The Effect of Compliant Layering on Damped Beams

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ABSTRACT

This paper reports the results of an analytical investigation into the effects of compliant layering on damped beams. The beams consist of laminated face sheets sandwiching a single damping layer. Compliant layering is introduced into this construction by making the extensional modulus of the inner layers of the face sheets substantially less than that of the outer layers. The analytical model, that is used to determine the mechanical response of this type of structure, is based upon a generalization of constrained layer theory. The analysis predicts that compliant layering can be used to reduce the forced response and improve the modal damping.

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1.0 INTRODUCTION

Damping treatments for bending components typically consist of adjacent layers of stiffness and damping materials. In these components the damping layers are sandwiched by the stiffness layers so that, when the stiffness layers deform under transverse loads, their bending will shear the damping layers. Because of their viscosity, the damping layers convert part of the strain energy of shearing into heat and thereby provide a means for dissipating the energies of shock and vibration [1, 2].

Any design approach that increases the rate or amount of shearing in the damping layers has the potential of improving the structural damping. Compliant layering, which in a layered design is the direct substitution of compliant material for stiffness material, offers such a possibility. The stiffness layers of conventional damping treatments consist of either monolithic isotropic, laminated quasi-isotropic or laminated unidirectional materials. This design practice results in in-plane moduli that are essentially constant over the depth of the stiffness layer. In these designs compliant layering would replace that part of the stiffness material that is adjacent to the damping layer with a material of lesser modulus. The in-plane modulus would no longer be constant over the stiffness layer and the in-plane extensional stiffness would be reduced. The hypothesis to be examined in this paper is that, under cyclic vibration, the use of compliant layers to reduce the in-plane extensional stiffness of damped treatments allows the stiffness layers on either side of the damping layer to undergo greater in-plane translations. This increases the rate of core shearing and thereby leads to higher levels of energy dissipation.

In a previous work [3, 4] a lamination theory was formulated that is applicable to a general class of damped bending structures, including structures with compliant layering. The lamination theory was used to examine the effects of stress coupling, lamination and compliant layering on damped plates. Here the original analytical theory is reduced for application to damped beams. Relevant parts of the previous analytical results are repeated and expanded here for the study of compliant layering in damped beams.

2.0 OUTLINE OF THE FORMULATION

The analytical model is a damped beam consisting of top and bottom face sheets sandwiching a single damping layer (see Figure 1). The face sheets are layered with a total of N^T layers in the top face sheet and N^B layers in the bottom face sheet. The thicknesses of the individual layers are designated by t_n^T for the top layers, t_n^B for the bottom layers and t^D for the damping layer. (Here the subscript n identifies individual stiffness layers while the superscripts T (top), D (damping), and B (bottom) refer to specific parts of the structure). The global coordinate system shown in Figure 1 and used in the development consists of the axial coördinate x_1 which is located in the mid-surface of the damping layer (the reference surface), and the transverse coordinate x_3.

To analytically model this structure the following assumptions are made:

1. The in-plane deformations of the face sheets vary linearly through the face sheet thickness;
2. The in-plane deformations of the damping layer vary linearly through it's thickness;
3. The in-plane displacement fields are continuous across the interfaces (perfect bonding);
4. The transverse displacement is the same for all parts of the cross section.
5. The moduli of all of the materials of construction can be treated by the Complex Modulus model;
6. The material model for the stiffness layers is transversely isotropic but neglects the thickness normal stresses. The axis of isotropy is parallel to the mid-surface;
7. The material model for the damping layer is isotropic but neglects all of the normal stresses.

Using assumptions 1 through 4, the motion of the structure can be expressed in terms of five displacement degrees of freedom (see Figure 2). These degrees of freedom are the reference surface displacements (u_1^0 and u_3^0), the rotation of the damping layer about the reference surface (α_1^D), and the rotations of the top and bottom face sheets (α_1^T and α_1^B). The degrees of freedom of this structural model are therefore a generalization of those found in constrained layer theory in that the top and bottom face sheets are allowed to rotate independently.

The displacements in terms of the degrees of freedom are

Top Face Sheet

$$u_1 = u_1^0(x_1,\tau) + \frac{1}{2}t^D\alpha_1^D(x_1,\tau) + (x_3 - \frac{1}{2}t^D)\alpha_1^T(x_1,\tau) \quad (1)$$

Damping Layer

$$u_1 = u_1^0(x_1,\tau) + x_3\alpha_1^D(x_1,\tau) \quad (2)$$

Bottom Face Sheet

$$u_1 = u_1^0(x_1,\tau) - \frac{1}{2}t^D\alpha_1^D(x_1,\tau) + (x_3 + \frac{1}{2}t^D)\alpha_1^B(x_1,\tau) \quad (3)$$

Complete Construction

$$u_3 = u_3^0(x_1,x_2,\tau) \quad (4)$$

where the symbol τ is used to refer to the time variable. From these assumed displacements, the strain fields are computed using the strain-displacement equations. The stress fields are then found by applying the constitutive laws.

