

**THE SPECTRAL DENSITY STUDY OF TRACKING PERFORMANCE**

**Part 1. The Effect of Instructions**

*Ezra S. Krendel*  
*The Franklin Institute*

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## FOREWORD

This report was prepared by The Franklin Institute with Ezra S. Krendel acting as Project Leader under USAF Contract No. AF 33(038)-10420 covering work on Human Frequency Response. The contract was initiated under a project identified by Research and Development Order No. 694-39, Servo Analysis of Human Control Systems, and was administered by the Psychology Branch of the Aero Medical Laboratory, Research Division, Wright Air Development Center, with Captain M. J. Warrick acting as Project Engineer.

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## ABSTRACT

Quantitative information about human frequency response functions in the control of a piloted aircraft would provide a rational basis for stability and control criteria. Were the pilot a linear, time invariant system, the ratio of the spectral densities of his output signal to his input signal would be the square of the transfer function describing the pilot. In addition, the computation of these spectral densities affords valuable insights into the response process under study. In an effort to examine the invariance of human system parameters an attempt was made to change an operator's tracking responses by using two different sets of instructions for the tracking of the same statistical input. Two subjects were used in a simple compensatory position tracking device. Although the amplitude response characteristics do not conclusively prove or disprove the invariance of the response patterns, the spectral densities of the signals provide insight into the manner in which the type of instructions affected the response. The value of future research is indicated.

## PUBLICATION REVIEW

Manuscript copy of this report has been reviewed and found satisfactory for publication.

FOR THE COMMANDING GENERAL:



ROBERT H. BLOUNT  
Colonel, USAF (MC)  
Chief, Aero Medical Laboratory  
Research Division

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## SECTION I

### INTRODUCTION

The purpose of this report is to illustrate an application of the spectral density approach to the study of human responses in a perceptual motor problem. [1] The particular aspect of this problem with which this report will deal is the influence of instructions on the performance of a tracking task. The type of data presentation resulting from the computation of the spectral density of the time functions in a tracking task would appear to be appropriate to the determination of differences in motor response patterns due to what are presumed to be differences in set to respond associated with different instructions to the subject. Knowledge of the spectrum of an input to a device under study and the resulting output's spectrum enable conclusions to be reached about the type of operation which the device performs on input signals.

The raw data for the spectral density analysis were obtained by means of a compensatory position tracking device, the control ratio of which was fixed at one centimeter of deflection on the scope face per degree of control stick motion. This device was designed to test the applicability of a spectral density analysis to the reduction of data from a dynamic aircraft simulator. Future and more ambitious experiments involving the methods outlined in this report will be performed with the dynamic simulator. It is hoped that these future tests will serve to shed more light on the response characteristics of pilots.

## SECTION II

### EXPERIMENTAL DETAILS

#### A. APPARATUS AND INPUT PROGRAM

The apparatus used in this experiment was a one-dimensional tracking device as illustrated in Figure 1. This device was designed to serve as a simplified approximation to a dynamic aircraft simulator for the purpose of providing a convenient means for planning research using the dynamic simulator. The display consisted of a cathode-ray 'scope mounted approximately 28 inches from the subject's eyes. A pip on this 'scope executed discrete left and right steps in a fixed horizontal line on the 'scope face. The subject's task was to center this pip on a vertical fiducial line. The joystick control permitted position tracking of this spot. The apparatus controls were designed to allow the programming of 20 square pulses of durations which could vary in eight discrete steps from 0.25 to 1.91 seconds. A program was devised so that the pulse durations were selected from a Poisson distribution of zero crossings, whereas the pulse amplitudes were alternately to the left and right with mean magnitudes of one centimeter in each direction. The average pulse duration was .75 seconds and the total length of the 20 pulse program was a little over 15 seconds. The individual amplitudes were selected from a Gaussian distribution of mean unity and standard deviation 0.25 cm.

Recordings were made on a Brush recorder of the control stick position,  $\theta_0$ , the instantaneous pip position,  $\epsilon$ , and the step function program,  $\theta_i$ . Figure 2 illustrates a segment of a typical record. The rounding-off of the square input pulses is due to the filtering action of the recorder's amplifiers.

