_y^{-7/9}$$

$$W: \quad T^{1/3} \quad M^{1/3} \quad (KE')^{2/9} \quad F_y^{-1/9}$$

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None	USAF AFFDL

13. ABSTRACT

A procedure for the design of minimum weight honeycomb structures for use as radiative heat shields is presented. Formulations are applicable to panels critical to intracell buckling and face compressive yield. Minimum weight is shown to occur at the condition where design variables produce a critical buckling stress equal to the compressive yield stress.

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