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## **Foreword**

This report was initiated by Fred W. Berner, Ph.D., Technical Advisor, and prepared in cooperation with Mr. Otto Schueller, Aerospace Medical Research Laboratory, in support of Project 7222, Combined Stress Effects in Aerospace Operations.

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This technical report has been reviewed and is approved.

C. H. KRATOCHVIL, Colonel, USAF, MC  
Commander  
Aerospace Medical Research Laboratory

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## Nomenclature and Units

<b>a</b>	Acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$
<b>a<sub>c</sub></b>	Coriolis acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$
<b>a<sub>L</sub></b>	Linear acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$
<b>a<sub>R</sub></b>	Radial acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$
<b>a<sub>T</sub></b>	Tangential acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$
<b>a<sub>t</sub></b>	Component of Coriolis acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$ due to change of tangential velocity $v_t$
<b>a<sub>u</sub></b>	Component of Coriolis acceleration in $\text{m/sec}^2$ or $\text{ft/sec}^2$ due to change of direction of velocity vector $u$
<b>F</b>	Force in kg-force (=kilopond kp) or lb
<b>F<sub>C</sub></b>	Coriolis Force in kp or lb
<b>F<sub>h</sub></b>	Resultant of precession and centripetal forces in kp or lb on mass $m$ , in horizontal fork position
<b>F<sub>l</sub></b>	Resultant of precession and centripetal forces in kp or lb on lower mass $m$ , in vertical fork position
<b>F<sub>P</sub></b>	Precession force in kp or lb
<b>F<sub>R</sub></b>	Radial (centripetal) force in kp or lb at radius $R$
<b>F<sub>r</sub></b>	Radial (centripetal) force in kp or lb at radius $r$
<b>F<sub>u</sub></b>	Resultant of precession and centripetal forces in kp or lb on upper mass $m$ , in vertical fork position
<b>g</b>	Gravitational acceleration = $9.80665 \sim 9.81 \text{ m/sec}^2$ or $= 32.1739 \sim 32.2 \text{ ft/sec}^2$
<b>I</b>	Moment of inertia or rotational inertia in $\text{kg m}^2$ or $\text{slug ft}^2$
<b>L</b>	Angular momentum in $\text{kg m}^2/\text{sec}$ or $\text{slug ft}^2/\text{sec}$
<b>m</b>	Mass element in kg mass (kg) or pound mass (=slug)
<b>M</b>	Total mass of spherical shell in kg or slug
<b>n<sub>R</sub></b>	Spin velocity in rpm around horizontal centrifuge arm $R$
<b>n<sub>V</sub></b>	Precession velocity in rpm around vertical centrifuge axis
<b>p</b>	Linear momentum in $\text{kg m/sec}$ or $\text{slug/sec}$
<b>r</b>	Spin radius of mass element $dm$ in m or ft
<b>R</b>	Radius of circular motion in m or ft
<b>s</b>	Distance in m or ft

## Nomenclature and Units (Cont.)

<b>t</b>	Time in seconds
<b>T</b>	Torque in m kp or ft lb
<b>T<sub>p</sub></b>	Precession torque in m kp or ft lb
<b>u</b>	Velocity of mass relative to rotating plane in m/sec or ft/sec
<b>u<sub>H</sub></b>	Horizontal component of spin velocity in m/sec or ft/sec
<b>u<sub>V</sub></b>	Vertical component of spin velocity in m/sec or ft/sec
<b>v</b>	Velocity in m/sec or ft/sec relative to ground
<b>v<sub>A</sub></b>	Velocity in m/sec or ft/sec of seat A relative to ground
<b>v<sub>B</sub></b>	Velocity in m/sec or ft/sec of body B relative to ground
<b>v<sub>t</sub></b>	Tangential velocity in m/sec or ft/sec
<b>W</b>	Weight in kg force (= kilopond kp)
<b><math>\alpha</math></b>	Angular acceleration in radians/sec <sup>2</sup>
<b><math>\hat{\alpha}</math></b>	Angle in radians between the radius vectors of seat A and body B
<b><math>\theta</math></b>	Angle in radians between spin radius vector r and the vertical
<b><math>\omega</math></b>	Angular velocity in radians/sec
<b><math>\omega_r</math></b>	Angular spin velocity in radians/sec around horizontal centrifuge arm R
<b><math>\omega_v</math></b>	Angular precession velocity in radians/sec around vertical centrifuge axis

### *Prefixes*

<b>d</b>	Differential
<b><math>\Delta</math></b>	Difference

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## Introduction

Periodic Coriolis and gyroscopic forces will appear on the test subjects and structure of human centrifuges, disorientation facilities, and rotating platforms and shakers, for simulating the dynamic effects occurring in aerospace vehicles and their escape systems. These periodic Coriolis and gyroscopic forces will combine with centrifugal, tangential, and oscillatory forces. The stresses caused by the vector resultant of these forces must be calculated and their effects evaluated before every experiment in view of possible hazards for the test subject as well as for the simulator structure. Careful consideration must also be given to the arrangement of the instrumentation on the proper locations. A profound understanding of all these inertia forces is, therefore, a necessity for the personnel responsible for planning and conducting human experiments with these facilities

In textbooks the Coriolis and gyroscopic effects are usually treated by abstract vector differential equations, and their physical meaning is sometimes only superficially interpreted when common language is used. However, the Coriolis force can be correctly explained in simple terms and be understood and visualized as easily as any other inertia force, such as the centrifugal force with which everybody is acquainted. The Coriolis force is named for its discoverer, the French engineer and physicist Gaspard G. Coriolis (1792-1843), who first drew attention to the deflecting force of the earth's rotation in 1835. Gyroscopic motions, in their general form are among the most difficult problems in the field of mechanics. The specific gyroscopic effects caused by the constrained motions of human centrifuges can also be explained in simple terms and be easily understood and visualized by everybody.

For a deeper understanding of the Coriolis and gyroscopic effects, it is necessary that one pay careful attention to the meaning of the terms velocity, acceleration, mass and force. The meaning of these terms in mechanics is somewhat different from their various and ambiguous meanings in everyday life. The following explanations, therefore, begin with a brief definition of these terms and a few simple basic laws of mechanics. Then they show their application to practical examples from everybody's experience, beginning with the simplest problems of rectilinear and circular motion, and progressing step by step to more complex motions which cause Coriolis and gyroscopic effects.

The simple explanations of these effects, as presented here, will not be found in textbooks. Although merely intended as an aid to new personnel working with human centrifuges, this treatise may be useful also for the application to other acceleration facilities with complex motions and to disorientation devices.

## Definitions and Basic Laws of Mechanics

### VELOCITY

Velocity is the distance traveled in a stated direction by a moving body in unit of time. In daily life it is sufficient to characterize velocity only by its average magnitude. For example, a car is driven with a velocity or speed of 60 miles per hour. In general, the velocity is not constant over a long interval of distance or time. In mechanics, therefore, the term *instantaneous velocity* is used as well as the term *average velocity*. The instantaneous velocity  $v$  is defined as the small element of distance ( $\Delta s$ ) passed in a small element of time ( $\Delta t$ ) during which the velocity can be assumed to be constant:

$$v = \lim \Delta s / \Delta t = ds/dt \text{ (read limit) measured;}$$

for example, in ft/sec or m/sec.

Stating the *magnitude* alone is not enough for a complete definition of the velocity. In addition to *magnitude*, *direction* of the velocity must be stated. The symbol for a directional magnitude is the vector; its length is a measure of the magnitude and the arrow indicates the direction. Vectors are added geometrically as every flyer knows from the determination of the travel velocity of an airplane as the resultant of the velocity of the airplane in the air and the wind velocity.

### ACCELERATION

Acceleration is the rate of change in a unit of time of velocity in *magnitude* or *direction*, or in *both*. As when talking about velocity, we have to distinguish between *average acceleration* and *instantaneous acceleration*. The instantaneous acceleration ( $a$ ) is:

$$a = \lim \Delta v / \Delta t = dv/dt$$

Acceleration is also a vector. If the velocity is measured in feet/second, the acceleration, that is, the change of velocity per second, is measured in feet/second per second, which is sometimes written as ft/sec $\cdot$ sec or ft/sec $^2$  (or m/sec $^2$  in the metric system).

### MASS

Mass in mechanics means two properties of every body: having *weight* and *inertia*. *Weight* is the force with which every body is attracted by the earth:

$$W = mg; \quad g = 32.2 \text{ ft/sec}^2 \text{ or } 9.81 \text{ m/sec}^2$$

$W$  in pounds or kg force (now called kilopond kp)

*Mass* is weight divided by the acceleration of the earth:

$$m = W/g \text{ in slugs or kg mass (now simply called kilogram kg)}$$

The mass of a body always remains the same whether the body is, for example, on the moon or on the earth. The weight of the same body on the moon, however, would be only one sixth of that on earth because of the lower mass attraction by the moon.

*Inertia* means that no body changes its velocity by itself.

*Any change of velocity in magnitude or direction requires the application of a force:*

$$F = m a \text{ in lb or kg force (now kilopond kp)}$$

This is the basic law from which all the following mass inertia effects are deduced. Force is also a vector.

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Force exertion always requires two bodies: an "acting" body, which exerts a force on another body, the "reacting" body. Therefore, forces always appear in pairs. Each of the two forces acts on the opposing body. The two forces are of equal magnitude but of opposite direction:

*action*=*reaction* (Newton)

This is true whether the bodies are at rest or moving.

Forces on resting bodies are called *static forces*. For example, when you press your hands against each other, the left hand acts on your right hand with the same force but in the opposite direction as the right hand acts or reacts on your left hand.

Forces that cause or change motion are called *dynamic forces*. For example, when you push your car on a snowy or icy surface you have to press your hands on the car and your feet firmly on the ground. Your hands act on the car with the same force but in the opposite direction as the car reacts on your hands. In the same way, your feet act against the ground with the same force, but in the opposite direction as the ground reacts on your feet. The resisting force of the car against being set in motion and accelerated consists mainly of its mass inertia reaction and a negligible amount of friction. To set the car in motion and accelerate it requires a much greater force than to keep it in motion. This leads frequently to an error of feeling, that an "unbalanced" or greater force is necessary than the resisting force of the car to set it in motion. However, any action on the car, your body, or the ground is at any instant balanced by an equal reaction on its counterpart. If you cannot find a firm place to stand and consequently slip on the icy ground, then you cannot apply the force on the car to set it in motion. On the other hand, when the car is once in motion, its mass inertia keeps it moving indefinitely at the same speed unless a counter-force, such as friction or braking, retards or stops its motion.

## STANDPOINT OF OBSERVATION

In the following examples all motions are observed from a *fixed reference point* on the *ground*. Whenever our reference point is changed, it is specifically indicated. Otherwise, great confusion would occur.

Applications of the definitions and basic laws of mechanics briefly reviewed in the foregoing section are demonstrated in the following sections, using commonplace examples. The examples may appear trivial; however, it should be remembered that our concept of force comes from our muscular sense of feeling, and our concept of velocity from our visual sense. For this reason the examples were selected according to the following concept: What we can feel and see we can understand. In the following examples assume that you are a passenger in a car, sitting relaxed at the right side of the driver.

## Linear Acceleration

### RECTILINEAR MOTION IN A PLANE WITH CONSTANT VELOCITY

If the car is driven straight ahead with constant velocity, your body is free of forces, except for its weight. It is the same as though the car were at rest. There is no *change* of velocity, neither in magnitude nor in direction. Therefore, there is no dynamic force.

### RECTILINEAR MOTION IN A PLANE WITH CONSTANT ACCELERATION (Fig. 1-A)

If the car is now accelerated with a constant acceleration  $a_L$  and continues to move straight ahead, it changes only the *magnitude* of its velocity, but not the direction. The force required to accelerate the car is produced by the motor and acts through the wheels on the ground. The force required to accelerate your body acts through the seat against your body and pushes you forward. The inertia force of your body reacts on the seat and pushes the seat backwards. You can feel these forces on your back and buttocks.