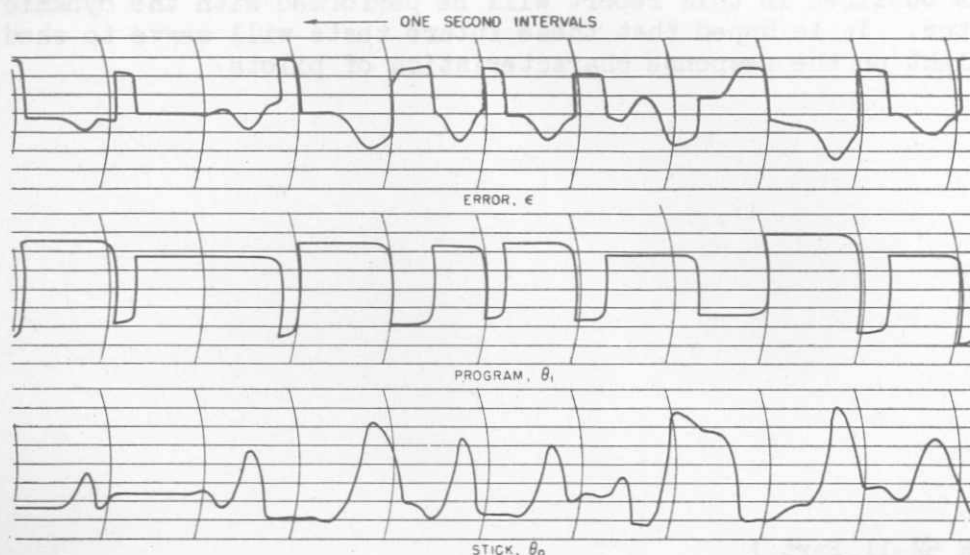


Figure 2. Ten Seconds of Recordings of  $\epsilon$ ,  $\theta_i$ , and  $\theta_0$  for Range 1.



Figure 1. Simplified Aircraft Simulator



The choice of the foregoing program was determined by a desire to present a visual display which would have a fairly smooth spectral density and which would provide a task requiring the operator to pay close attention to the pip on the scope face. Figure 3 is a plot of the spectral density of the selected program of square pulses.

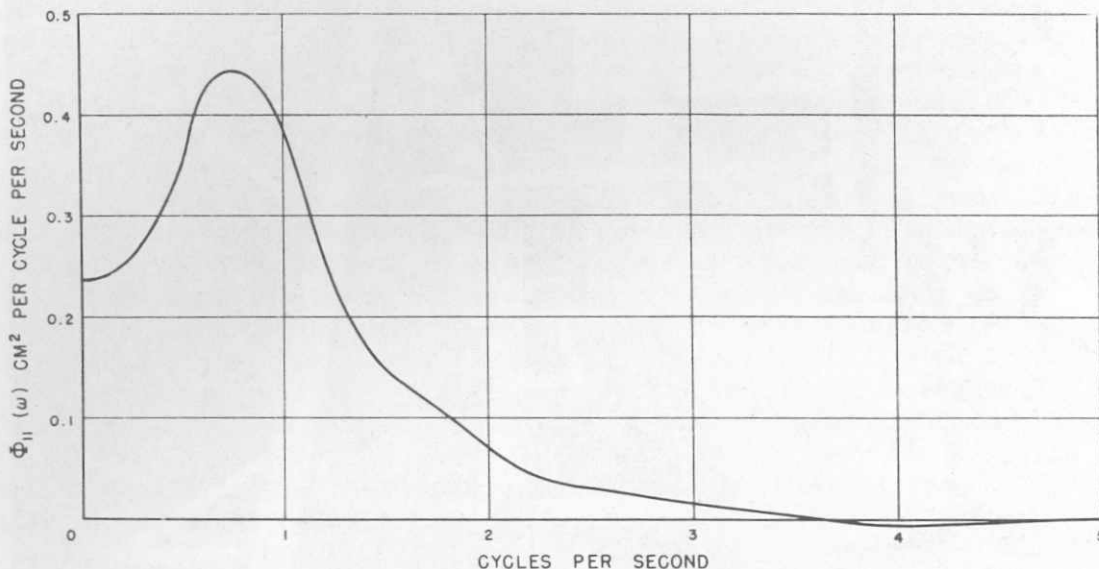


Figure 3. Spectral Density for the Program,  $\theta_i$ .

#### B. EXPERIMENTAL PROCEDURE

Two male subjects were used in this experiment. Subject #1 was a retired military man who had at least thirty years of flying experience. Subject #2 was a man in his early twenties who had no flight experience. The same program of square pulses was presented to each subject, and each subject was given intensive training in the tracking problem. The experimental variable was the subject's instructions. On the first day of the experiment subject #1 was instructed as follows: "It is of crucial importance to this study that you pay careful attention to these instructions. Return the pip to the vertical line as accurately as you can." On the same

day subject #2 was given instructions which differed from the foregoing only in that the word "rapidly" was substituted for the word "accurately". Prior to recording the three time functions,  $\theta_0$ ,  $\theta_1$ ,  $\epsilon$  for each subject, the subjects received a familiarization training of a few minutes with the simplified simulator. Following this session, the subject underwent four training sessions of five minutes each, separated by intervals of about one hour. During these sessions each subject repeatedly tracked the same program under his particular instructions, which were repeated at the onset of each training period. An hour after the last training session the subject was once again given the appropriate instructions and presented a succession of five 20-step programs. The three measured signals were recorded for the last two of these programs, and these recorded signals were considered to be representative of a trained subject.

