# ACTIVE DAMPING OF A CANTILEVER BEAM SYSTEM

H. V. Vu and S. E. Husher
Department of Mechanical Engineering

D. E. Zimmerman
Department of Engineering and Industrial Technology

California State University, Long Beach
Long Beach, CA 90840

## Abstract

The control-structure interaction of a flexible structure, namely a cantilever beam, and a reaction mass actuator (RMA) is investigated. Mathematical model, in the form of differential equations and transfer functions, is obtained. The study is broken into two steps: (1) open loop and (2) closed loop. Within the open loop part, the RMA is broken into two sub-steps: (a) dead RMA and (b) passive RMA. In the closed loop part, negative feedback of the beam tip velocity is used for active RMA. Transient responses and root loci are given.

## Introduction

The system under consideration is a cantilever beam with a RMA (reaction mass actuator), also called PMA (proof mass actuator) attached to the tip of the beam (Figure 1). The RMA consists of two mechanical components: the magnet-shaft assembly of mass m and the housing of mass $m_h$. When the magnet-shaft assembly is fixed to the housing, the RMA is called "dead RMA," and when the assembly is free, is is called "passive RMA." When the control loop is closed, the RMA is called "active RMA." The control-structure interaction (CSI) of this electromechanical system will be analyzed in the following steps:

1) Open loop
   a) Dead RMA. The simplest model is a single-degree-of-freedom (SDOF) system. The undamped natural frequency is determined, and the beam tip response, which is obtained experimentally, is presented.
   b) Passive RMA. The simplest model is a two-degree-of-freedom (TDOF) system. The undamped natural frequencies are determined, and the beam tip response which is obtained experimentally, is presented.
2) Closed loop
   Active RMA. The velocity of the beam tip is used for negative feedback. The control-structure interaction is investigated. The transient responses and root loci are shown.

## System Dynamics

### 1) Open Loop

The governing differential equation of the beam, using Euler-Bernoulli model, can be shown as

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=f(x,t) \qquad 0<x<l \tag{1}$$

where E, I, $\rho$, A, $l$ are the Young's modulus, area-moment of inertia, density, cross-sectional area, and length, respectively.

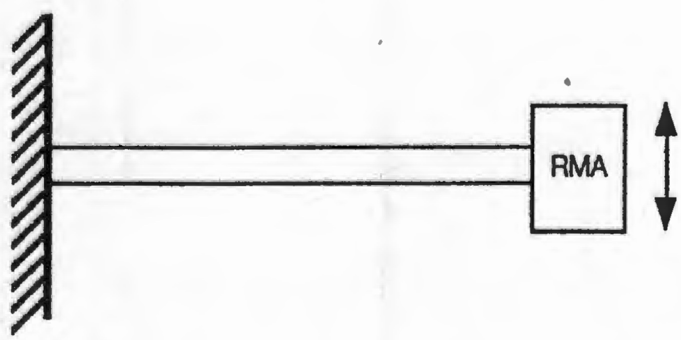



**Figure 1** Cantilever beam system with RMA (Reaction Mass Actuator)

### a) Dead RMA

The system consists of a cantilever beam with a concentrated mass at the beam tip. The frequency equation of the system can be shown, see [1] for example, as

$$1+\frac{1}{\cos\lambda L\cosh\lambda L}-\frac{M}{\rho AL}\lambda L(\tan\lambda L-\tanh\lambda L)=0 \qquad (2)$$

The transcendental equation (2) must be solved numerically to yield the eigenvalues $\lambda_i$, then the natural frequencies are given as

$$\omega_i=\sqrt{\frac{EI}{\rho AL}}\lambda_i^2 \qquad i = 1, 2, ..., \infty \qquad (3)$$

Since the beam model given by Eq. (1) yields infinite degrees of freedom, the control-structure interaction of the beam and the RMA is difficult to analyze. The problem is more tractable if the system with dead RMA is modeled as SDOF for the fundamental mode. Figure 2 shows this model with K, M, and b are the equivalent stiffness, equivalent mass, and equivalent damper, respectively. The mass and stiffness can be calculated from physical properties, but the damping must be determined experimentally.