For example: If the car is accelerated with a constant acceleration in two seconds from a velocity  $v_0=20$  miles/hr to a velocity  $v_1=50$  miles/hr, the change in velocity is  $50 - 20 = 30$  miles/hr in two seconds, that is a change of 15 miles/hr per second. Since 1 mile/hr = 1.467 ft/sec, the acceleration  $a_L = \Delta v / \Delta t = 15 \times 1.467 = 22$  ft/sec $\cdot$ sec. One g-unit is 32.2 ft/sec $\cdot$ sec. The acceleration of the car, therefore, is approximately 0.68 g-units. This means that the force pushing you forward is approximately 68% of your body weight. The acceleration is in line with the velocity and in the same direction as indicated on the left side of Fig 1 by a thick arrow.

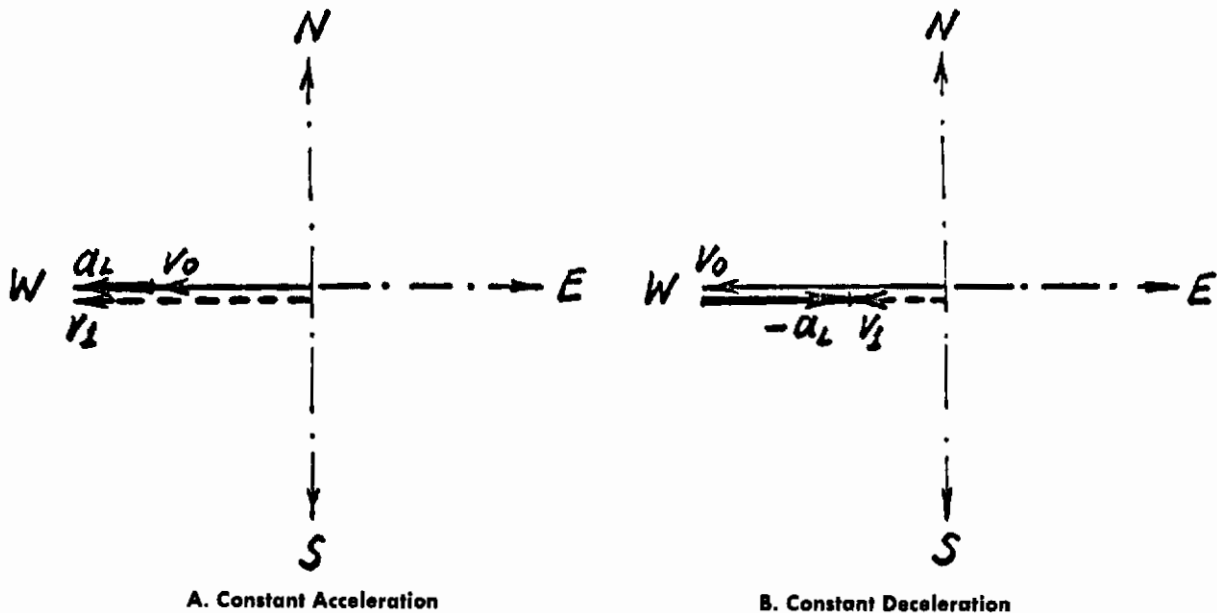


Figure 1, Rectilinear Motion in a Plane

### RECTILINEAR MOTION IN A PLANE WITH CONSTANT DECELERATION (Fig. 1-B)

When the brakes are suddenly applied and the velocity of the car is instantly reduced, your relaxed upper body, as a passenger, is in the first instant free of forces. Due to its inertia it continues to move forward with the original velocity of the car until your hands push against the glove compartment or, if you are not alert enough, your head bumps against the windshield. When you press

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your hands against the glove compartment the acceleration force acts on your hands and pushes you backwards. The inertia force of your body reacts on the glove compartment, pushing it forward. You can feel these forces again.

For example: If the car comes from a velocity  $v_0=50$  miles/hr to a full stop ( $v_1=0$ ) within 3.5 seconds, the average change in velocity  $a_L=\Delta v/\Delta t$  is 50 miles/hr divided by 3.5 seconds = 14.3 miles/hr per second. With 1 mile/hr = 1.467 ft/sec, the average deceleration  $-a_L=14.3 \times 1.467=21$  ft/sec $\cdot$ sec or about 0.65 g-units. This means your hands and feet have to press against the front with a force of about 65% of your body weight to keep your body in the seat. The distance traveled until the car comes to a stop will be equal to the average velocity multiplied by the time passed from applying the brakes to the stop. The instantaneous velocity at the beginning of the deceleration  $v_0=50$  miles/hr, that is  $50 \times 1.467=73.4$  ft/sec. The instantaneous velocity  $v_1$  at the stop is, of course, zero. The average velocity, therefore is  $\frac{1}{2} \times 73.4=36.7$  ft/sec, and the braking distance  $s=36.7$  ft/sec  $\times$  3.5 sec = 128 feet.





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also has the tendency to continue its motion in the straight direction of the instantaneous velocity of the car. The centripetal force required for accelerating your body towards the center of the turning circle acts on your body through the right door and seat and pushes you inwards. The inertia force of your body reacts on the right door and seat and pushes it outward, away from the center of the turning circle. Therefore, the reaction force of your body is called *centrifugal force*. If the door were to open you would fall out of the car (be "fugitive"). However, the instant the door gives, the centripetal force and its reaction, the centrifugal force, vanish and your body is free of forces (except, of course, its weight). You will not fall out of the car in radial direction, but, due to the inertia of your body, tangentially to the turning circle in the instantaneous direction of the velocity of the car.

The following relations can easily be seen in figure 2: The instantaneous velocity vector  $v$  changes its direction with the angular velocity  $\omega$  of the car around its turning center. The angular velocity can be considered as the circumferential velocity at a radius of unit length. The angular velocity is measured in radians/second. One radian is an angle, the arc of which is equal to its radius, that is, an angle of  $57^\circ 17' 45''$ . If we chose one foot as radius, then the length of the arc in feet is at the same time a measure of the angle in radians. If the circumferential velocity at the radius one is  $\omega$ , the circumferential velocity  $v$  at the radius  $R$  is (see fig. 2b)

$$v = \omega \times R \quad (1)$$

Since the magnitude of the circumferential velocity and the instantaneous or tangential velocity is the same, equation 1 holds also for the instantaneous or tangential velocity vector.

If the velocity *change* at the radius one is one  $\omega$ , the velocity change of the vector  $v$  is the radial acceleration  $a_R$  (see fig. 2c)

$$a_R = \omega \times v \quad (2)$$

Placing the value of  $v$  from equation 1 into equation 2 gives

$$a_R = \omega^2 \times R \quad (3)$$

Or replacing  $\omega$  in equation 2 by  $\omega = v/R$  from equation 1 gives

$$a_R = v^2/R \quad (4)$$

The centripetal and centrifugal force is

$$F_R = m a_R \quad (5)$$

wherein the force  $F_R$  is in lb or kg force (now kilopond kp), the mass  $m$  in slugs or kg mass, and the centripetal or centrifugal acceleration  $a_R$  in  $\text{ft}/\text{sec}^2$  or  $\text{m}/\text{sec}^2$ . If the acceleration  $a_R$  is expressed in gravitational units, the force  $F_R$  becomes

$$F_R = W a_R \quad (6)$$

wherein  $W$  is the weight in lb or kg (kp), and  $a_R$  in g-units.

Example: The car makes a left turn with a constant velocity of 20 miles per hour at a radius of the turning circle of 60 feet:

$$1 \text{ mile/hr} = 1.467 \text{ ft/sec}; \quad 20 \text{ miles/hr} = 29.4 \text{ ft/sec}$$

$$\text{From equation 4: } a_R = v^2/R = 29.4^2/60 = 14.4 \text{ ft/sec}^2$$

$$1 \text{ g-unit} = 32.2 \text{ ft/sec}^2, \quad 14.4/32.2 = 0.45 \text{ g-units}$$

This means your body will push against the right door and side during the turn with a force equal to 45% of your body weight. The angular velocity of the car around its turning center follows from equation 1 with

$$\omega = v/R = 29.4 \text{ ft/sec} \div 60 \text{ ft} = 0.49 \text{ radians/sec}$$

This means, if the circumferential velocity  $v$  at a radius of 60 feet is 29.4 ft/sec, the circumferential velocity at a radius of one foot will be 0.49 ft/sec. With this value of the angular velocity, the centripetal acceleration  $a_R$  follows from equation 3 again as

$$a_R = \omega^2 R = 0.49^2 \times 60 = 14.4 \text{ ft/sec}^2 \text{ as before.}$$

### CIRCULAR MOTION IN A PLANE WITH CONSTANT TANGENTIAL ACCELERATION (Fig. 3)

Let us now assume the car is accelerated with a constant tangential acceleration while in a left turn. The car changes now not only the *magnitude*, but also the *direction* of its velocity. Change of magnitude of velocity is always in line with its instantaneous direction, and change of direction is always perpendicular to its instantaneous direction. The tangential or linear acceleration force acts through the seat against your body and pushes you forward as it did during accelerated rectilinear motion. The centripetal force acts through the right door and the seat against your body, pushing you inwards. The resultant of these two forces acts from the right corner of

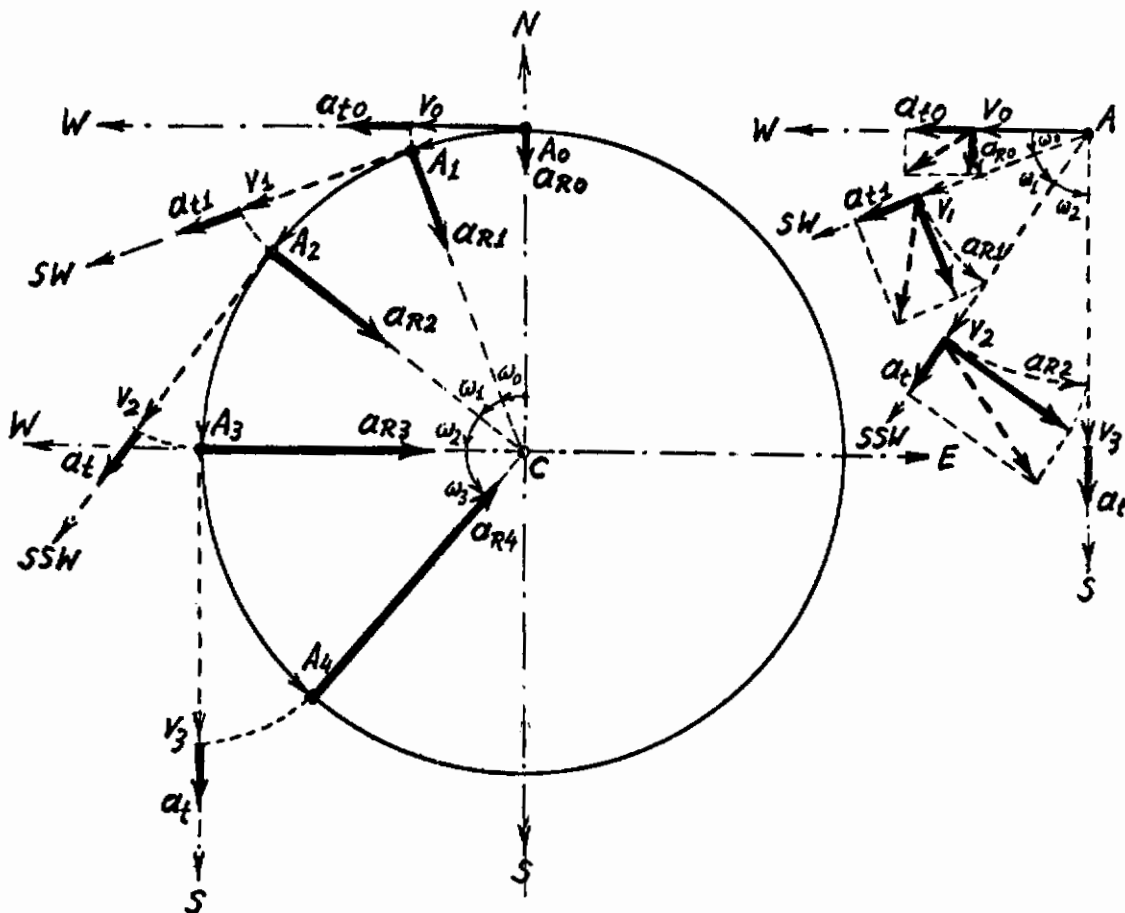


Figure 3. Circular Motion with Constant Tangential Acceleration



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the seat and door and pushes you diagonally forward to the left. The inertia forces of your body react against the tangential acceleration on the seat pushing it backward, and against the radial acceleration as centrifugal force on the right door pushing it outwards. The resultant inertia force of your body reacts on the right corner of your seat and the right door, pushing them diagonally backwards and outwards.