The equations of motion for the damped beam structure are derived using Hamilton's Principle in conjunction with Reissner's Variational Theorem. Since Hamilton's Principle is only applicable to conservative systems, the material properties are initially treated as being purely elastic without any damping. The energy integrals, the integrands of which are formed from the field variables, are then minimized for this provisional, fully elastic system. The stress resultants are included by performing the thickness integration of these integrals. Taking the variation of the integrals with respect to the generalized displacements and forces and setting the coefficients of like variations to zero yields the governing system of differential equations. These equations include the force-displacement relations, the boundary conditions and the following equations of motion

$$-F^T_{11,1} - F^B_{11,1} - P_1 + M\ddot{u}^0_1 + I^D_1 \ddot{\alpha}^D_1 + I^T_1 \ddot{\alpha}^T_1 + I^B_1 \ddot{\alpha}^B_1 = 0 \quad (5)$$

$$-F^T_{13,1} - F^B_{13,1} - F^D_{13,1} - P_3 + M\ddot{u}^0_3 = 0 \quad (6)$$

$$\frac{1}{2} t^D (-F^T_{11,1} + F^B_{11,1}) + F^D_{13} + I^D_1 \ddot{u}^0_1 + I^D_2 \ddot{\alpha}^D_1 + I^T_2 \ddot{\alpha}^T_1 + I^B_2 \ddot{\alpha}^B_1 = 0 \quad (7)$$

$$\frac{1}{2} t^D F^T_{11,1} + F^T_{13} - M^T_{11,1} + I^T_1 \ddot{u}^0_1 + I^T_2 \ddot{\alpha}^D_1 + I^T_3 \ddot{\alpha}^T_1 = 0 \quad (8)$$

$$-\frac{1}{2} t^D F^B_{11,1} + F^B_{13} - M^B_{11,1} + I^B_1 \ddot{u}^0_1 + I^B_2 \ddot{\alpha}^D_1 + I^B_3 \ddot{\alpha}^B_1 = 0 \quad (9)$$

in which the F^T_{ij}, F^D_{ij}, F^B_{ij}, M^T_{11}, M^D_{11} and M^B_{11} are the face sheet and damping layer force and moment stress resultants, the P_i are the applied tractions and the M, I^D_1, etc. are inertial constants.

At this point the force-displacement relations are substituted into the equations of motion. This yields a set of five displacement-equilibrium equations the unknowns of which are the five functional displacement degrees of freedom. Solutions to specific problems are found by applying the appropriate set of boundary conditions and solving these equations. In matrix notation these equations take the form

$$[M][\ddot{u}] + [D][u] = [P] \quad (10)$$

where $[M]$ is the mass matrix, $[D]$ is a differential operator matrix, $[u]$ is a vector of unknown displacement functions and $[P]$ is a load vector.

Once an elastic solution is obtained, damping can be introduced by invoking the Correspondence Principle in which the elastic moduli are replaced by the complex viscoelastic moduli of the Complex Modulus model. Application of the damped beam model is therefore limited to steady state harmonic vibrations.

3.0 SOLUTION FOR SIMPLY SUPPORTED BEAMS

Consider a beam of length a in the x_1 direction. On the $x_1=0$ and $x_1=a$ edges the beam is simply supported. For these boundary conditions the Fourier series method can be applied to solve equation (10) using the following series expansions for the displacement degrees of freedom

$$u^0_1 = \sum_{m=1}^{\infty} U^m_1 \cos\left(\frac{m\pi x_1}{a}\right) e^{i\Omega\tau} \quad (11)$$

$$u^0_3 = \sum_{m=1}^{\infty} U^m_3 \sin\left(\frac{m\pi x_1}{a}\right) e^{i\Omega\tau} \quad (12)$$

$$\alpha_1^D = \sum_{m=1}^{\infty} A_1^{mD} \cos(\frac{m\pi x_1}{a}) e^{i\Omega\tau} \quad (13)$$

$$\alpha_1^T = \sum_{m=1}^{\infty} A_1^{mT} \cos(\frac{m\pi x_1}{a}) e^{i\Omega\tau} \quad (14)$$

$$\alpha_1^B = \sum_{m=1}^{\infty} A_1^{mB} \cos(\frac{m\pi x_1}{a}) e^{i\Omega\tau} \quad (15)$$

In these equations the superscripted constants are Fourier coefficients and Ω is the frequency of the steady state excitation.

The harmonically varying excitations (with respect to time) are also expressed in terms of Fourier series expansions

$$P_1(x_1,\tau) = \sum_{m=1}^{\infty} P_1^m \cos(\frac{m\pi x_1}{a}) e^{i\Omega\tau} \quad (16)$$

$$P_3(x_1,\tau) = \sum_{m=1}^{\infty} P_3^m \sin(\frac{m\pi x_1}{a}) e^{i\Omega\tau} \quad (17)$$

where the P_i^m are the Fourier coefficients determined from the Fourier formulaes.

Substituting the above expansions into equation (10) results in an infinite number of uncoupled equations that can be grouped into sets by common indicial values. Thus a set of five equations and five unknowns is obtained for each indicial value where the unknowns of these equations are the Fourier coefficients of the displacement series. Expressing these equations in matrix form leads to the following general expression for each indicial value

$$-\Omega^2[M][U^m] + [B_m][U^m] = [P^m] \quad (18)$$

where $[U^m]$ is a vector of Fourier displacement coefficients, $[B_m]$ is a modal stiffness matrix whose elements are determined by the material and geometric properties of the structure, and $[P^m]$ is a vector of the Fourier loading coefficients.

The analysis can be completed in several ways depending upon the type of information desired. For instance, the dynamic response of a damped beam to a specific excitation can be found through the direct solution of equation (18). If however, the modal loss factors are to be determined then the Forced Mode Method [5] is applied.

4.0 APPLICATIONS

4.1 STRUCTURAL DESCRIPTION

The beam examined in this analytical study has a length of 25.4 cm. The top and bottom face sheets of the beam consist of 6 stiffness layers with each layer having a thickness of 0.1725 mm. The damping layer has a thickness of .0965 mm. The stiffness layers consist of IM6/3501-6 carbon-epoxy with a fiber volume fraction of 60%. The properties of this material are shown in Table 1 where the disparity in the axial and transverse extensional moduli should be noted. The damping layer consists of ISD 112 Scotchdamp SJ2015x. The frequency dependence of the storage and loss moduli of this material are accounted for in the analysis. The mass density of the damping material is .98 gm/cc.