On the second day of the experiment the same procedure was adhered to except that subject #1 was instructed to track for speed and subject #2 was instructed to track for accuracy.

### C. DATA REDUCTION

In order to convert the time function records to spectral densities, it was necessary to obtain their autocorrelation functions. This process was accomplished by means of a mechanical analogue device at the MIT Servomechanisms Laboratory. [2] The cosine transform was obtained by means of a 24 harmonic analyzer at the MIT Instrumentation Laboratory. The details of the data reduction procedure, as well as an outline of the theoretical background of the procedure is to be found in AFTR-6723. [1]

## SECTION III

### RESULTS

Figures 4 and 5 show the spectral density plots for  $\theta_0$ ,  $\theta_1$ , and  $\epsilon$  for subject #1 under instructions for accuracy and instructions for speed. Figures 6 and 7 are analogous plots for subject #2. The confidence limits denoted by the vertical lines represent pessimistic approximations to spreads of  $2\sigma$  on either side of the mean due to errors in the mechanical determination of the autocorrelation function, and do not include the errors due to the sampling and truncation inherent in the spectral density computations. The spectral densities have been smoothed according to Tukey's procedure. [1,3]

It will be noticed from Figures 4 and 5 that there is little difference between the response of subject #1 under instructions for accuracy and under instructions for speed. The explanation of this fact may conceivably be based on the following: subject #1 may have interpreted the words accuracy and speed in the framework of his past experience with tracking devices, and he may have realized that in order to achieve a minimum error he would have to track as rapidly as possible.

Subject #2, on the other hand, gives evidence of a different type of response for instructions to track for accuracy as compared with instructions to track for speed. This can be seen by comparing Figure 6, which represents tracking for accuracy, with Figure 7, which represents tracking for speed. In Figure 6 the error signal's spectral density is predominantly in the low frequency range with a peak at about  $3/4$  cycles per second; thus it might be implied by comparing the error spectral density with the program's spectral density at higher frequencies that the tracker was not introducing any of the higher frequencies into the error signal. Subject #2 was, in effect, smoothing his response and lumping his output in the lower frequencies when instructed to track for accuracy. This gives insight into subject #2's interpretation of the word "accuracy". On the other hand, Figure 7, which is subject #2's response pattern under instructions for speed, demonstrates that when instructed to respond for speed, subject #2 effectively eliminated the smoothing of his response and redistributed his output over higher frequencies with somewhat of a peak at a frequency of one cycle per second. Subject #2 might be thought of as attempting to square up his response to the step inputs by introducing more high frequency components in his output.