For example: As in the previous example, the car makes a left turn at a radius of 60 feet of the turning circle, and is now accelerated with a constant acceleration from a velocity  $v_0=20$  miles/hr to a velocity  $v_1=30$  miles/hr in 1.5 seconds.

$$1 \text{ mile/hr} = 1.467 \text{ ft/sec} \quad 20 \text{ miles/hr} = 29.4 \text{ ft/sec}$$

$$25 \text{ miles/hr} = 36.7 \text{ ft/sec}$$

$$30 \text{ miles/hr} = 44.0 \text{ ft/sec}$$

The change of the tangential velocity  $\Delta v = 44.0 - 29.4 = 14.6$  ft/sec in 1.5 seconds. The average tangential acceleration

$$a_T = \Delta v / \Delta t = 14.6 \text{ ft/sec} : 1.5 \text{ sec} = 9.7 \text{ ft/sec}^2$$

$$1 \text{ g-unit} = 32.2 \text{ ft/sec}^2 : 9.7 / 32.2 = 0.3 \text{ g-units} = \text{constant}$$

The centripetal acceleration  $a_R = v^2 / R$

$$\text{at } 20 \text{ miles/hr} = 29.4 \text{ ft/sec} \quad a_R = 29.4^2 / 60 = 14.4 \text{ ft/sec}^2 = 0.45 \text{ g-units}$$

$$\text{at } 25 \text{ miles/hr} = 36.7 \text{ ft/sec} \quad a_R = 36.7^2 / 60 = 22.4 \text{ ft/sec}^2 = 0.7 \text{ g-units}$$

$$\text{at } 30 \text{ miles/hr} = 44.0 \text{ ft/sec} \quad a_R = 44.0^2 / 60 = 32.3 \text{ ft/sec}^2 = 1.0 \text{ g-unit}$$

The tangential or linear acceleration remains constant during the turn and pushes you forwards with 0.3 g-units, that is, with a force of 30% of your body weight. The centripetal force, however, increases with the square of the tangential velocity from 0.45 g at 20 miles/hr to 0.7 g at 25 miles/hr, and would increase to 1.0 g at 30 miles/hr if the car were still in the turn. One g is approximately the utmost limit of acceleration and deceleration applicable under normal road and tire conditions at which the car will begin to skid or roll.

## Coriolis Acceleration and Force

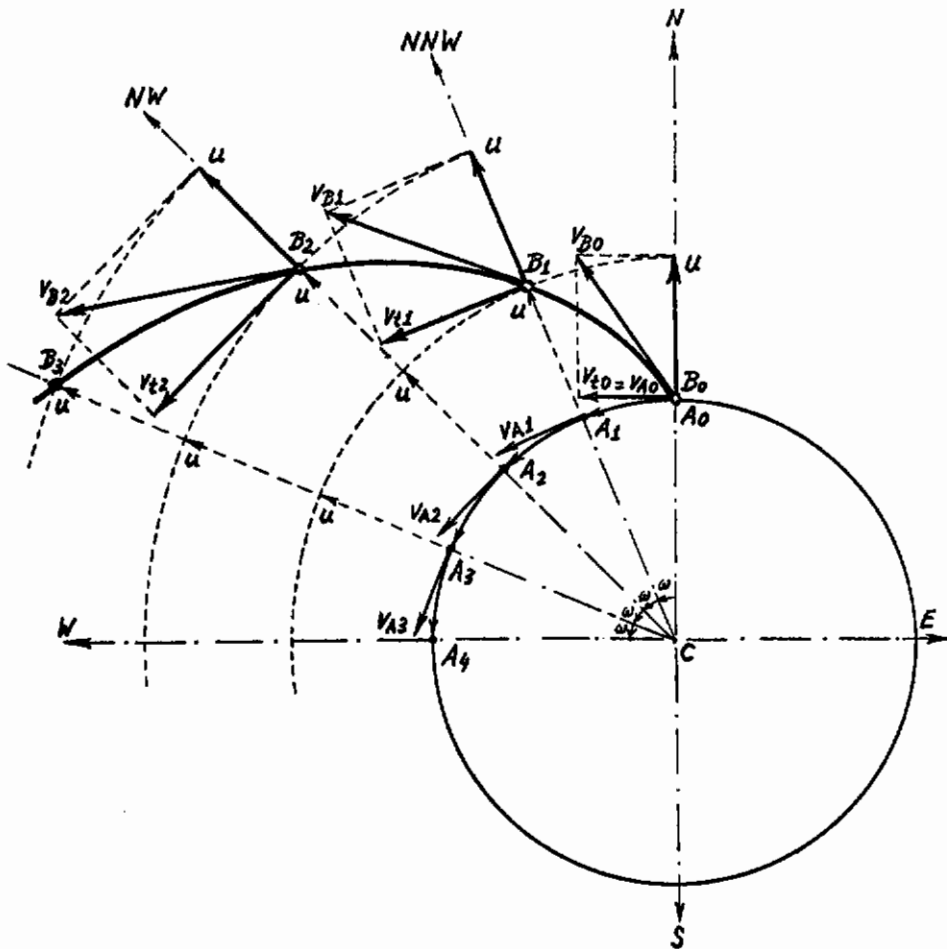
Let us now assume you move your head or body with constant velocity forward, backward, or sideward (not up and downward) with respect to your seat while the car makes a left turn with constant velocity. Your body motions, of course, make the turn with the car as everything else in the car. The velocity of your body motion constantly changes direction with respect to the ground with the car. However, the velocity of your body motion changes not only its direction but also its magnitude with respect to the ground. For example, when you move your head or body straight forward towards the windshield, your head or body moves in this instant somewhat ahead of the car. Your body has a somewhat higher tangential velocity than the car and therefore requires a somewhat higher centripetal acceleration than the car. When you move your body sideward towards the right door you move it to a somewhat greater radius of the turning circle where again the tangential velocity is somewhat higher. When you move your body backwards, you are moving somewhat behind the car with respect to the ground and decrease your tangential velocity. When you move to the left, you move to a smaller radius of the turning circle where the tangential velocity is lower. In whatever direction you move, you add to the constant centripetal acceleration of your body due to the turn of the car, an additional change of velocity in magnitude and direction due to your own body motion. This additional change of velocity due to your own body motion within the turning car is called *Coriolis acceleration*, and the force required to produce it, and its reaction force, are called *Coriolis forces*.

The Coriolis effect appears at first very strange. When you move something on a rotating platform, it seems to be pushed sideward. For example, if you want to move your body straight ahead towards the windshield it seems to be pushed sideward to the right during a left turn. You have to push it to the left to get it straight ahead toward the windshield. But this effect appears quite natural if you consider your motion with respect to the ground. Due to its mass inertia your body has the tendency to move straight ahead in the instantaneous tangential direction of the car. The car makes the turn underneath your body to the left. Hence, to change the direction of your body motion together with the car you have to apply the necessary centripetal force now with your muscles, which otherwise the seat and right door applies to your body when you are sitting still.

A frequent misunderstanding is that the Coriolis effect occurs only during motion in radial direction. Figures 4 through 7 illustrate that the Coriolis effect occurs during motion in any direction in the plane of rotation. When moving during a turn in the car your body moves on the first part of a spiral with respect to the ground. For clarity, the motion is extended in the figures far out on a rotating platform or merry-go-round.

### RADIAL MOTION ON A ROTATING PLATFORM

Figure 4 illustrates the change of velocity  $v_B$  of your body with respect to the ground during radial motion. At time zero your body  $B_0$  begins to move from the seat  $A_0$  with a constant velocity  $u$  with respect to the platform on a straight line radially away from its center of rotation. The platform rotates in a left turn with a constant angular velocity  $\omega$ , the seat  $A$  with a constant circumferential or tangential velocity  $v_A$ . After one second the seat has moved on a circle from  $A_0$  to  $A_1$ . Your body has simultaneously moved on the platform in a radial direction away from the seat by the distance  $u$  to  $B_1$ . After two seconds the seat is at  $A_2$  and your body has moved by a distance of two  $u$  to  $B_2$  in radial direction away from the seat, and so on. At time zero the velocity vector  $u$  of your body motion and the radius vector of the seat  $A_0$  show to the North. After one second both have changed their direction by the angle  $\omega$  to NNW, after two seconds to NW, and so on. At time



**Figure 4. Radial Motion on Rotating Platform**

zero the tangential velocity  $v_{t0}$  of your body was the same as the tangential velocity  $v_A$  of the seat  $A_0$ . After one second the tangential velocity of your body has increased proportionally to the greater radius from  $v_{t0}$  to  $v_{t1}$  after two seconds to  $v_{t2}$ , and so on. The resultant  $v_B$  of the radial velocity  $u$  and the tangential velocity  $v_t$  of your body changes during the same time intervals from  $v_{B0}$  to  $v_{B1}$ ,  $v_{B2}$ , and so on, in magnitude as well as in direction. While you are moving on a straight line in radial direction on the platform, an outside observer from a fixed standpoint sees you moving in a spiral and the seat in a circle with respect to the ground. The resultant  $v_B$  of your body velocity is at any point tangential to the spiral shaped path of your motion.

### TANGENTIAL MOTION ON A ROTATING PLATFORM

Figure 5 illustrates the change of your body velocity  $v_B$  during tangential motion. The radius vector and, with this, the tangential velocity  $v_t$  of your body increase now slower than during radial motion. However, the velocity  $u$  of your body motion is now added to the tangential velocity  $v_A$  of the seat. Therefore, you are moving ahead of the radius vector of the seat. For example, after 3 seconds your seat is at  $A_3$  on the circle, while your body is already ahead by the angle  $\hat{\alpha}$  at  $B_3$ .