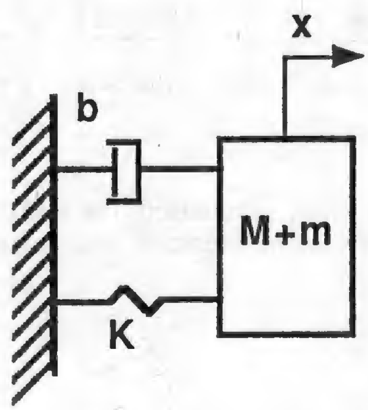



**Figure 2** A simple model of the system with dead RMA

It can be found in vibration texts, see [2] for example, that

$$K=\frac{3EI}{L^3} \qquad M=m_h+0.236\rho AL \qquad (4)$$

(m and $m_h$ are the masses of the RMA magnet-shaft assembly and housing, respectively.)

An experiment was performed, where the physical parameters of the tested beam (Aluminum 6061-T6) are

$$L=30.75\ in. \qquad A=3\ in.\times 0.25\ in. \qquad E=10\times10^6\ psi \qquad \rho=0.2588\ \frac{lb_f s^2}{in^4}$$

Thus, the equivalent stiffness and equivalent mass are calculated to be

$$K=4.03\ \frac{lb}{in} \qquad M=6.33\times10^{-3}\frac{lb_f s^2}{in} \qquad m=6.47\times10^{-4}\frac{lb_f s^2}{in}$$

The natural frequency is calculated and observed to be 3.8 Hz and 3.5 Hz, respectively. The response at the beam tip of the system with dead RMA is shown in Figure 3.

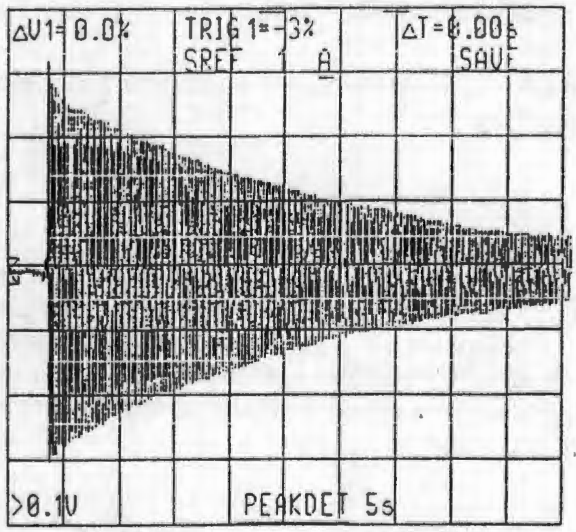



**Figure 3** Response at the beam tip with dead RMA

b) Passive RMA

When the moving part of the RMA is released, the RMA acts as a passive vibration absorber (Figure 4). When b = 0, the system becomes the classical Den Hartog's vibration absorber problem [3].

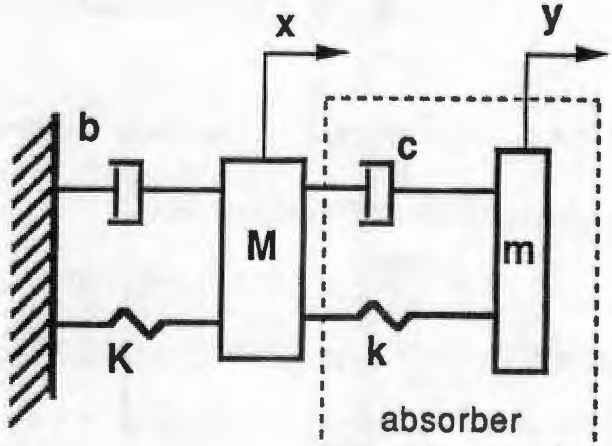



**Figure 4** System with vibration absorber (passive RMA)

The differential equations are

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} b+c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} K+k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (5)$$

where the undamped natural frequencies can be obtained as

$$\omega_1^2 = \frac{(M+m)k+mK}{2Mm} - \sqrt{\left(\frac{(M+m)k+mK}{2Mm}\right)^2 - \frac{Kk}{Mm}}$$

$$\omega_2^2 = \frac{(M+m)k+mK}{2Mm} + \sqrt{\left(\frac{(M+m)k+mk}{2Mm}\right)^2 - \frac{Kk}{Mm}} \qquad (6)$$

The response at the beam tip with passive RMA is shown in Figure 5.