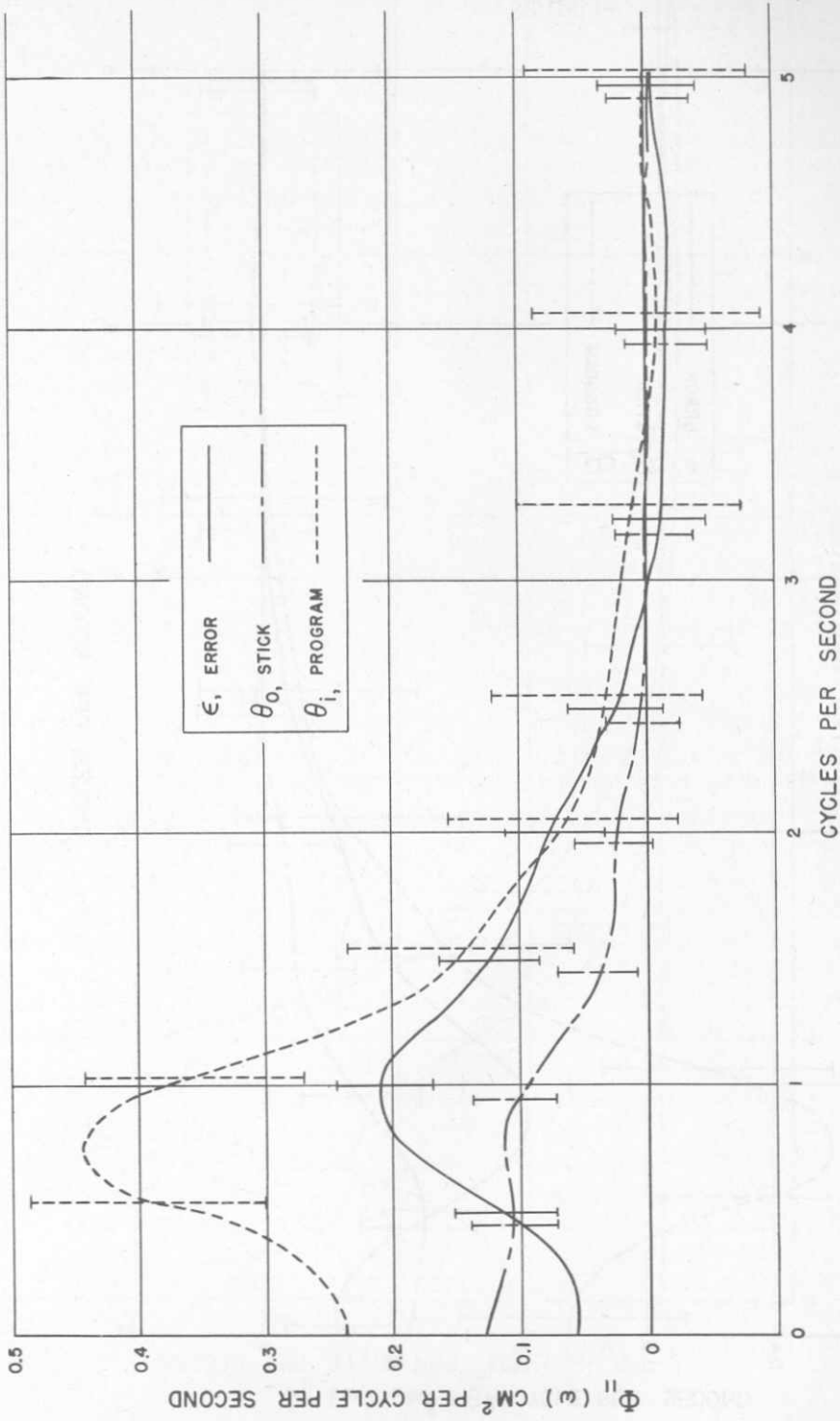


Figure 4. Spectral Densities of  $\epsilon$ ,  $\theta_0$ ,  $\theta_i$ , for Subject 1, Instructed for Accuracy.