# Contrails

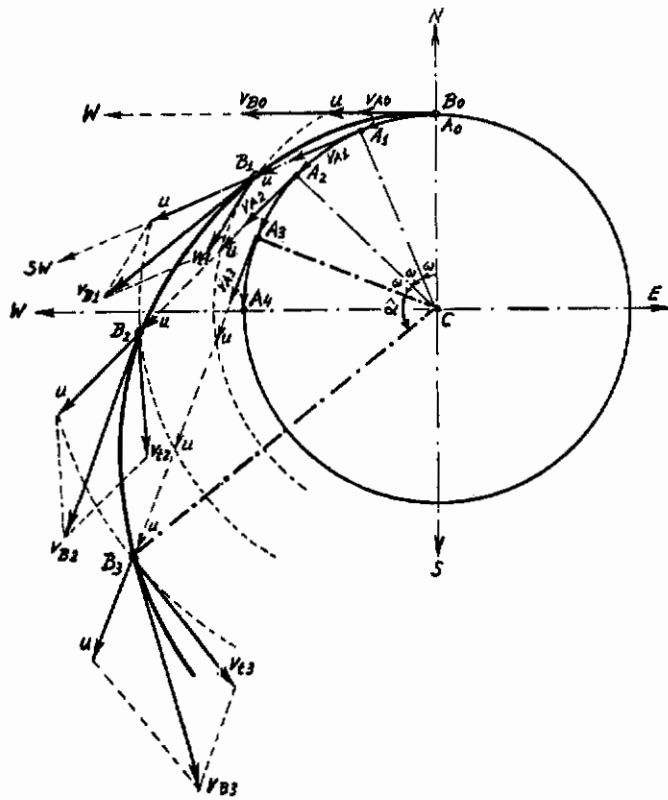


Figure 5. Tangential Motion on Rotating Platform

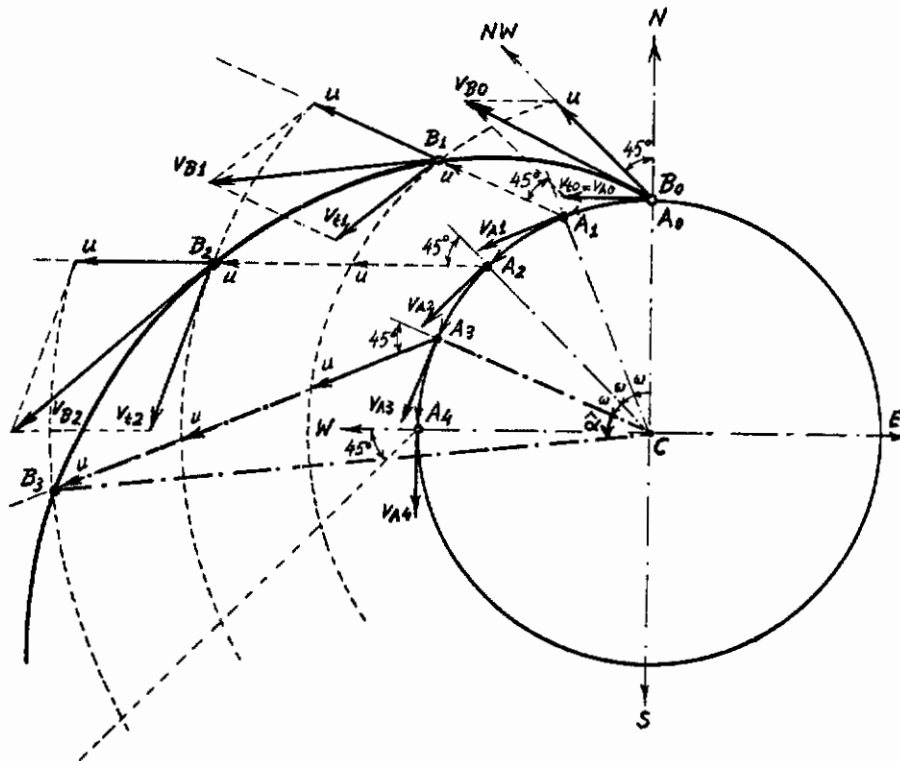


Figure 6. Motion in General Direction on Rotating Platform  
(For Example at 45° to Radius)

## MOTION IN A GENERAL DIRECTION ON A ROTATING PLATFORM

Figure 6 illustrates the change of your body velocity  $v_B$  during motion in a general direction within the plane of rotation, for example at  $45^\circ$  to the radius vector. The increase of the radius vector and tangential velocity  $v_t$  of your body lies now between that during radial and tangential motion, and so does the angle  $\hat{\alpha}$  of your advanced motion with respect to the seat.

Figure 7 shows for comparison the path and velocity change of your body with respect to the ground during motion in radial, tangential, and general direction (e.g.,  $45^\circ$ ) within the plane of rotation.

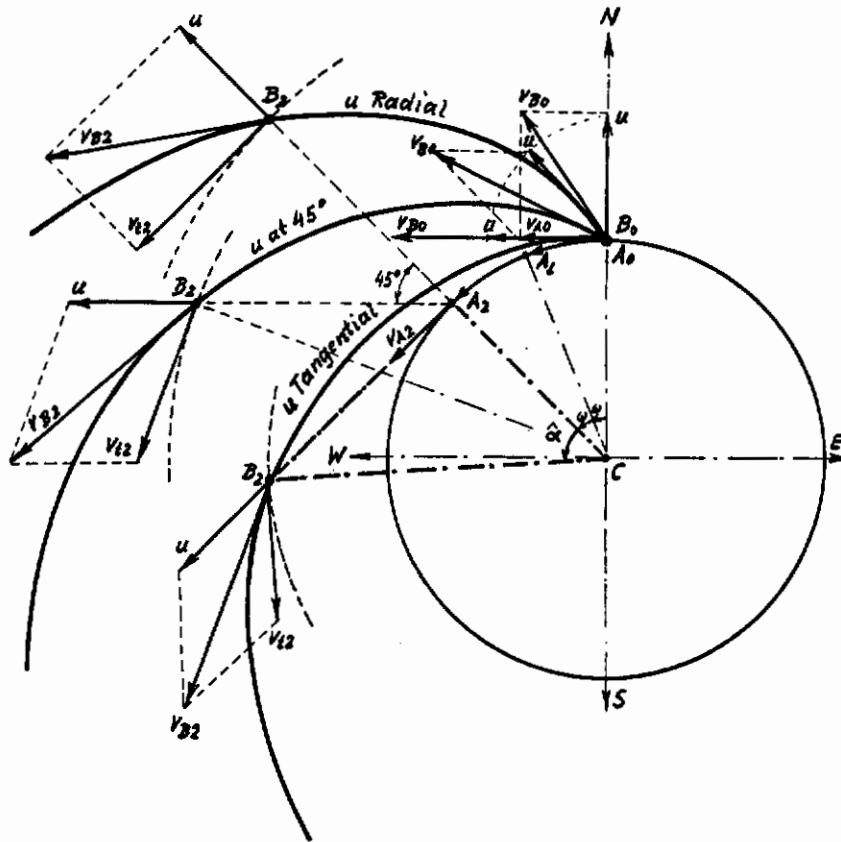


Figure 7. Motion on Rotating Platform in Various Directions

## DERIVATION OF EQUATIONS FOR CORIOLIS ACCELERATION AND FORCE

The following paragraph and figures 8 through 10 show that the change of the velocity  $v_B$  in magnitude and direction with respect to the ground is the same regardless of the direction of the motion in the plane of rotation.

Figure 8 illustrates the two components of Coriolis acceleration during radial motion, and shows their calculation by a simple geometric consideration.

The first component of Coriolis acceleration consists of the *change in direction* of the velocity vector  $u$  of your body motion:

$$\Delta u / \Delta t = a_{c1} = \omega \times u$$

# Coriails

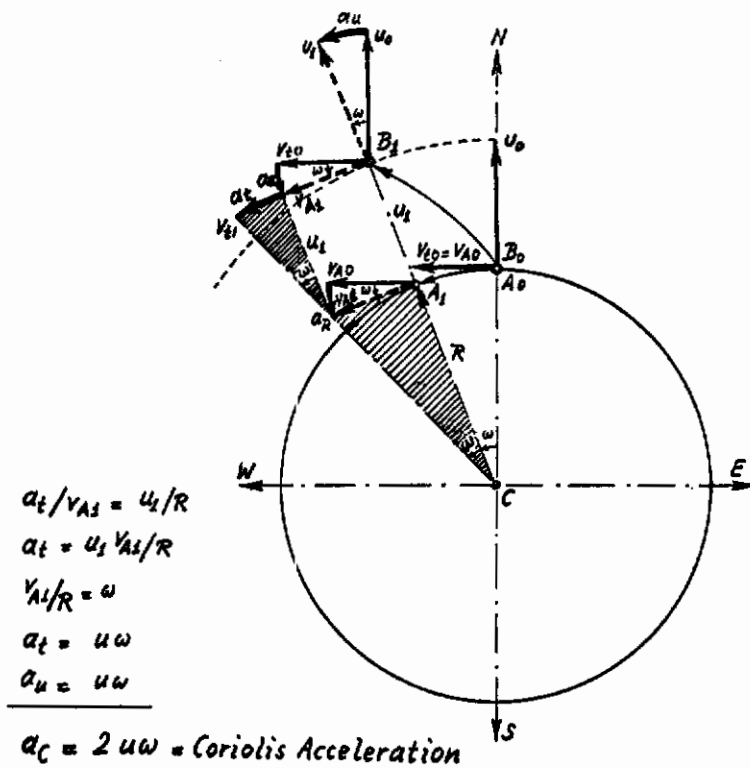


Figure 8. Coriolis Acceleration During Radial Motion on Rotating Platform

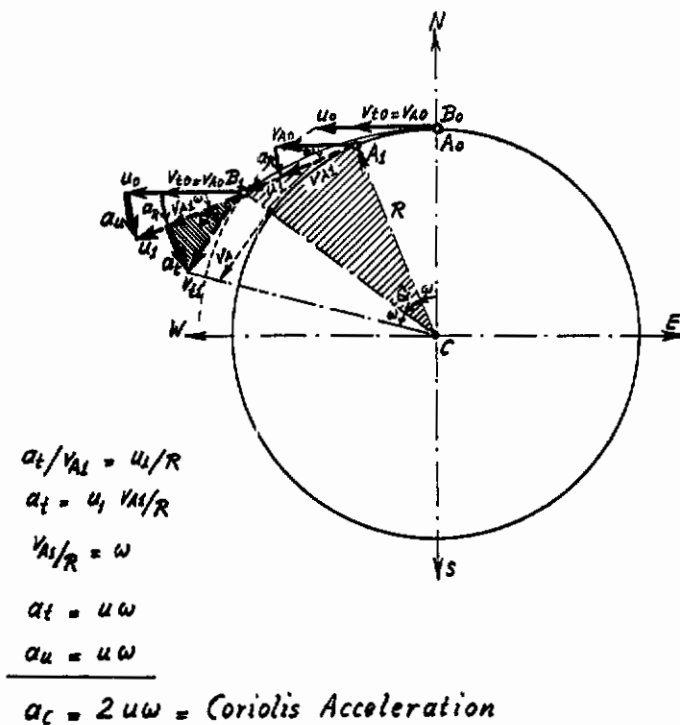


Figure 9. Coriolis Acceleration During Tangential Motion on Rotating Platform



# Contrails

The second component consists of a *change in magnitude* and *direction* of the *tangential* velocity  $v_t$  of your body motion. The shaded triangles in figure 8 are similar, thus their sides are proportional as follows:

$$\Delta v_t \Delta t = a_t = \omega \times u$$

The direction of both components is the same and is perpendicular to the velocity vector  $u$ . The total change of the velocity  $v_B$  of your body motion in addition to the centripetal acceleration  $a_R$ , that is, the Coriolis acceleration  $a_C$ , therefore, is the sum of  $a_n$  and  $a_t$ :

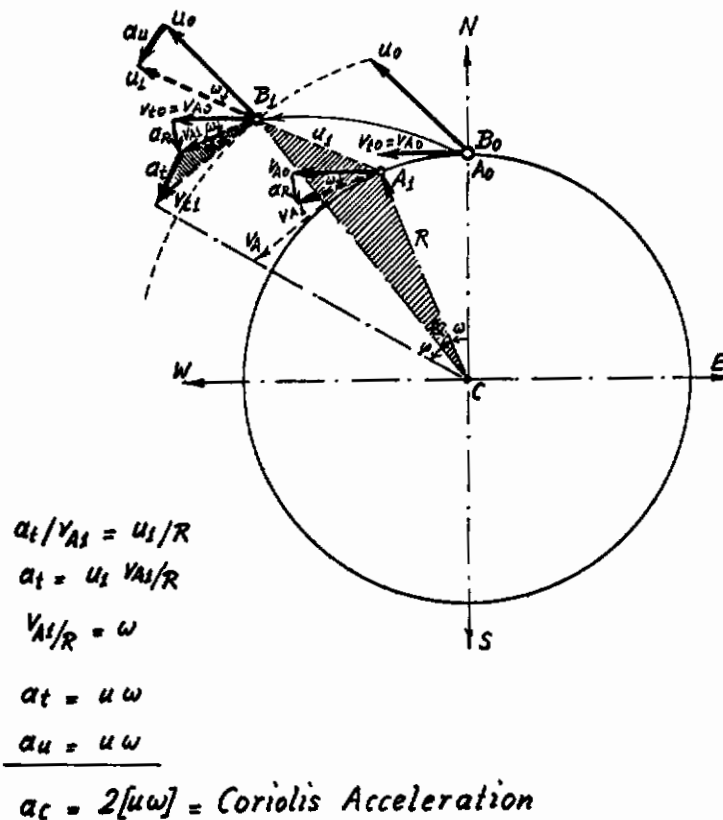
$$\Delta v_B / \Delta t = a_C = a_n + a_t = 2(\omega \times u) \tag{7}$$

The Coriolis force  $F_C$ , of course, is

$$F_C = 2m(\omega \times u) \tag{8}$$

Figures 9 and 10 illustrate the same relations for motion in tangential and in general direction within the plane of rotation.