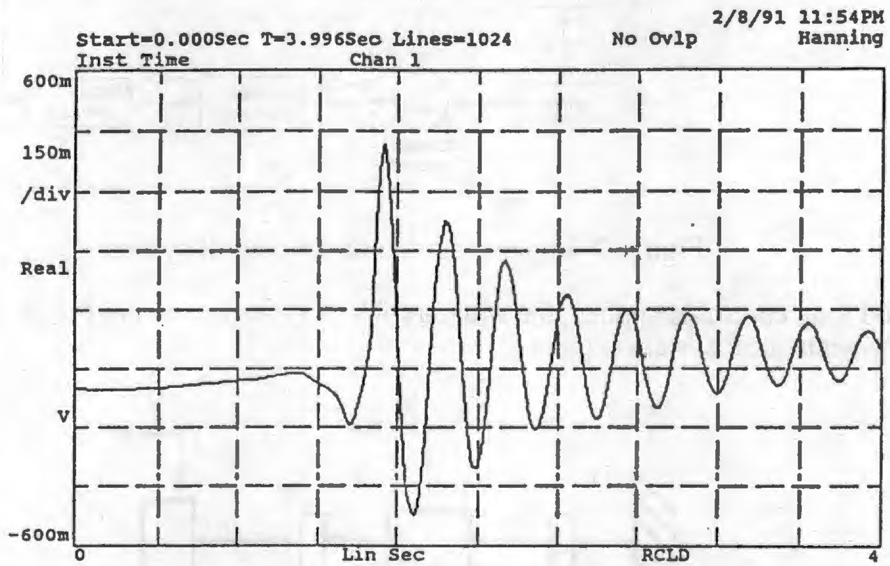



**Figure 5** Response at the beam tip with passive RMA

2) <u>Closed Loop - Active RMA</u>

The closed loop control utilizes the beam tip velocity $\dot{x}$ for negative feedback and the system can be *conceptualized* as shown in Figure 6.

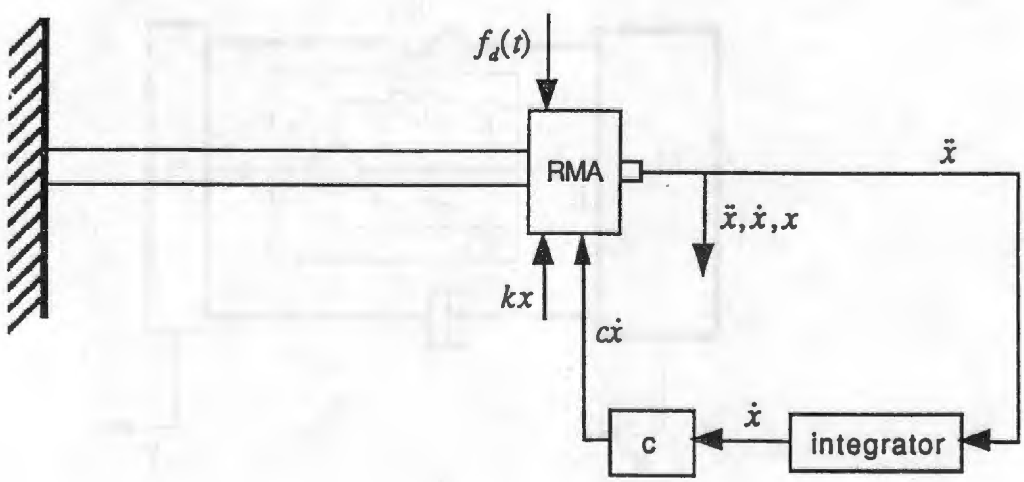



**Figure 6** Conceptualized control scheme

For physical implementation, an *actual* system can be shown as in Figure 7.