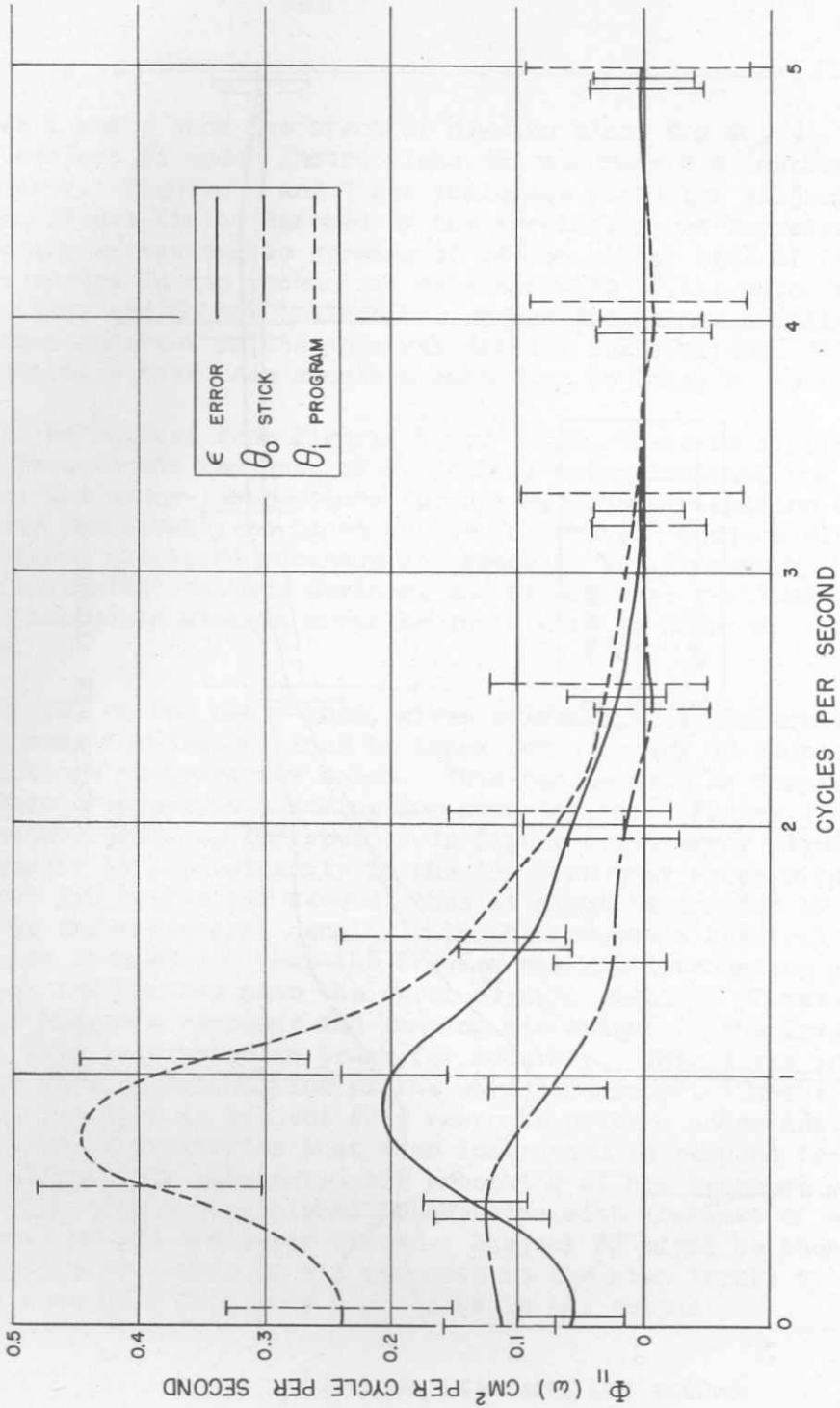


Figure 5. Spectral Densities of  $\epsilon$ ,  $\theta_0$ ,  $\theta_i$ , for Subject 1, Instructed for Speed.