The Coriolis force and its inertia reaction force always act perpendicular to the direction of the velocity vector  $u$  of your body motion, as the centripetal and centrifugal force always act perpendicular to the direction of the velocity vector  $v$  of the instantaneous velocity of the car during



**Figure 10. Coriolis Acceleration During Motion in General Direction on Rotating Platform**

a turn. You can find the direction of the required Coriolis force by turning the velocity vector  $u$  of your body motion  $90^\circ$  in the sense of the direction of the rotating system. The reaction force to the applied Coriolis force due to the mass inertia of your body reacts in the opposite direction, as the centrifugal force reacts in the opposite direction to the centripetal force. For example: As in the example on page 7 the car makes a left turn with a constant velocity of 20 miles per hour at a radius of the turning circle of 60 feet. You make a quick body movement toward the windshield of about 1.5 feet in a tenth of a second, that would be a velocity of  $u = 1.5 \text{ feet}/0.1 \text{ sec} = 15 \text{ ft/sec}$ . The tangential velocity  $v_A$  of the car and your seat is

$$20 \text{ miles/hr} \times 1.467 \text{ ft/sec per mile/hr} = 29.4 \text{ ft/sec}$$

The angular velocity of the car around its turning center is

$$\omega = v_A/R = 29.4 \text{ ft/sec}/60 \text{ ft} = 0.49 \text{ radians/sec}$$

Hence the Coriolis acceleration  $a_c$  becomes

$$a_c = 2\omega \times u = 2 \times 0.49 \text{ radians/sec} \times 15 \text{ ft/sec} = 14.7 \text{ ft/sec}^2$$

$$1 \text{ g-unit} = 32.2 \text{ ft/sec}^2 \quad a_c = 14.7/32.2 = 0.46 \text{ g-units}$$

This means, in addition to the centripetal force, you have to push your upper body with a force of 46% of its weight to the left simultaneously with your forward motion, otherwise it would move to the right against the right door due to its mass inertia. In other words, if you were not to apply the Coriolis force and, of course, the centripetal force, your body would move straight ahead in the instantaneous direction tangential to the turning circle, and the car would turn underneath your body until the right door hits your body and pushes it into the turn.



## Gyroscopic Effects

In the previous sections we saw how centrifugal and Coriolis forces arise as an effect of inertia, inertia being the resistance of the velocity vector of a mass against any change, either in its magnitude or in its direction, or in both. In this section we demonstrate how inertia also affects the two fundamental characteristics of the gyroscope, its directional stability and precession.

### DIRECTIONAL STABILITY

On a spinning wheel each mass particle must continuously change the direction of its velocity vector within the plane of rotation and is therefore subjected to a radial force, the centripetal force. The centripetal forces are transferred to the mass particles by internal stresses in the wheel. When we tilt the axis of the spinning wheel we change the direction of the plane of rotation and with it the instantaneous direction of the velocity vectors of the mass particles. Tilting of a spinning wheel, therefore, requires the application of an external force or torque, respectively, which can be many times greater than the force or torque required to tilt the wheel when it is at rest. As we know that no mass can change its velocity vector by itself, a fast spinning wheel, therefore, maintains the orientation of its spin-axis in space unless an external force is applied. The French physicist and astronomer Jean B. L. Foucault (1819-1868) demonstrated in 1851 the rotation of the earth with his pendulum (as can be seen in the Smithsonian Institution in Washington, DC), A year later he demonstrated it again with a gyroscope. The gyro-rotor was carried in gimbal rings so that its axis had freedom to assume any direction in space. The vertical gimbal ring was suspended by a wire to reduce friction to a minimum. Because of the gyroscopic inertia the rotor maintained the direction of its axle and of the vertical gimbal ring stabilized in space, while the base of the instrument and a scale revolved slowly with the earth. By a long pointer attached to the vertical gimbal ring one could view the revolution of the earth proceeding with the scale. Therefore, he called the instrument *gyroscope*, from the Greek words *gyros*=revolution and *skopein*=to view.

A special application of the gyroscope, using its stabilizing effects is sometimes called *gyrostat* (from the Greek word *statikos*=stationary), used for example as a ship or monorail stabilizer. Because of its directional stability and the exactly predictable precession effect, which are explained in the following paragraph, the gyroscope has found many applications in flight instruments, compass, autopilot and guidance systems during the recent decades.

### PRECESSION FORCE

Any rotating or spinning mass has gyroscopic properties. It is not necessary that the mass be balanced or symmetrical around the spin-axis. Following is a discussion of the precession forces on a human centrifuge, shown schematically in figures 11 to 18. Precession is the angular motion of the spin-axis as the effect of an applied force or torque, respectively. In figures 11 to 18 the spherical cabin shell and its supporting fork is spinning around the horizontal axis X-X with a constant angular spin velocity  $\omega_R$ , while simultaneously the horizontal spin axis X-X is rotating or *precessing* around the vertical centrifuge axis Z-Z with a constant angular precession velocity  $\omega_V$ . We will investigate the precession effect by two different methods. First, by an analytical method on a single small mass at the ends of the fork, and then by the usual axiomatic method on the spherical cabin shell deducing the effect from the axiom of "conservation of angular momentum." Finally we will see that both methods lead to the same result on the spherical shell.

### PRECESSION EFFECT ON A SINGLE SMALL MASS $dm$

First, consider a single small mass  $dm$  at the upper fork end the instant the fork is passing through the vertical position (Fig. 11). In this instant the vector  $u$  of the spin velocity of the mass

particle around the horizontal axis X-X coincides with the plane of rotation or precession of the spin axis around the vertical centrifuge axis Z-Z. Hence, the spin velocity vector  $u$  of the mass particle  $dm$  at the upper fork end is in this instant added to the circumferential velocity of the mass around the centrifuge axis and is at the same time changing its direction with the angular precession velocity  $\omega_v$ . In other words, the mass particle  $dm$  this instant is in a tangential motion with the velocity  $u$  to the circle of rotation around the precession axis Z-Z. As we know from the previous section, any motion in any direction within the plane of rotation of a mass  $m$  requires the application of a Coriolis force  $F_C$  according to equation 8 on page 15 (see also fig. 9):

$$F_C = 2m(\omega \times u)$$

Hence the Coriolis force required for the single mass particle  $dm$  at the upper fork end -- which we call  $dF_p$  -- in this instant is

$$dF_p = 2 dm (\omega_v \times u) \tag{9}$$

The Coriolis force  $dF_p$  required for the single mass particle  $dm$  at the lower fork has the same magnitude, but opposite direction because the spin velocity vector  $u$  of this mass particle must be subtracted from the circumferential velocity around the centrifuge axis. As we know we can find the direction of the required Coriolis force by turning the velocity vector  $u$  in the sense of the angular velocity  $\omega_v$ . The vector  $dF_p$  of the mass  $dm$  at the upper fork end points towards the precession axis Z-Z, and that of the mass  $dm$  at the lower fork end points away from it. Each of the two mass particles  $dm$  will require the application of a precession torque  $dT_p$

$$dT_p = r \times dF_p \tag{10}$$

around an axis Y-Y which is perpendicular to both the spin axis X-X and the precession axis Z-Z (see figure 11).

Let us now assume that after one or more revolutions of the spin axis around the centrifuge axis the fork passes through a position under an angle  $\theta$  with the vertical as shown in figure 12. In this instance only the horizontal component

$$u_H = u \cos\theta$$

of the spin velocity vector  $u$  coincides with the plane of the precessional rotation and changes its direction with the angular precession velocity  $\omega_v$ , therefore producing a Coriolis Force

$$dF_p = 2 dm (\omega_v \times u_H) = 2 dm (\omega \times u \cos\theta) \tag{11}$$

Each of the two mass particles  $dm$  again requires the application of a precession torque  $dT_p$ :

$$dT_p = r \times dF_p$$

but now in a plane inclined by the angle  $\theta$  against the vertical (see figure 12). The vertical component  $u_v$  of the spin velocity vector does not change its direction and, therefore, will not produce any Coriolis effect.

Let us finally assume that after one or more additional revolutions of the spin axis around the centrifuge axis, the fork is now passing through its horizontal position as shown in figure 13. In this instant the spin velocity vectors  $u$  of the two mass particles  $dm$  at the fork ends are pointing vertically upwards and downwards. As we know, motion perpendicular to the plane of rotation or parallel to the axis of rotation does not produce any Coriolis effect because the velocity vectors do not change their direction in this instant.