To study the effects of compliant layering on structural damping, the fiber reinforced layers adjacent to the damping layer are given a 90 degree off-axis orientation with respect to the x_1 coordinate direction. The off-axis orientation of the inner layers makes these layers compliant with respect to the x_1 coordinate direction. Therefore this particular type of lamination serves as a compliant layer design.

The notation used to specify the structural arrangement of the damped beams is identical to that used for laminations of advanced composites except for the addition of the symbol d which will indicate the presence of a damping layer. For instance, the baseline structure for this study, so called because it does not include compliant layering effects, is designated $0_6 / d / 0_6$.

4.2 NUMERICAL RESULTS

Figure 3 shows the loss factors of four different damped beams for the first five bending modes of vibration (Figures 3 to 6 repeat results that can be found in References 3 and 4). Here it is seen that there is little or no gain in damping for the fundamental mode but that in the higher modes the compliant layered laminates have significantly greater loss factors. (The matching of natural frequencies of the beams indicates that the gain in damping is not due to changing material properties.)

The goal of a damping design is to reduce resonant stresses and displacements. This is achieved by increasing the structural loss factor which in a compliant layer design is accomplished by sacrificing static stiffness (i.e. through the use of 90 degree layer orientations). It is necessary then to verify that the structural response actually decreases in the highly damped but more flexible compliant layer designs. To analytically test the response, the structures are subjected to forcing functions that approximately excite the resonant response (the approximation is introduced by not accounting for the negligible moment and in-plane components of the load vector that are required by the Forced Mode method for a strict proportionality to the inertia loading). Figure 4 shows the result of this computation where the amplitude of the transverse displacements have been normalized with respect to the modal response of the baseline beam. Except for the fundamental mode where virtually no improvement is achieved, the analysis predicts reduced resonant responses. (The failure of complaint layering to aid in controlling the response of the fundamental mode is attributed to the dimensions of the particular configuration being examined.)

The controlling parameter in increasing the damping in the compliant layered designs is the extensional modulus of the compliant layers. This is seen in Figure 5 where the modulus of the inner layers is varied parametrically as a percentage of the modulus of the outer layers. The loss factor directly increases with decreasing modulus. This modulus also controls the phase lag between the damping layer rotation (α_1^D) and the other displacement degrees of freedom (which respond approximately in-phase). Figure 6 shows that this phase lag increases with decreasing modulus.

To test the hypothesis that complaint layering leads to higher energy dissipation through greater in-plane translations of the face sheets, the following ratios are formed

$$R_i^D = \frac{|\alpha_1^D|_i}{|U_3^0|_i} \tag{19}$$

$$R_i = \frac{|U_3^0|_i}{|U_3^0|_{Baseline}} \tag{20}$$

in which the subscript i is used to refer to a particular design and the vertical bars indicate the amplitude of the listed degree of freedom. The ratio R_i^D is a measure of the amount of core rotation (shearing) that occurs per transverse displacement. The ratio R_i is a relative measure of the resonant response. For the first four modes of response Table 2 shows these ratios and the corresponding loss factors for the baseline beam and three compliant layer designs. In each mode it is seen that the design that leads to the highest R_i^D also has the lowest resonant response and the highest loss factor. This indicates that compliant layering affects the response by increasing the rate of core shearing.

It can be argued that the relationship between the material properties, the structural configuration and the dynamic response is very complex and that the benefits in mechanical behavior obtained in the compliant layer design can be attributed to reaching an optimum balance of conventional design parameters rather than to the compliant layering. Since in the previous analysis the thicknesses of the stiffness and damping layers were restricted to commercially available sizes this may very well be the case. To examine this issue an additional analytical test is performed. For an excitation that excites specific modes of response, fix the thicknesses of the face sheets and vary the thickness of the damping layer until the response is minimized The result is an optimized damping design for that specific excitation using conventional design practice. At this point compliant layering is introduced to see if a further reduction in response can be achieved. Table 3 shows the results of such an analysis for each of the first four modes of response. In each mode the compliant layering design yields an improvement over the optimized conventional design. Figures 7 and 8, which show this information plotted against the resonant frequency, indicate that the improvements are not due to changes in the amplitude of the forcing function or to changes in the frequency dependent material properties.

5.0 CONCLUSIONS

In order to examine the use of compliant layering in damped structures a structural theory was developed and applied to a simple but representative structural system. The analytical study revealed that compliant layering can increase the efficiency of damping designs by increasing the modal damping and reducing the forced response. The work presented here supports the following conclusions that were previously reported in References 3 and 4.

Compliant layering, which is the replacement of face sheet material with a less stiff material at the interface of the face sheets and the damping layer, affects the dynamic response of the beam through the alteration of in-plane extensional stiffness properties. This creates a mechanism for increasing the rate of shearing in the damping material by increasing the relative in-plane displacements of the face sheets. The rate of shearing and the associated energy dissipation were found to increase as the modulus of the compliant layer was reduced. However, there is a limitation to this process since the moduli of the compliant layer must be high enough to confine the shear deformation to the damping layer.

Compliant layering can also be used to reduce the weight of damped structures since compliant materials are generally less massive than stiff materials. For instance, metallic face sheets that incorporate a glass-epoxy compliant layer can have improved dynamic resistance at a reduction in weight. This same effect can be achieved by merely removing some of the material on the inner side of the face sheets through grooving, waffling or scoring this surface.

Compliant layering introduces challenges to the fabrication process since it involves either the mating of dissimilar materials [6] or the unbalancing of quasi-isotropic laminates. Also, there will be additional steps in the laminate fabrication which will add to the cost of building these components. Nevertheless, depending upon the total cost of construction, compliant layering offers an important design option in the use of damped bending structures.