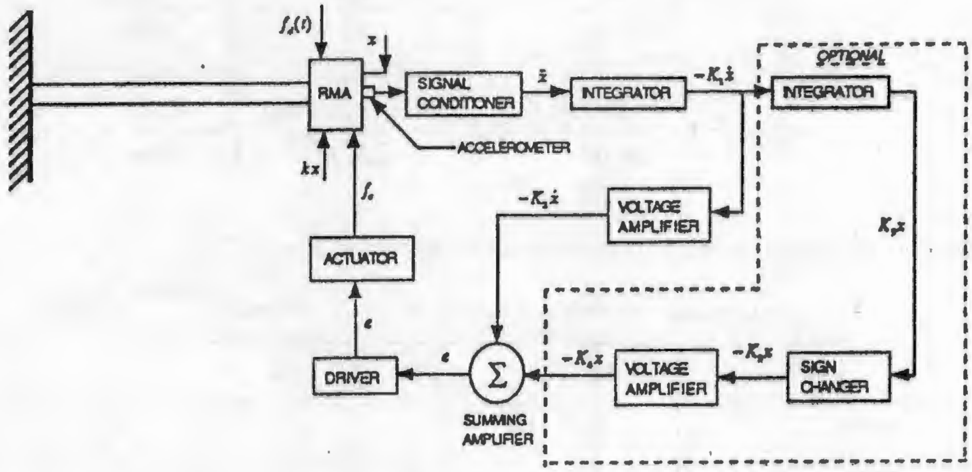



**Figure 7** Implementation of the control system

When closed loop control is applied, the structure-RMA system shown in Figure 7 can be modeled as an electromechanical system (Figure 8).

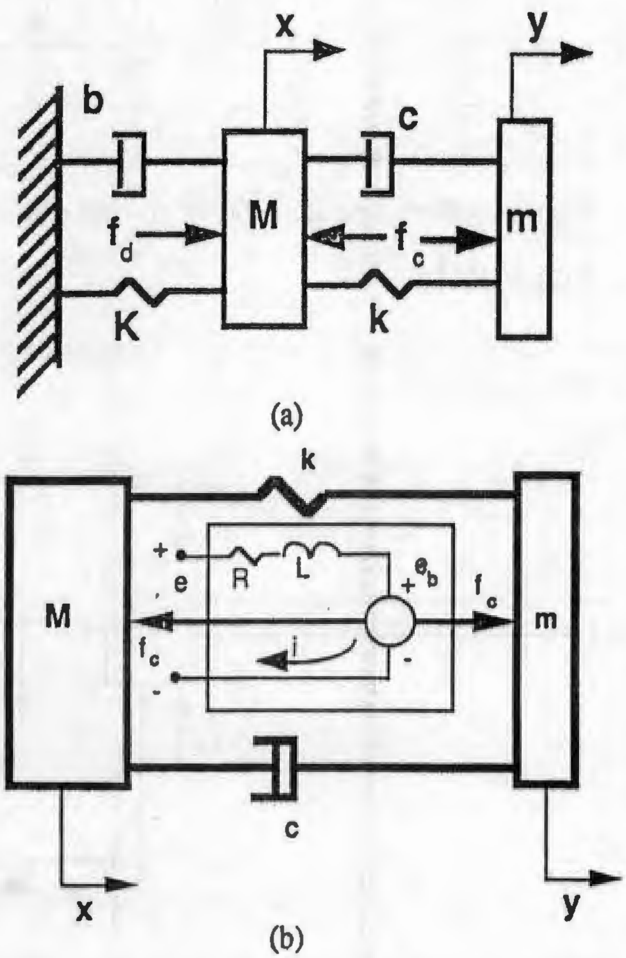



**Figure 8** Electromechanical system: (a) mechanical and (b) electrical

The governing differential equations for the mechanical part can be obtained as

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix}+\begin{bmatrix} b+c & -c \\ -c & c \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix}+\begin{bmatrix} K+k & -k \\ -k & k \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}=\begin{Bmatrix} f_d-f_c \\ f_c \end{Bmatrix} \tag{7a}$$

where $f_d$ and $f_c$ are the disturbance force and control force, respectively. The differential equation for the electrical part is