# Contrails

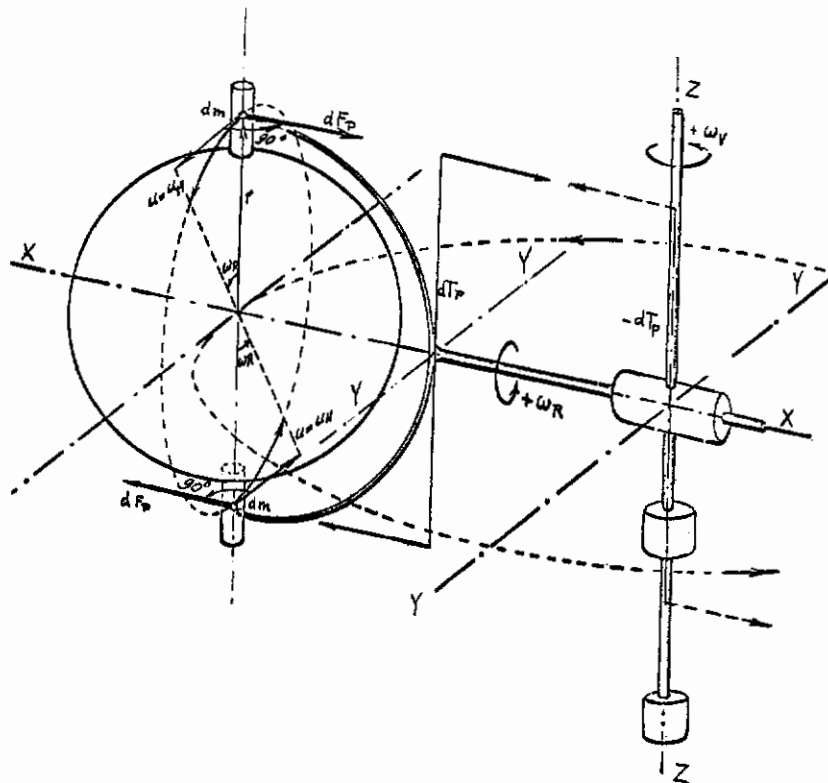


Figure 11. Precession Force on Single Mass, Centrifuge Fork Vertical

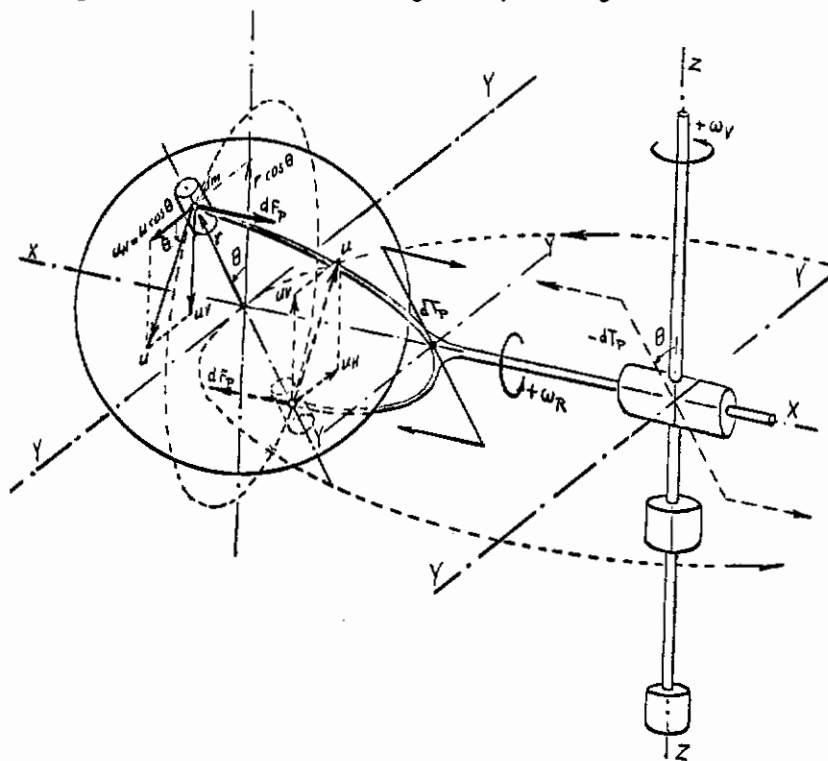


Figure 12. Precession Force on Single Mass, Centrifuge Fork Inclined

In summary, we have shown that a single mass particle  $dm$  spinning with a constant angular velocity  $\omega_R$  around an axis X-X, which is simultaneously rotating or "precessing" with a constant angular velocity  $\omega_V$  around an axis Z-Z perpendicular to the spin axis, is subjected to an oscillating Coriolis force  $dF_p$  which follows a cosine law according to equation 11

$$dF_p = 2 dm (\omega_V \times u \cos\theta)$$

With  $u = \omega_R \times r$ , equation 11 becomes

$$dF_p = 2 dm r \cos\theta \omega_V \omega_R \quad (13)$$

Each mass particle produces an oscillating torque

$$dT_p = r \times dF_p \quad (14)$$

which gives with  $dF_p$  from equation 13

$$dT_p = 2 dm r^2 \cos\theta \omega_V \omega_R \quad (15)$$

These oscillating forces or torques, respectively, must be applied to effect the "precession" of the spin axis and are therefore called *precession forces* or *precession torques*. Their inertia reactions, indicated in figures 11 and 12 in dashed lines *resist* the precession and, therefore, are called *gyroscopic resistance* or *gyroscopic torque*.

An example for illustration:

Consider a relatively small mass of 1 kg weight (now called 1 kilopond = 2.2 pounds) at each fork end, concentrated at a radius  $r = 2.1$  m (about 7 feet) from the spin axis X-X (fig. 14). The centrifuge radius from the center of the cabin to the centrifuge axis Z-Z is  $R = 5.8$  m (19 feet). The fork with the cabin is spinning with  $n_R = 30$  rpm, and the centrifuge rotating (precessing) with  $n_V = 55$  rpm.

The precession force  $F_p$  of each single weight will be according to equation 13

$$F_p = 2 m r \cos\theta \omega_V \omega_R$$

The maximum of the precession force,  $F_{p \max}$ , occurs when the spin velocity vector  $u$  coincides with the plane of precession, that is, when the fork is passing through the vertical position as shown in figures 11 and 14. In this instant the angle  $\theta = 0$  and  $\cos\theta = 1$ , hence

$$F_{p \max} = 2 m r \omega_V \omega_R \quad (16)$$

The mass of 1 kilopond (2.2 lb) weight is

$$m = 1 \text{ kp/g} = 1 \text{ kp}/9.81 \text{ m/sec}^2 = 0.102 \text{ kg (mass)}$$

The angular precession velocity

$$\omega_V = 2\pi n_V/60 = \pi n_V/30 = 3.14 \times 55/30 = 5.76 \text{ radians/sec}$$

The angular spin velocity

$$\omega_R = 2\pi n_R/60 = \pi n_R/30 = 3.14 \times 30/30 = 3.14 \text{ radians/sec}$$

Hence, the maximal precession force

$F_{p \max} = 2 \times 0.102 \text{ kg} \times 2.1 \text{ m} \times 5.76 \text{ rad/sec} \times 3.14 \text{ rad/sec} = 7.76 \text{ kp (17 lb)}$ , that is, almost eight times the weight of the mass.



The oscillating Coriolis force is superimposed over the centripetal force  $F_R$  around the centrifuge axis Z-Z, and a further centripetal force  $F_r$  around the fork axis X-X.

The centripetal force  $F_R$  around the centrifuge axis Z-Z in the vertical position of the fork is according to equations 3 and 5.

$F_R = m a_R = m \omega v^2 \times R = 0.102 \times 5.76^2 \times 5.8 = 20 \text{ kp (44 lb)}$ ,  
that is, twenty times the weight of the mass.

The centripetal force  $F_r$  around the fork axis X-X is

$F_r = m \omega_R^2 \times r = 0.102 \times 3.14^2 \times 2.1 = 2.1 \text{ kp (4.6 lb)}$ ,  
that is, about twice the weight of the mass.

The vectors of these three forces must be added geometrically as shown in figure 14. On the weight in the upper position the precession force  $F_{p \text{ max}}$  and the centripetal force  $F_R$  act in the same direction radially towards the centrifuge axis Z-Z and must be added. On the weight in the lower position the precession force  $F_{p \text{ max}}$  acts in radial direction away from the centrifuge axis and must be subtracted from the centripetal force  $F_R$ . The centripetal forces  $F_r$  act radially towards the fork axis X-X and perpendicular to the other two forces  $F_{p \text{ max}}$  and  $F_R$ .

The resultant force  $F_u$  on the upper weight is

$$F_u = \sqrt{(F_{p \text{ max}} + F_R)^2 + F_r^2} = \sqrt{(7.76 + 20)^2 + 2.1^2} = 27.8 \text{ kp (61 lb)}$$

The resultant force  $F_l$  on the lower weight is

$$F_l = \sqrt{(F_R - F_{p \text{ max}})^2 + F_r^2} = \sqrt{(20 - 7.76)^2 + 2.1^2} = 12.4 \text{ kp (27 lb)}$$

In the horizontal position of the fork (fig. 15) the precession force  $F_p$  is zero. The centripetal force  $F_R$  fluctuates somewhat because of the varying radius R between the masses m and the centrifuge axis Z-Z. The radius R and with it the centripetal force  $F_R$  will have their maximum in the horizontal position of the fork:

$$R_{\text{max}} = \sqrt{R^2 + r^2} = \sqrt{5.8^2 + 2.1^2} = 6.18 \text{ m}$$

The maximal centripetal force  $F_{R \text{ max}}$  therefore is

$$F_{R \text{ max}} = 20 \times 6.18 / 5.8 = 21.3 \text{ kp (47 lb)}$$

The centripetal force  $F_r$  around the spin axis X-X is constant since the radius r does not vary.

$$F_r = 2.1 \text{ kp (4.6 lb)} = \text{constant}$$

The resultant force  $F_h$  in the horizontal position of the fork will be the same on both weights

$$F_h = \sqrt{F_{R \text{ max}}^2 + F_r^2} = \sqrt{21.3^2 + 2.1^2} = 21.4 \text{ kp (47.2 lb)}$$

- Each of the two weights will produce an oscillating precession torque according to equation 15.

$$T_p = 2m r^2 \cos \theta \omega v \omega_R$$

The maximal torque produced by both weights or masses m together occurs in the vertical position of the fork and is

$$T_{\text{max}} = 2 F_{p \text{ max}} \times r = 2 \times 7.76 \times 2.1 = 32.6 \text{ m kp (236 ft lb)}$$