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Axial Extensional Modulus	148. GPa
Transverse Extensional Modulus	8.96 GPa
Axial Poisson's Ratio	.35
Axial Shear Modulus	4.48 GPa
Transverse Shear Modulus	2.07 GPa
Axial Loss Factor	.00128
Transverse Loss Factor	.0110
Shear Loss Factor	.0110
Mass Density	1.52 gm/cc

Table 1 Material Properties of IM6/3501-6 Carbon-Epoxy

Beam Structure		Mode 1			Mode 2		
Design i	Lay-Up Specifications	R_i^D / R_1^D	R_i	η_S	R_i^D / R_1^D	R_i	η_S
1	$0_6/d/0_6$ (*Baseline*)	1.00	1.00	.26	1.00	1.00	.33
2	$0_5/90/d/90/0_5$	1.04	0.94	.28	1.08	0.88	.39
3	$0_4/90_2/d/90_2/0_4$	1.02	0.98	.26	1.10	0.85	.40
4	$0_3/90_3/d/90_3/0_3$	0.92	1.18	.23	1.06	0.92	.37

Beam Structure		Mode 3			Mode 4		
Design i	Lay-Up Specifications	R_i^D / R_1^D	R_i	η_S	R_i^D / R_1^D	R_i	η_S
1	$0_6/d/0_6$ (*Baseline*)	1.00	1.00	.34	1.00	1.00	.30
2	$0_5/90/d/90/0_5$	1:11	0.85	.43	1.12	0.83	.42
3	$0_4/90_2/d/90_2/0_4$	1.17	0.78	.48	1.21	0.73	.50
4	$0_3/90_3/d/90_3/0_3$	1.16	0.79	.46	1.24	0.70	.52

Measure of the Rate of Core Rotation $$R_i^D = \frac{|\alpha_1^D|_i}{|U_3^0|_i}$$

Measure of the Relative Resonant Response $$R_i = \frac{|U_3^0|_i}{|U_3^0|_{Baseline}}$$

The Modal Structural Loss Factor η_S

Table 2 Core Rotation per Transverse Deflection for Four Damping Designs

Mode	Design (1)	t^1 mm	t^2 mm	t^d mm	η_s	Displacement (2)
1	B	.000	1.035	.185	.36	1.00
	C	.310	.725	.185	.42	.86
2	B	.000	1.035	.085	.32	1.00
	C	.290	.745	.085	.37	.87
3	B	.000	1.035	.050	.29	1.00
	C	.270	.765	.050	.34	.88
4	B	.000	1.035	.035	.28	1.00
	C	.290	.745	.035	.33	.87

Notation:

t^1 The total thickness of the layers with a 90 degree orientation (Compliant Layer).

t^2 The total thickness of the layers with a 0 degree orientation.

t^d The thickness of the viscoelastic layer.

(1) Structural design;

B = Optimized design using conventional design practice.

C = Optimized design using compliant layering.

(2) The amplitude of the transverse displacement is normalized with respect to the response found for the conventional design.