$$e = Ri + L\frac{di}{dt} + e_b \tag{7b}$$

The electromechanical coupling is given by

$$f_c = k_m i \qquad e_b = k_m(\dot{y}-\dot{x}) \tag{8}$$

If the beam tip velocity is used as negative feedback for the active RMA,

$$e = k_g \dot{x} \tag{9}$$

where $k_g$ is the gain. Then, combining Eqs. (7-9) yields the closed-loop system equations as

$$\begin{bmatrix} M & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{i} \end{Bmatrix}+\begin{bmatrix} b+c & -c & 0 \\ -c & c & 0 \\ -(k_g+k_m) & k_m & L \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{i} \end{Bmatrix}+\begin{bmatrix} K+k & -k & k_m \\ -k & k & -k_m \\ 0 & 0 & R \end{bmatrix}\begin{Bmatrix} x \\ y \\ i \end{Bmatrix}=\begin{Bmatrix} f_d \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

Taking the Laplace transform,

$$\begin{bmatrix} Ms^2+(b+c)s+K+k & -(cs+k) & k_m \\ -(cs+k) & ms^2+cs+k & -k_m \\ -(k_g+k_m)s & k_m s & Ls+R \end{bmatrix}\begin{Bmatrix} X(s) \\ X(s) \\ I(s) \end{Bmatrix}=\begin{Bmatrix} F_d(s) \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

The transfer functions relating x, y, i, and $f_d$ are given by

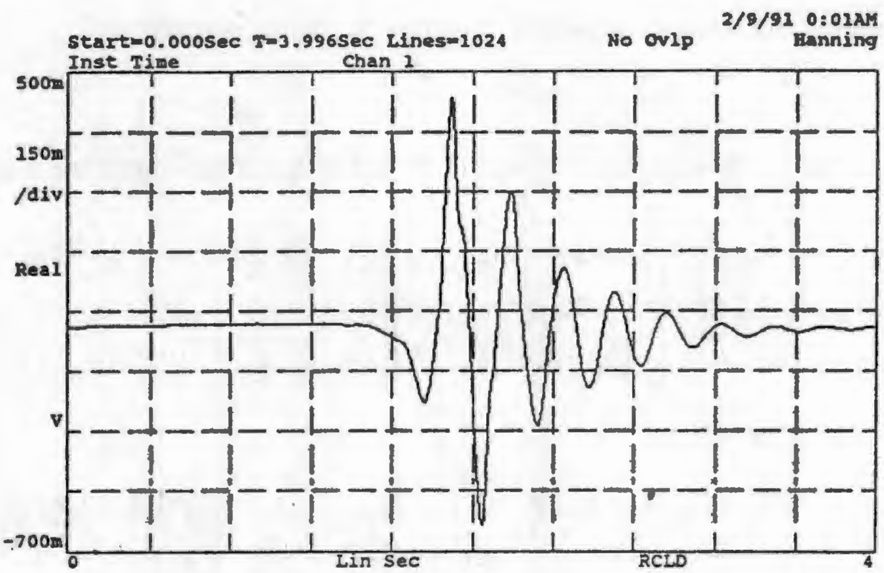
$$\begin{Bmatrix} X(s) \\ Y(s) \\ I(s) \end{Bmatrix}=\begin{Bmatrix} H_1(s) \\ H_2(s) \\ H_3(s) \end{Bmatrix}F_d(s) \tag{12}$$

where the following are obtained with the aid of Mathematica [4]