# Contraails

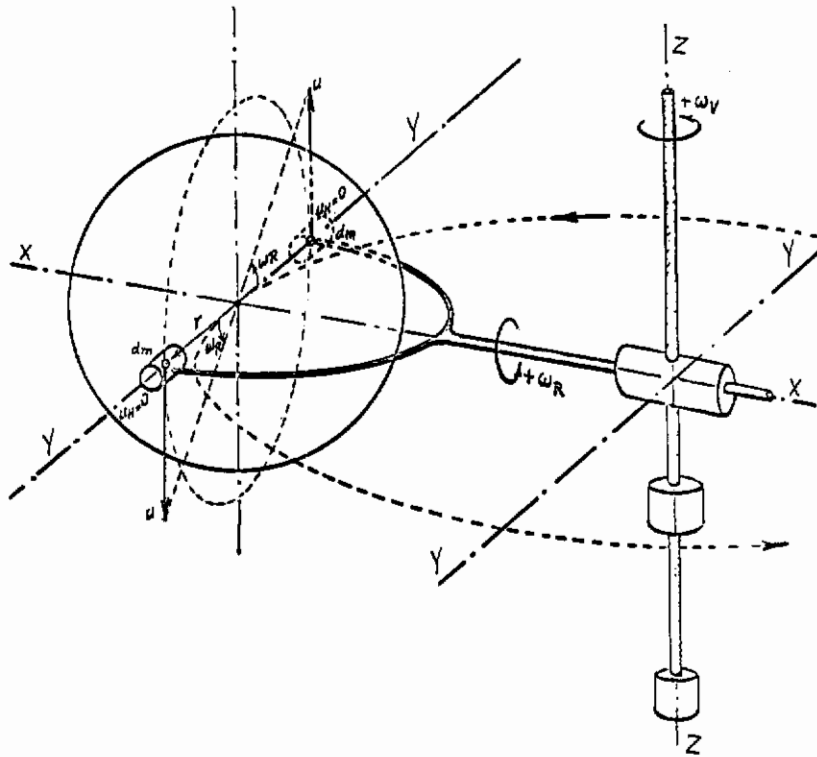


Figure 13. Precession Force on Single Mass, Centrifuge Fork Horizontal

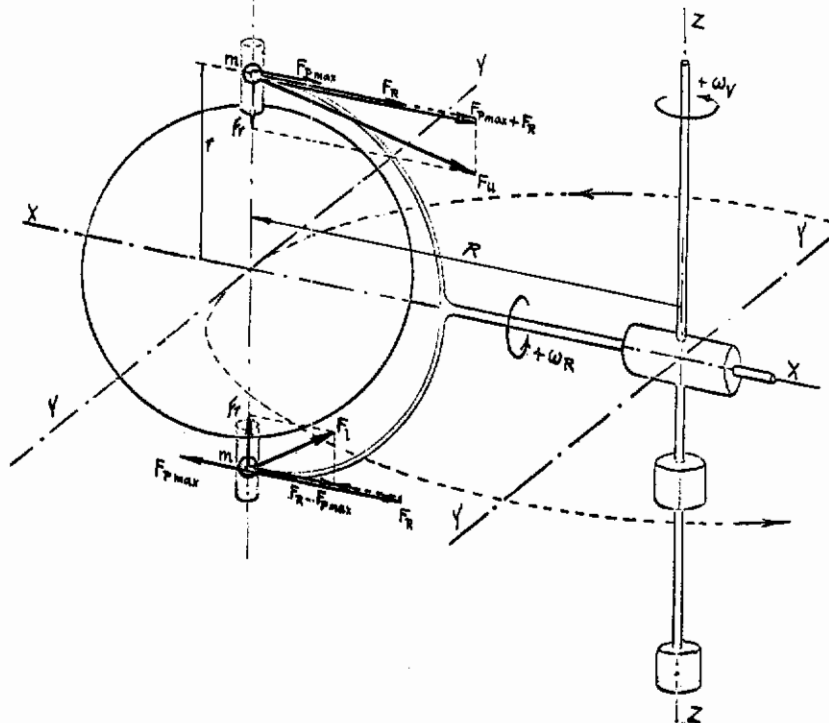


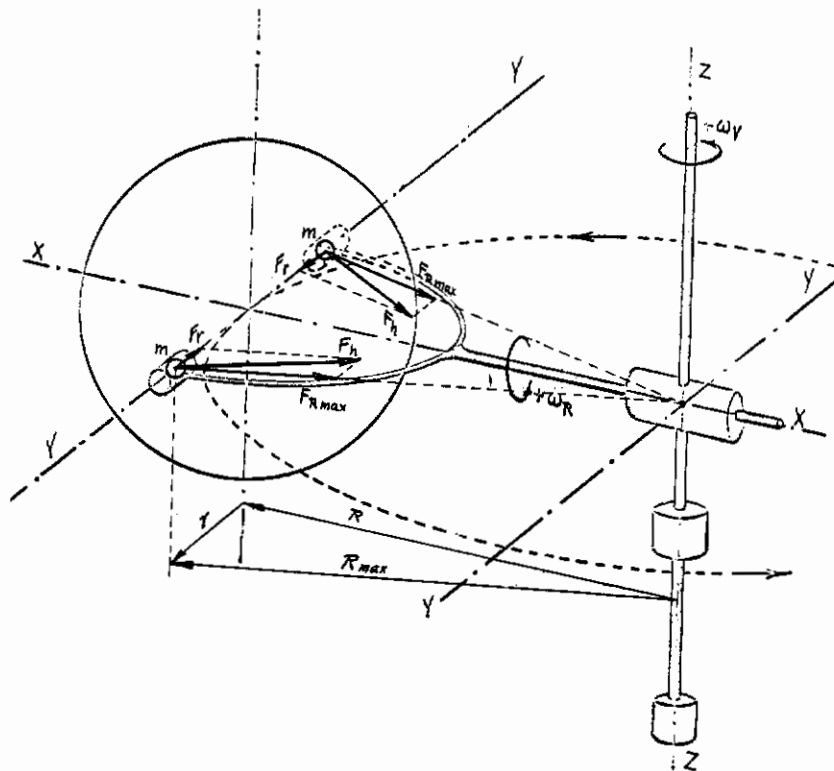
Figure 14. Resultant of Precession and Centripetal Forces on Single Mass, Centrifuge Fork Vertical

This oscillating torque acts on the fork and must be transferred over the centrifuge axis and bearings to the base structure and finally to the foundation.

The contribution of the weight of the masses to the resultant force is negligible in the above examples and has not been considered to avoid unnecessarily complicating the problem. For completeness, however, note that the vector of the weight should be added geometrically to the vectors of the reaction forces, that is, the *centrifugal* and Coriolis reaction force, and not to the vectors of the external forces, namely the centripetal and external Coriolis force. The weight is a force exerted by the mass to its supporting structure as the centrifugal and the Coriolis reaction force, while the centripetal and external Coriolis force act inversely by the structure on the mass, forcing it into its curved path.

In summary, we have shown that any single mass element spinning around an axis X-X with a constant angular velocity  $\omega_R$ , while the spin axis X-X is simultaneously rotating or "precessing" around an axis Z-Z with a constant angular velocity  $\omega_V$ , is subjected to the vector resultant of three forces:

- A precession force  $F_p$  oscillating perpendicular to the plane of spin around the spin axis X-X.
- A somewhat fluctuating centripetal force  $F_R$  directed radially towards the precession axis Z-Z, and
- A constant centripetal force  $F_r$  directed radially towards the spin axis X-X.



**Figure 15. Resultant of Precession and Centripetal Forces on Single Mass, Centrifuge Fork Horizontal**

**PRECESSION EFFECT ON A SPHERICAL SHELL DERIVED FROM THE AXIOM OF THE "CONSERVATION OF ANGULAR MOMENTUM"**

In textbooks, gyroscopic effects are usually derived from the axiom of *conservation of angular momentum*. Gyroscopic problems are easier to solve with this axiomatic method in most applications where rotationally symmetric rotors are used. However, a critiqueless application of the axiomatic method to gyroscopic problems on rotationally nonsymmetric masses, such as the human body, could easily lead to wrong conclusions and severe consequences. Therefore, the axiomatic derivation of the precession effect is also briefly reviewed and compared with the analytic derivation of the previous section.

The linear momentum  $p$  of a mass particle in motion is the product of its mass  $m$  and its velocity vector  $v$ :

$$p = m v$$

The angular momentum  $L$  of a mass particle in rotational motion is the product of its linear momentum  $p$  and the radius  $r$  of its rotation:

$$L = r \times p = r \times m v \tag{17}$$

Since no mass can change its velocity vector by itself, the linear and angular momentum must be constant unless an external force is applied. This is the axiom of *conservation of momentum*. In textbooks, this axiom is used also for the deduction of the directional stability of the gyroscope.

To change the linear momentum of a mass, a force  $F$  must be applied

$$F = m \, dv/dt = dp/dt$$

To change the angular momentum of a mass particle a torque  $T$  must be applied around the axis of rotation:

$$T = r \times F = r \times m \, dv/dt = dL/dt \tag{18}$$

This is the basic equation usually applied for solving problems in gyroscopic mechanics. Inserting the value for the circumferential velocity  $v = r \times \omega$  into equation 17 gives:

$$L = m r^2 \omega \tag{19}$$

Equation 18 was derived for a single mass particle. For a body of extended dimensions the total angular momentum will be the sum or the integral of the momenta of the single mass particles  $dm$ :

$$L = \int dm \, r^2 \omega \text{ which can be written}$$

$$L = \omega \int r^2 \, dm \text{ since } \omega \text{ is a constant.} \tag{20}$$

The integral

$$I = \int r^2 \, dm \tag{21}$$

is the *rotational inertia* or *moment of inertia* around the axis of rotation. The values for this integral are listed in mechanics book tables for various bodies, such as discs, cylinders, spheres, etc.

With the symbol  $I$  for the moment of inertia, equation 20 for the angular momentum  $L$  simplifies to

$$L = I \omega \tag{22}$$



in analogy to the equation for the linear momentum

$$p = m v$$

and equation 17 for the torque T, required to change the angular momentum, simplifies to

$$T = dL/dt = I d\omega/dt = I\alpha \quad (23)$$

wherein  $\alpha$  is the angular acceleration, in analogy to the equation for the force

$$F = dp/dt = m dv/dt = m a$$

The angular momentum L is the product of the scalar I and the vector  $\omega$ . The angular momentum, therefore, is also a vector quantity pointing in the same direction as the vector of the angular velocity, that is, in line or parallel to the axis of rotation.

The application of equations 22 and 23 is illustrated on the spherical cabin shell of a human centrifuge (see fig. 16 to 19), assuming a uniform mass distribution over the entire shell. The angular momentum of the cabin spinning around the fork axis X-X with an angular velocity  $\omega_R$ , is

$$L = I\omega_R$$

The vector of the angular momentum L is in line with the spin axis X-X. If the spin axis is now rotating (precessing) around the vertical centrifuge axis Z-Z, the vector of the angular momentum L of the cabin is forced to change its direction also with the angular precession velocity  $\omega_V$ . Hence, the rate of change dL/dt of the angular momentum, that is, the required precession torque  $T_p$ , will be according to equations 18 and 23

$$T_p = dL/dt = \omega_V \times L$$

which gives with  $L = I\omega_R$  from above

$$T_p = I\omega_V \omega_R \quad (24)$$

Equation 24 is the equation for the precession torque of the gyroscope usually found in textbooks. An example for its application:

Let us assume the cabin shown in figures 16 to 19 is a thin spherical shell with a radius of 1.8 m (about 6 feet), weighing 1,000 kp (2,200 lb), spinning with  $n_R = 30$  rpm around the horizontal fork axis and precessing with  $n_V = 55$  rpm around the vertical centrifuge axis as in the example on page 20.

The moment of inertia of a thin spherical shell around an axis through its center is

$$I = 2 Mr^2/3$$

wherein M is the total mass of the shell,

$$M = 1,000 \text{ kp} / 9.81 \text{ m/sec}^2 = 102 \text{ kg (mass)}$$

$$I = 2 \times 102 \text{ kg} \times (1.8 \text{ m})^2 / 3 = 215 \text{ kg m}^2 (= \text{kp m sec}^2)$$

As in the previous example:

$$\omega_V = \pi n_V / 30 = 3.14 \times 55 / 30 = 5.76 \text{ radians/sec}$$

$$\omega_R = \pi n_R / 30 = 3.14 \times 30 / 30 = 3.14 \text{ radians/sec}$$

$$T_p = I\omega_V \omega_R = 215 \text{ kpm sec}^2 \times 5.76 \text{ 1/sec} \times 3.14 \text{ 1/sec} = 3,900 \text{ kpm}$$

$$3,900 \text{ kilopondmeter (kpm)} = 28,000 \text{ foot pound}$$

The moment of inertia  $I$  and the angular velocities  $\omega_v$  and  $\omega_R$  in equation 24 all have constant values. The precession torque  $T_p$  of the spherical shell, therefore, is also of constant magnitude. In the following section figures 16 to 19 show how and why a rotationally-symmetric mass, such as the spherical cabin shell, produces a constant precession torque, while any single mass element, such as the single weights at the fork ends in the previous example, produces an oscillating precession force and torque.

### PRECESSION EFFECT ON A SPHERICAL SHELL DERIVED FROM THE CORIOLIS EFFECT

Figures 16 and 17 show a number of mass particles  $dm$  with their spin velocity vectors  $u$  and their precession force vectors  $dF_p$  on various parallel circles around the spin axis of the spherical shell. Figure 16 shows the cabin in a position with the fork vertical, figure 17 with the fork horizontal. Regardless of the position of the spherical shell, there is always a mass particle  $dm$  on every point of every parallel circle of the spherical shell. Every mass particle will be subjected to an oscillating precession force  $dF_p$  according to equation 13:

$$dF_p = 2 \, dm \, r \, \cos\theta \, \omega_v \, \omega_R$$

Figure 18 shows the distribution of the precession forces  $dF_p$  of the mass particles  $dm$  around a great circle in a perspective view. Figure 19 shows the same in orthogonal projection with their spin velocity vectors  $u$  and their horizontal and vertical components  $u_H$  and  $u_V$  in a side view. The magnitude of the precession forces  $dF$  of the mass particles  $dm$  increases linearly with their distance  $r \cos\theta$  from the axis Y-Y. Each mass particle produces a precession torque  $dT_p$  around the axis Y-Y, which also increases with the distance  $r \cos\theta$  from the axis Y-Y:

$$\begin{aligned} dT_p &= r \cos\theta \times dF_p = r \cos\theta \times 2 \, dm \, r \cos\theta \, \omega_v \, \omega_R = \\ dT_p &= 2 \, dm \, r^2 \cos^2\theta \, \omega_v \, \omega_R \end{aligned} \tag{25}$$

The vectors of the precession forces  $dF_p$  of the mass particles on the semicircle above the axis Y-Y all point in the direction towards the centrifuge axis Z-Z, those of all mass particles on the semicircle below the Y-Y axis point away from the centrifuge axis. All mass particles  $dm$  on the spherical shell together produce a total torque  $T_p$  around the Y-Y axis, which is the sum or integral of the torques  $dT_p$  of the single mass particles  $dm$ :

$$T_p = \int_0^{2\pi} dT_p = \int_0^{2\pi} 2 \, dm \, r^2 \cos^2\theta \, \omega_v \, \omega_R$$

which can be written

$$T_p = \omega_v \, \omega_R \int_0^{2\pi} r^2 \cos^2\theta \, dm = \omega_v \, \omega_R \int_0^{2\pi} I^2 \, dm$$

since  $\omega_v$  and  $\omega_R$  are constant.