Table 3 Optimized Designs

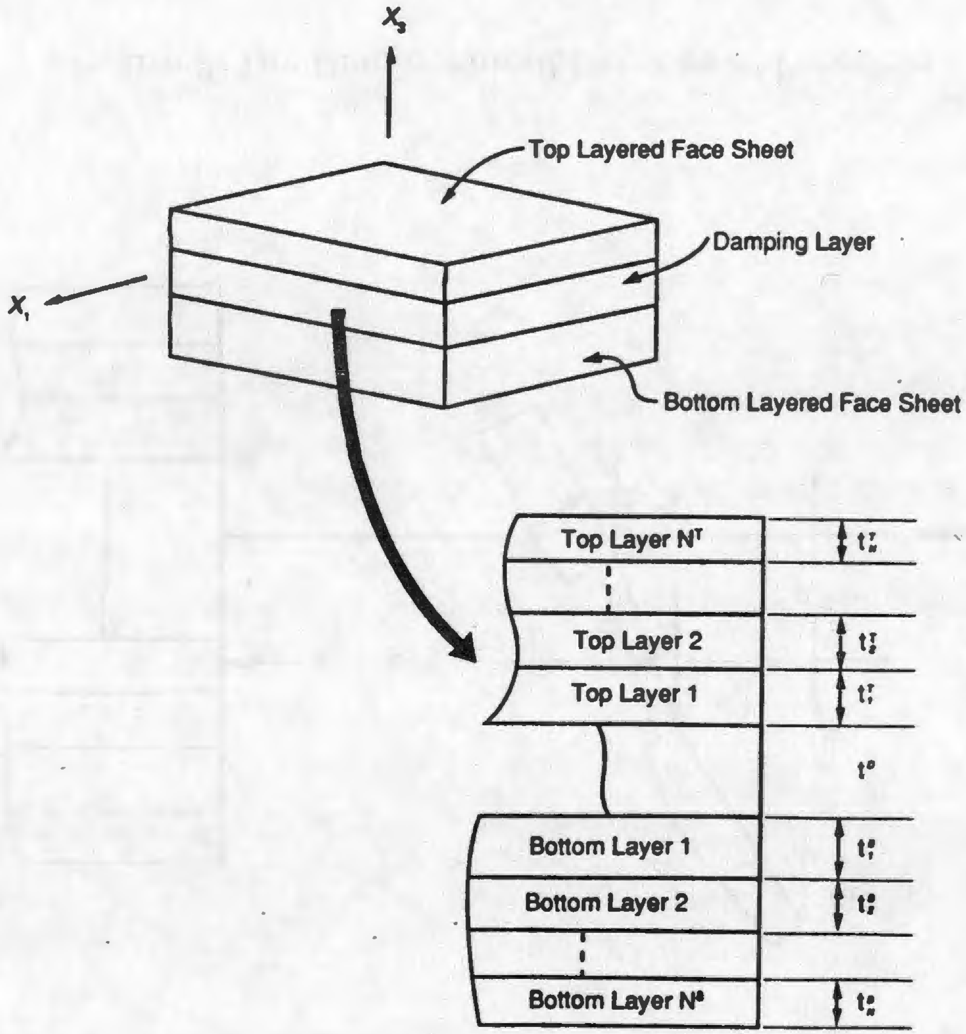

Figure 1 Damped Sandwich Beam

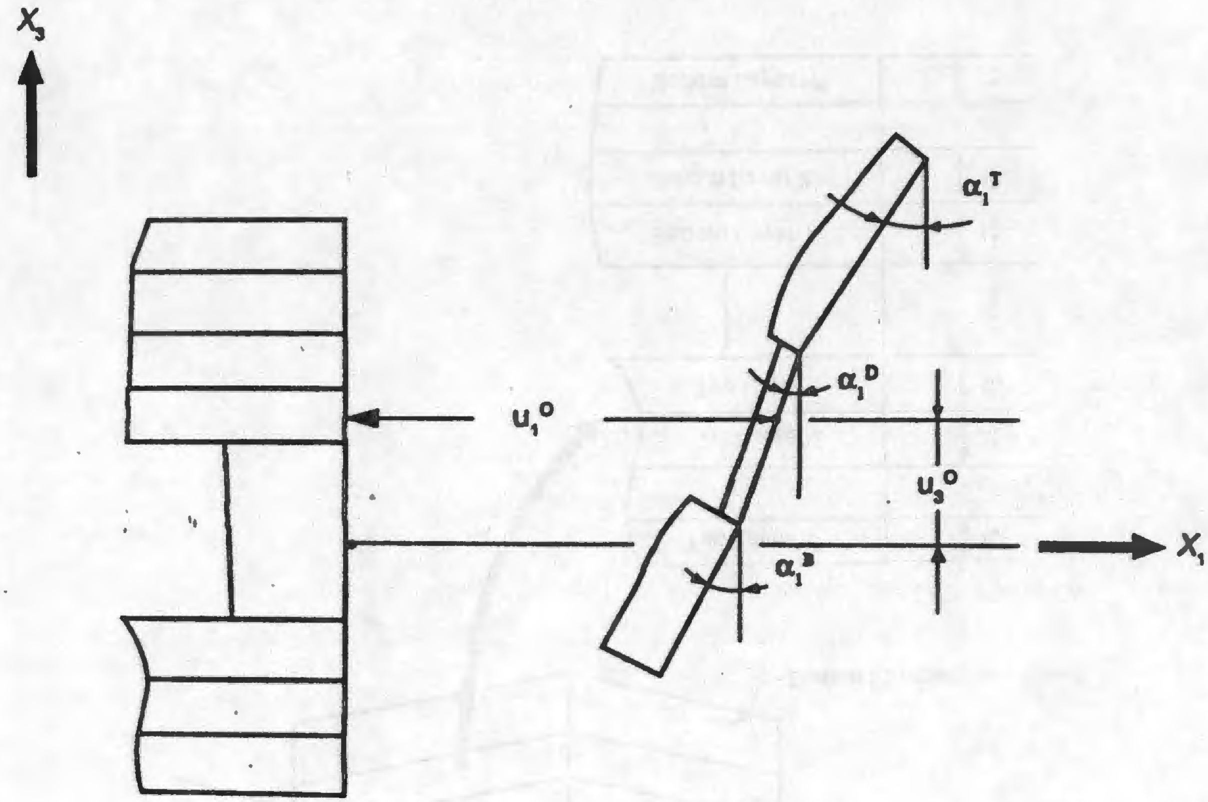

Figure 2 The Displacement Degrees of Freedom