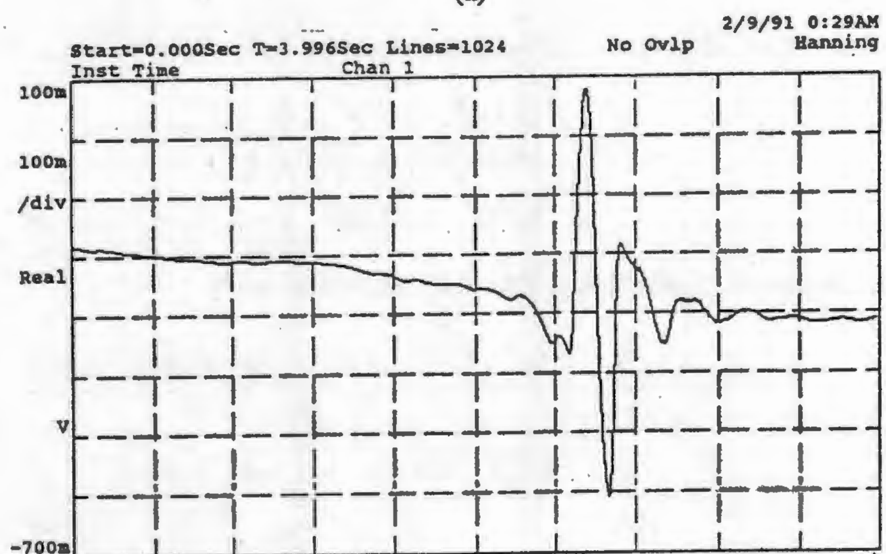
$$\begin{aligned} H_1(s) &= \frac{X(s)}{F_d(s)} = \frac{mLs^3+(mR+cL)s^2+\left(cR+kL+k_m{}^2\right)s+kR}{\Delta(s)} \\ H_2(s) &= \frac{Y(s)}{F_d(s)} = \frac{cLs^2+\left(cR+kL+k_m k_g+k_m{}^2\right)s+kR}{\Delta(s)} \\ H_3(s) &= \frac{I(s)}{F_d(s)} = \frac{s\left[\left(k_m+k_g\right)ms^2+ck_g s+kk_g\right]}{\Delta(s)} \end{aligned} \tag{13}$$

$$
\begin{aligned}
\Delta(s) = {} & \{MmL\}s^5 \\
& +\{(MR+Lb)m+(M+m)cL\}s^4 \\
& +\{(M+m)cR+(Mk+mK+mk+bc)L+(M+m)k_m{}^2+(Rb+k_g k_m)m\}s^3 \\
& +\{(Mk+mK+mk+bc)R+(Kc+bk)L+bk_m{}^2\}s^2 \\
& +\{(cR+Lk+k_m{}^2)K+Rbk\}s \\
& +\{KkR\} \qquad (14)
\end{aligned}
$$

The response at the beam tip, with active RMA, for different values of gain is shown in Figure 9.




(a)




(b)

**Figure 9** System response with the active RMA: (a) moderate gain and (b) high gain

It is interesting to note that, for an otherwise stable control system, by simply switching the electrical leads of the RMA, the system becomes unstable or self-excited vibration is induced (Figure 10).