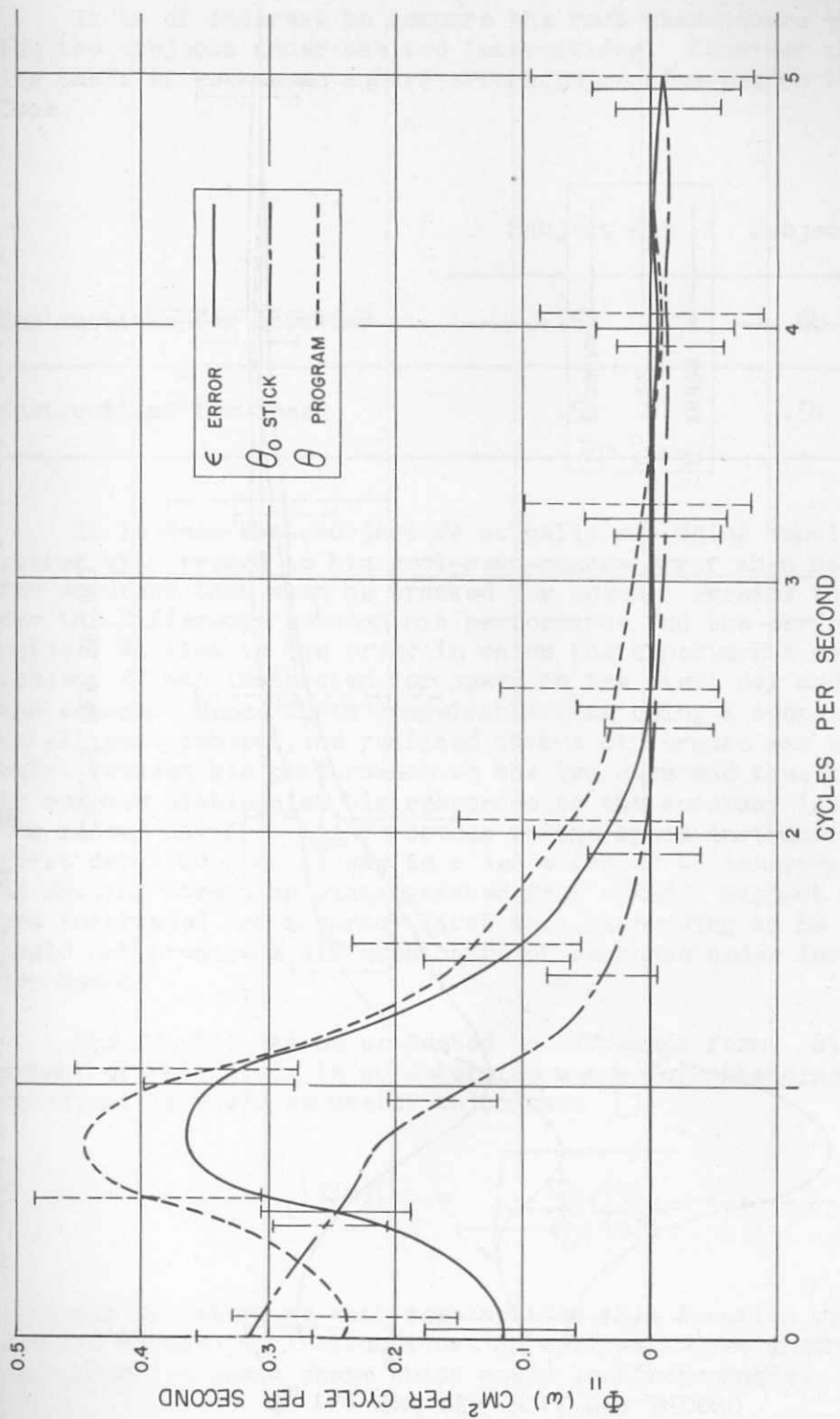


Figure 6. Spectral Densities of  $\epsilon$ ,  $\theta_0$ ,  $\theta$ ; for Subject 2, Instructed for Accuracy.