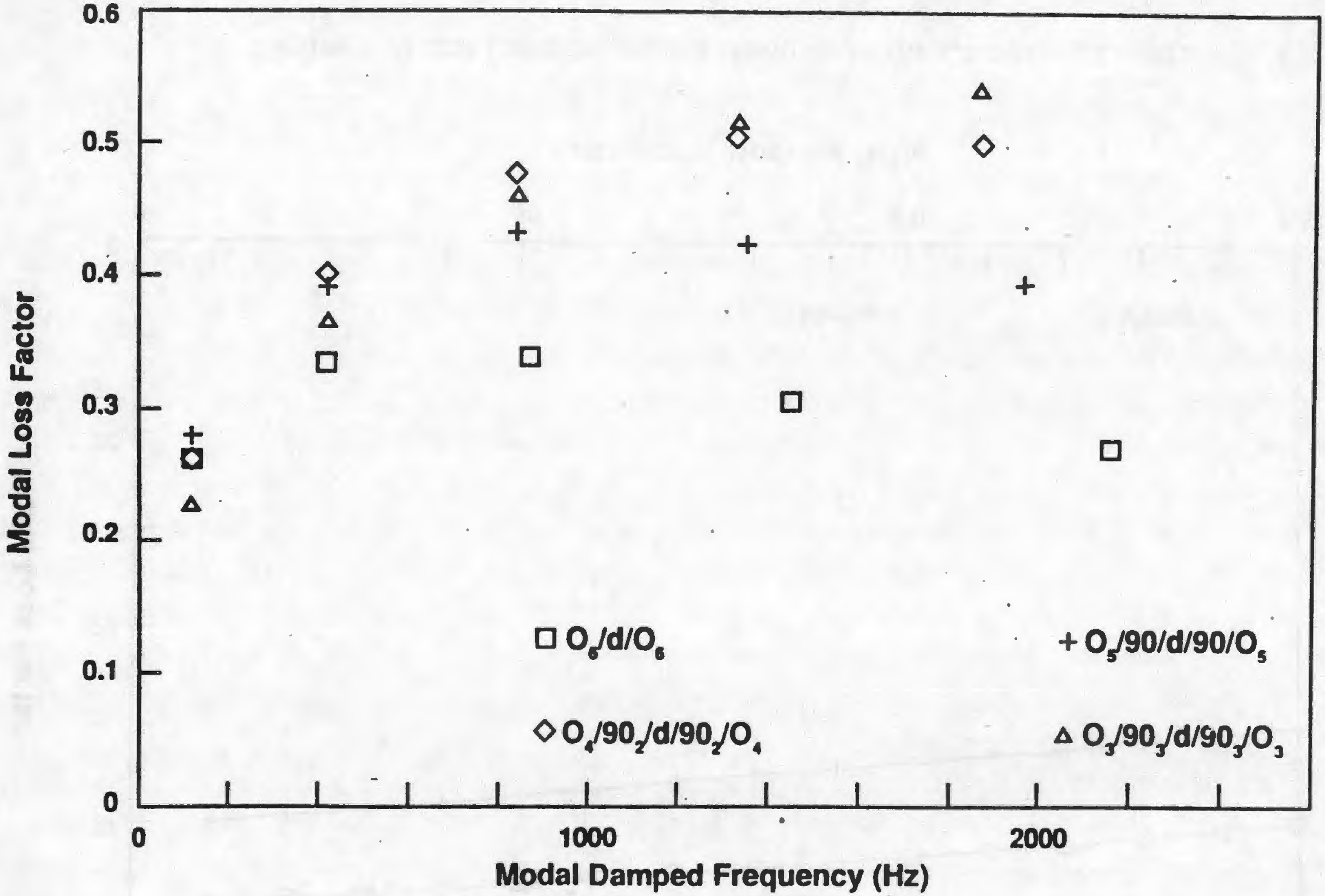

Figure 3 Modal Damping for Bending Modes 1 to 5

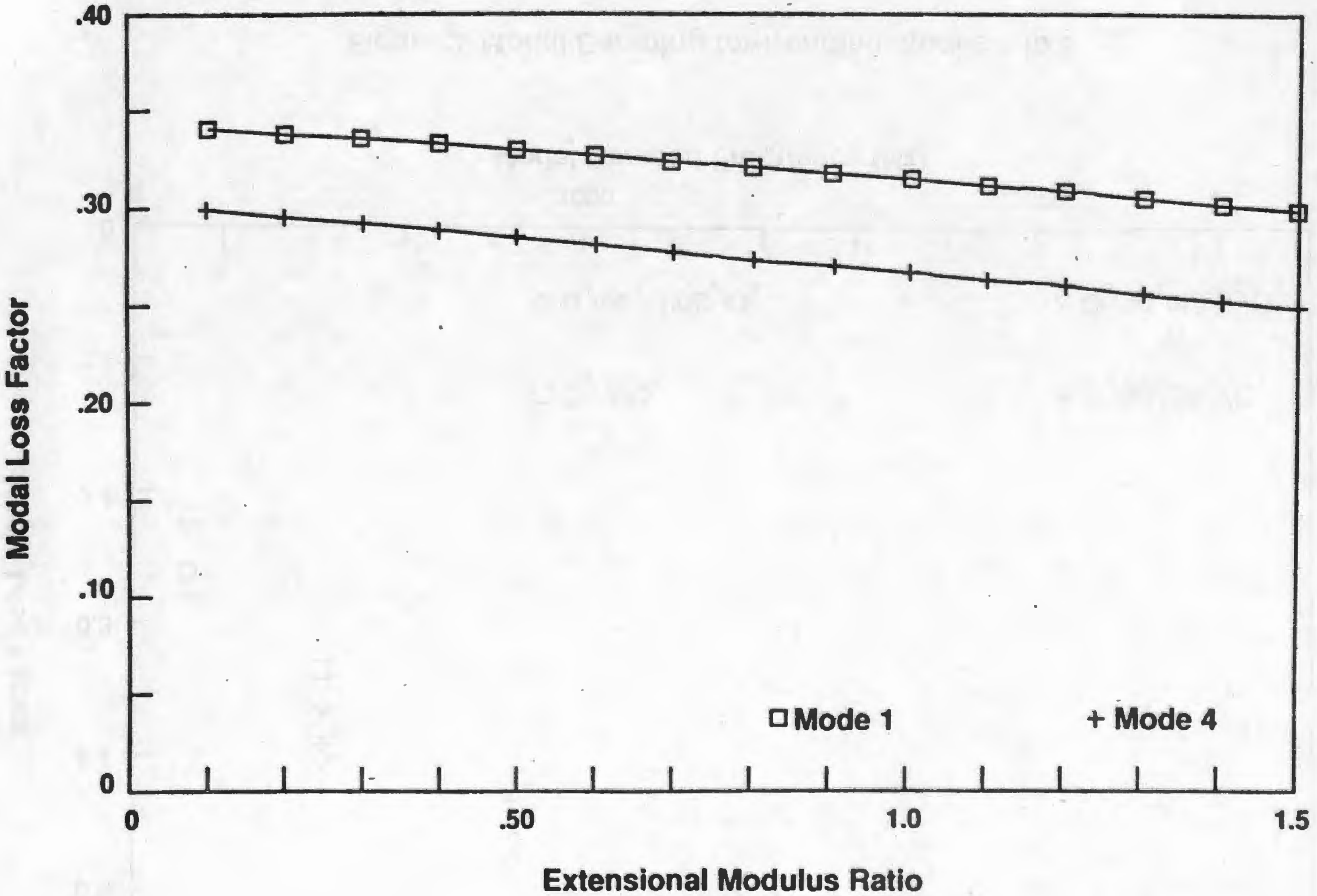