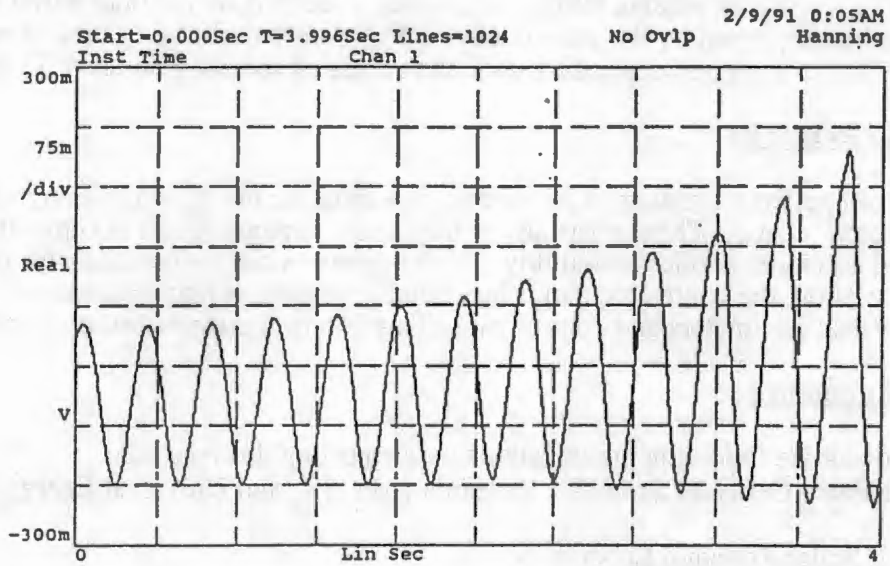
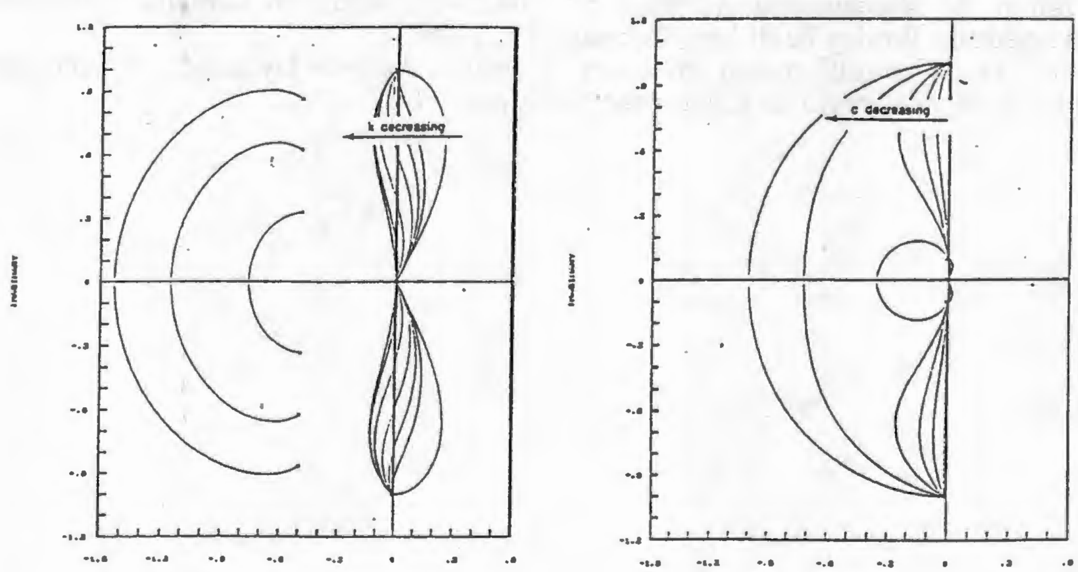



Figure 10 System response to positive feedback

The stability behavior of the controlled system, as k and c of the RMA are varied, can be seen in Figure 11.




(a)

(b)

Figure 11 Root loci: (a) decreasing k and (b) decreasing c

## Discussion

From the experimental results it can be seen that for the uncontrolled system (with dead RMA), the beam tip vibrates in excess of 45 seconds (Figure 3). The system's ability to dampen out vibration is improved by the use of passive RMA. It is about 4 seconds or 10 times faster (Figure 5). The system is further improved by the use of active RMA where the settling time is anywhere from 2 seconds to less than 1 second depending upon the values of control gain used (Figure 9).

## Concluding Remarks

Active control applied to structures provides a powerful means of suppressing vibrations, but it also incurs some "costs." These costs are mainly: more expense; more complexity in electronics, hardware and software; and less reliability. With negative velocity feedback for the configuration under consideration, the control system is less reliable because it may become unstable, for certain values of physical parameters and control gain. This fact is also discussed by Inman [5].

## Acknowledgements

We wish to thank the following organizations for supporting this research:

- US Air Force Office of Scientific Research (AFOSR) and Universal Energy Systems (UES), Inc.
- Frank J. Seiler Research Laboratory

## References

1. Shaw, O.G. and H. V. Vu, "Modal Analysis and Active Vibration Control of a System of a Cantilever Beam and a Reaction-Mass Actuator," *Proceedings of the AIAA Guidance, Navigation, and Control Conference,* 1990, pp. 73-78.
2. Steidel, R.F., *An Introduction to Mechanical Vibrations.* New York: John Wiley & Sons, 1989.
3. Den Hartog, J. P., *Mechanical Vibrations.* New York: Dover Publications, Inc., 1985
4. Wolfram, S., *Mathematica: A System for Doing Mathematics by Computer.* Redwood City, CA: Addison-Wesley Publishing Company, Inc., 1991.
5. Inman, D., "Control/Structure Interaction: Effects of Actuator Dynamics," *Proceedings of the AIAA Dynamics Specialist Conference,* 1990, pp. 311-321.