The integral  $I = \int_0^{2\pi} r^2 \, dm$  is the already known rotational inertia or moment of inertia. Thus we arrive in this way at the same equation 24 of the precession torque:

$$T_p = I \, \omega_v \, \omega_R$$

as we did by its derivation from the axiom of conservation of momentum in the foregoing section.

# Contraails

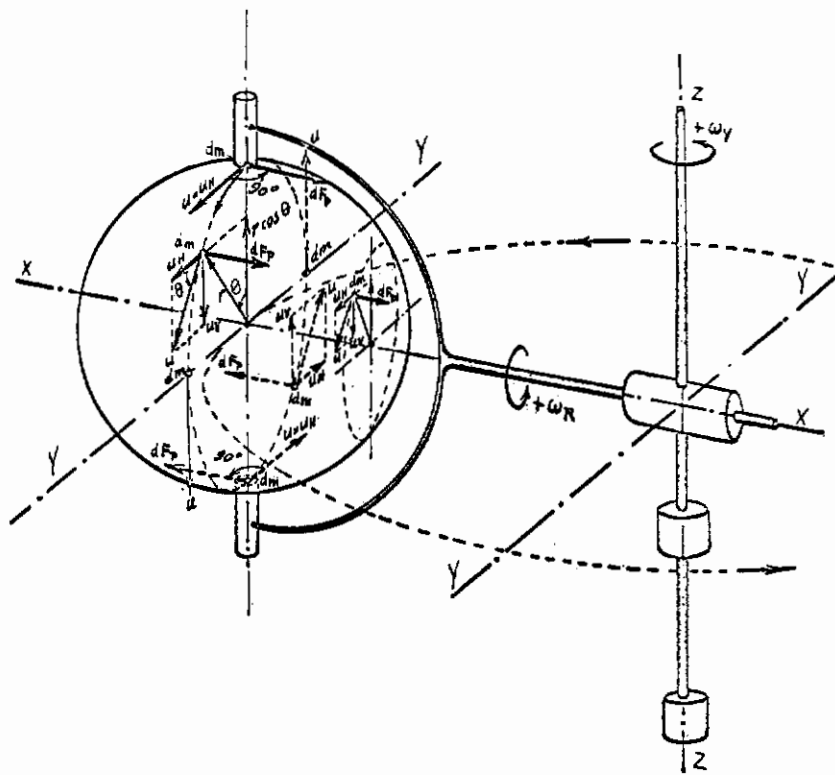


Figure 16. Precession Forces on Spherical Shell, Centrifuge Fork Vertical

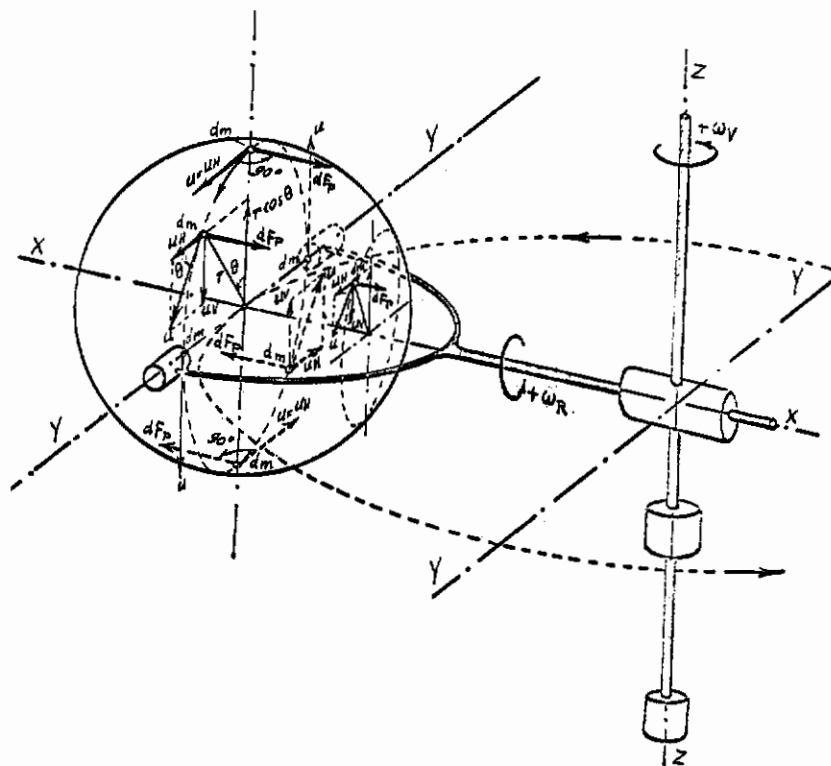


Figure 17. Precession Forces on Spherical Shell, Centrifuge Fork Horizontal

# Contraails

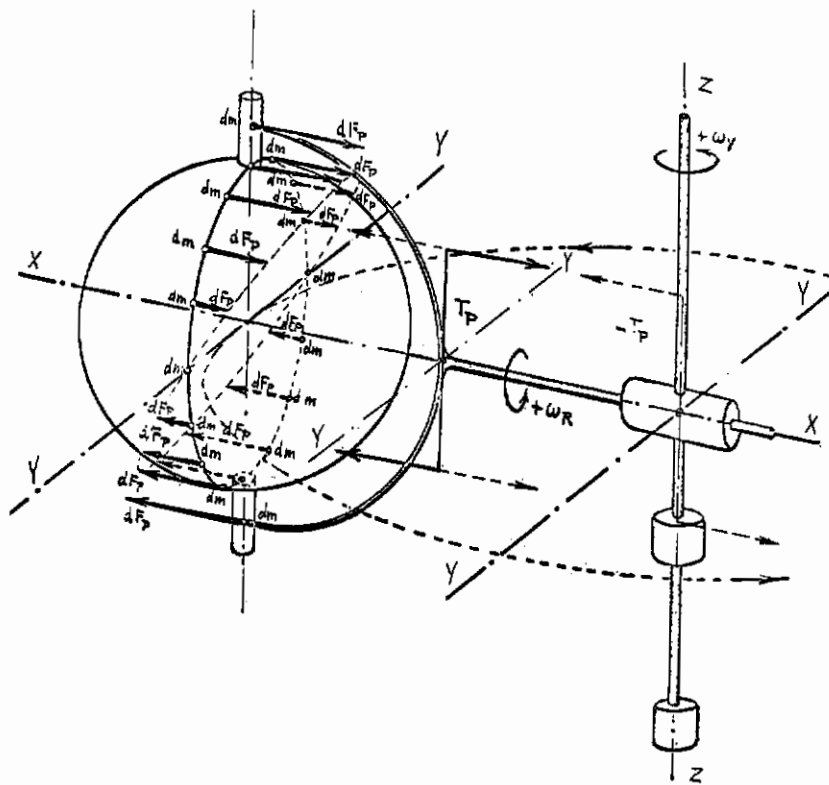


Figure 18. Distribution of Precession Forces on Great Circle of Spherical Shell, in Perspective View

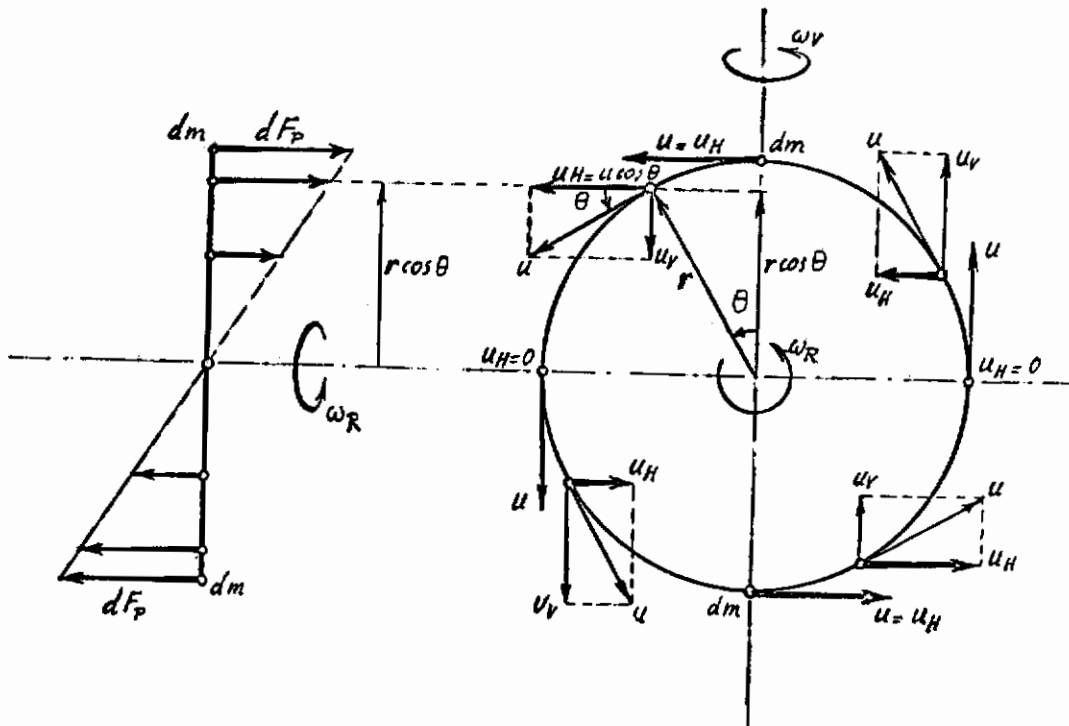


Figure 19. Distribution of Precession Forces and Velocity Components on Great Circle of Spherical Shell in Orthogonal Projection

Again, we see that the spherical shell produces a precession torque of constant magnitude. We can also see that the inertia reaction to the precession torque, the gyroscopic resistance indicated in figure 18 in dashed lines, has the tendency to erect the spin axis so that it is finally parallel with the precession axis and that both rotations occur in the same sense.

In summary, the purpose of these lengthy derivations and comparison of the analytical and axiomatic method for calculating the precession torque was to show that the simple equation 24

$$T_p = I \omega_v \omega_R$$

cannot be applied to the human body. This equation was derived for a rotationally symmetric rigid body and gives a constant precession torque. The human body, however, is neither rotationally symmetric nor rigid. It cannot be overemphasized that for the evaluation of the inertia effects on the human body, the oscillatory character of the precession force must be taken into consideration. During gyroscopic motions, such as tumbling, every cell and organ of the test subject's body is exposed to a different oscillating precession force. Therefore, the precession force must be calculated for each of the softly connected body organs separately, applying equations 13 and 16:

$$dF_p = 2 \, dm \, r \, \cos\theta \, \omega_v \, \omega_R$$

$$dF_{p \, \text{max}} = 2 \, dm \, r \, \omega_v \, \omega_R$$

For calculating the total resultant inertia force (the resultant of the precession force and the centripetal forces around the spin axis and the precession axis) on a single body organ, for example the heart, the analytical method is the method of choice as demonstrated on the example of a single weight at the fork end of the centrifuge on page 20.