Figure 5 Modal Damping vs the Modulus of the Compliant Layer

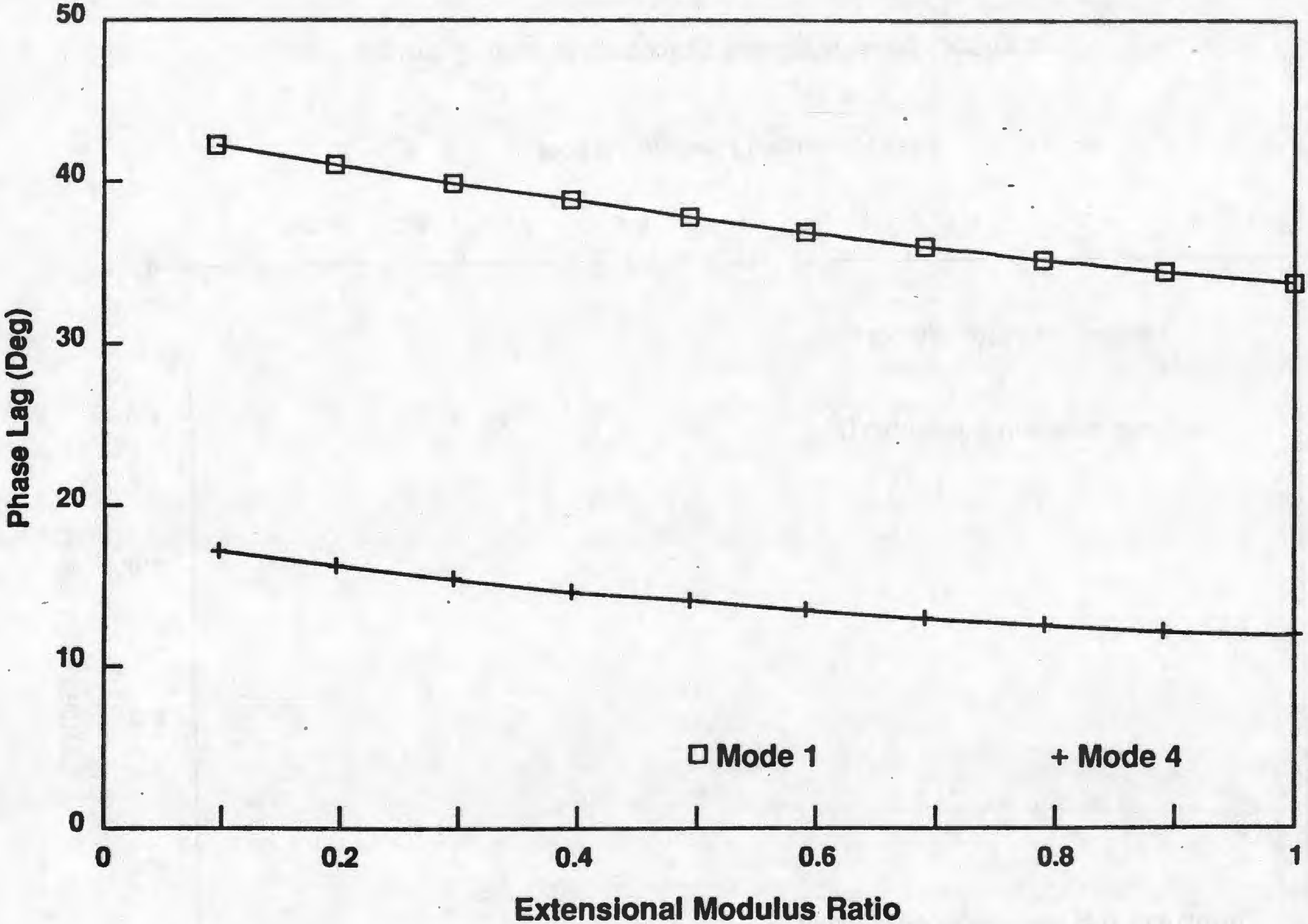

Figure 6 The Phase Lag of the Core Rotation vs the Modulus of the Compliant Layer

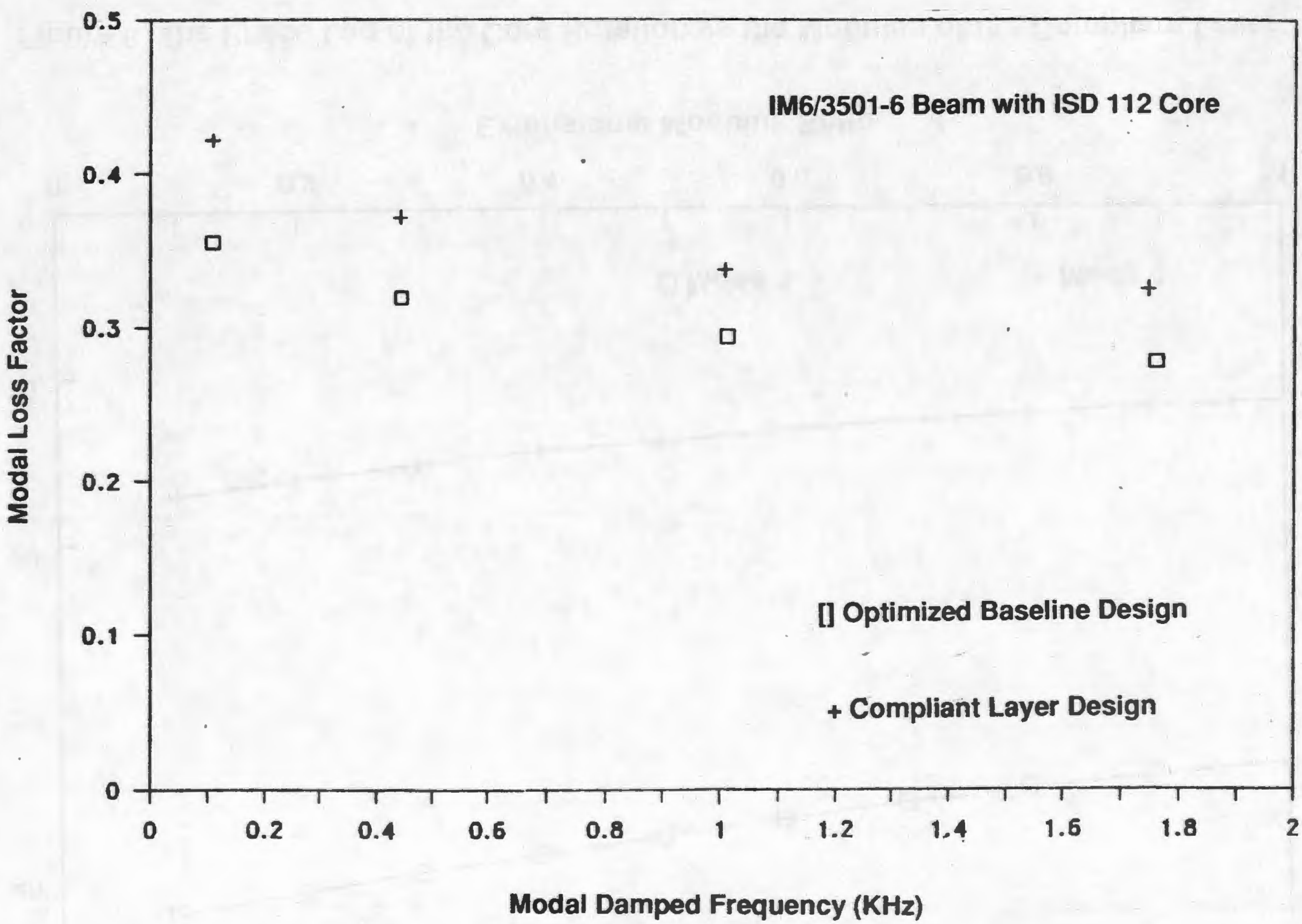

Figure 7 Modal Damping for Optimized Designs

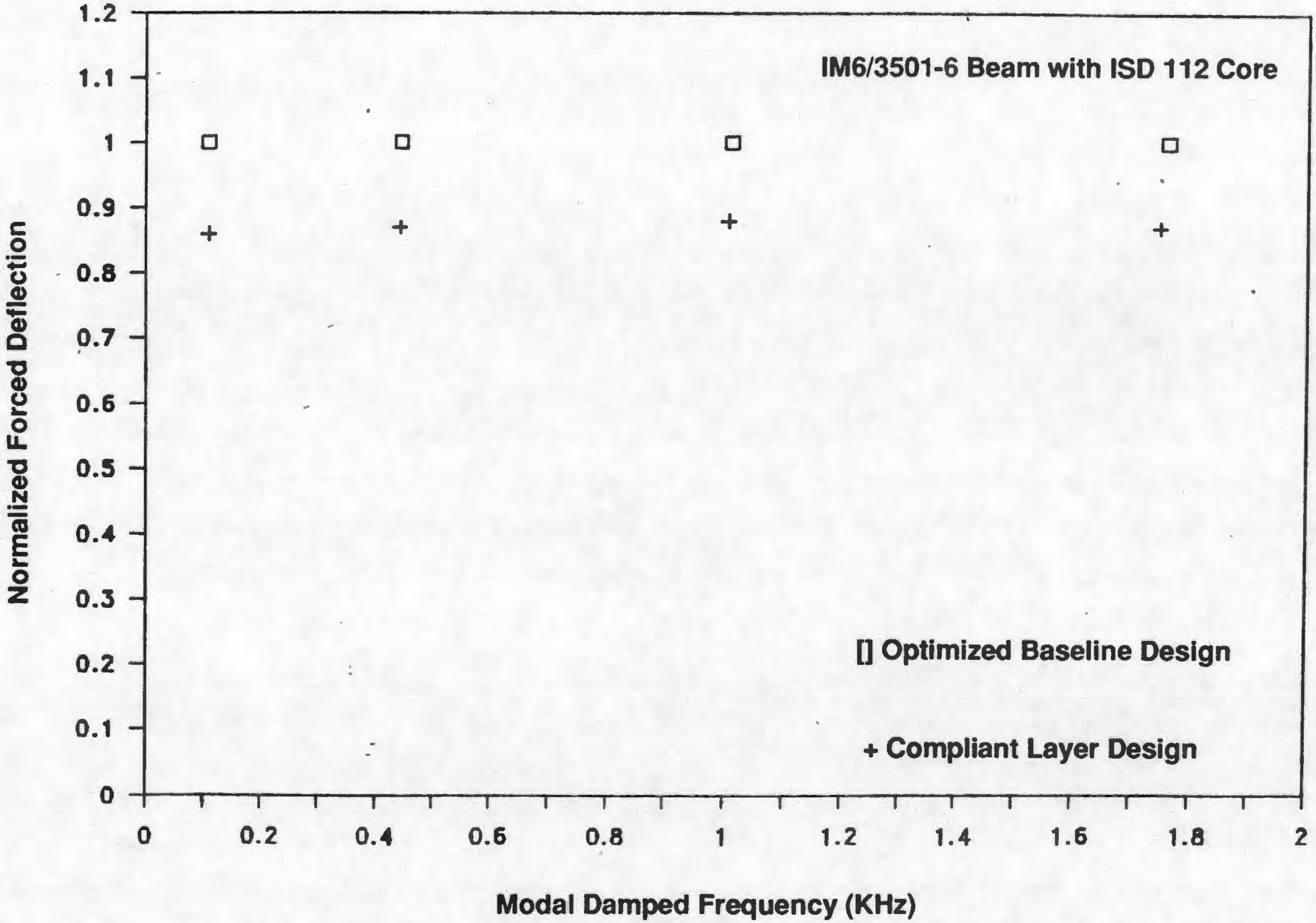

Figure 8 Normalized Forced Deflection for Optimized Designs