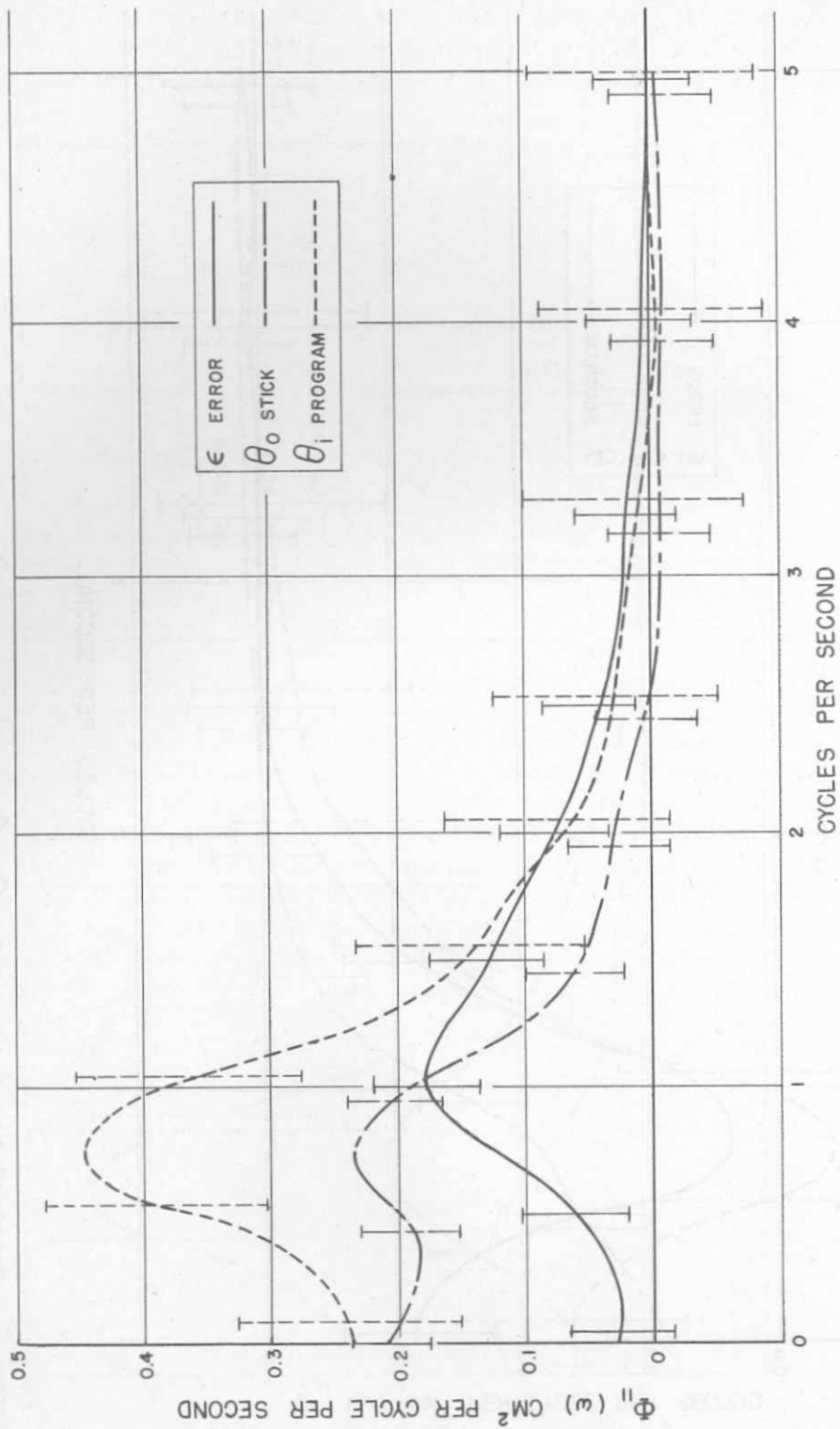


Figure 7. Spectral Densities of  $\epsilon$ ,  $\theta_0$ ,  $\theta_1$  for Subject 2, Instructed for Speed.

It is of interest to compare the root-mean-square error for the two subjects under the two instructions. Consider the following table of root-mean-square errors in centimeters on the scope face.

	Subject #1	Subject #2
Instructions for Accuracy	.53	.64
Instructions for Speed	.54	.54

It is seen that subject #2 actually was doing considerably poorer with regard to his root-mean-square error when he tracked for accuracy than when he tracked for speed. Perhaps the reason for the difference between his performance and the performance of subject #1 lies in the order in which the experiments were performed. Subject #2 was instructed for speed on the first day and accuracy on the second. Hence it is conceivable that being a cooperative and intelligent subject, he realized that a difference was suspected to exist between his performance on the two days and thus he attempted to somehow distinguish his responses to the accuracy instructions on the second day from his responses to the speed instructions on the first day. Subject #2 was in a sense forced to interpret accuracy as meaning something distinguished from speed. Subject #1, however, was instructed for accuracy first and, responding as he did, he could not produce a different type of response under instructions for speed.

The results can be presented in different form. Since the intent of this study is to determine means for obtaining a transfer function, it would be useful to compute [1]

$$|Y(p)| = \sqrt{\frac{\Phi_{oo}(\omega)}{\Phi_{ii}(\omega)}} \quad , \quad (1)$$

in order to determine what regularities this function preserves despite changes in instructions, as well as to see whether or not this function has a shape which would look reasonable. There are



two independent ratios which can be computed, but the computation of three ratios in all serves to present the results more clearly. Consider Figure 8 which is a simplification of the tracking problem:

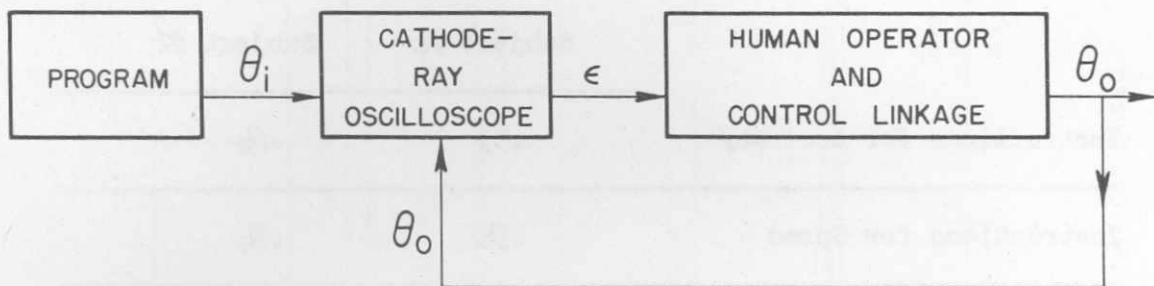
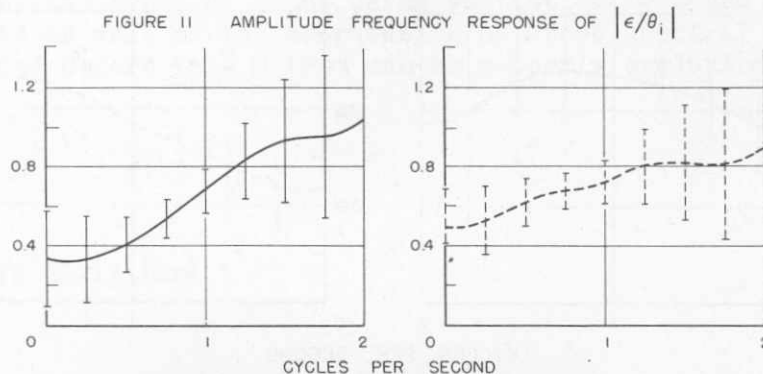
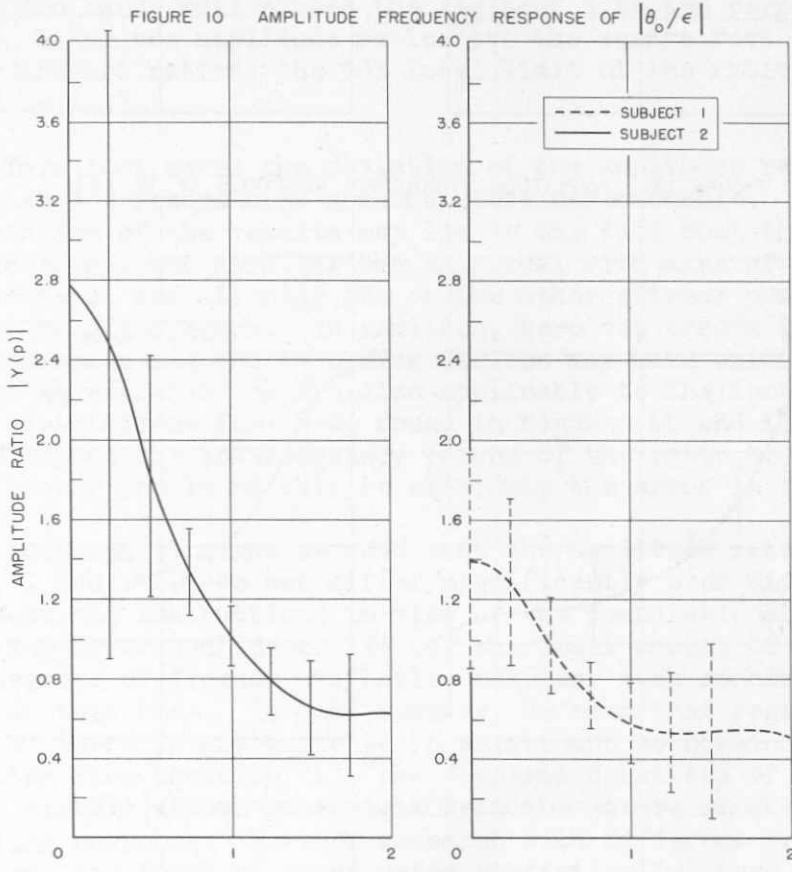
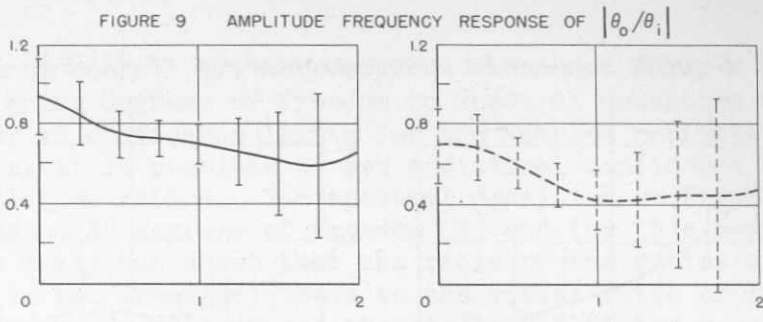


Figure 8. Simplified Diagram of Tracking Problem.

$\theta_o/\theta_i$  is the amplitude ratio across the open tracking loop,  $\theta_o/\epsilon$  is the amplitude ratio across the human operator's section of the loop and  $\epsilon/\theta_i$  is the amplitude ratio for the closed loop.

Figures 9, 10, and 11 present the foregoing ratios for subjects #1 and #2 under instructions for speed; whereas Figures 12, 13, and 14 are these ratios for subjects #1 and #2 under instructions for accuracy. The fiducial limits represent  $2\sigma$  confidence limits based on the confidence limits in Figures 4, 5, 6, and 7. [1] The bandwidths over which the amplitude ratios were plotted were limited so that the  $2\sigma$  confidence limits did not become so large as to render the plots meaningless.

The amplitude ratios present certain features which demand explanation. The amplitude ratio  $|\theta_o/\theta_i|$  in Figures 9 and 12 would reasonably be expected to equal unity for low frequencies, since one would expect that the tracker could position a pip perfectly under d-c conditions. Figures 9 and 12 depart from unity to an extent which cannot be explained in terms of the computational error



CYCLES PER SECOND  
TRACKING UNDER INSTRUCTION FOR SPEED

**Tracking Under Instruction for Speed.**

FIGURE 12 AMPLITUDE FREQUENCY RESPONSE OF  $|\theta_o/\theta_i|$

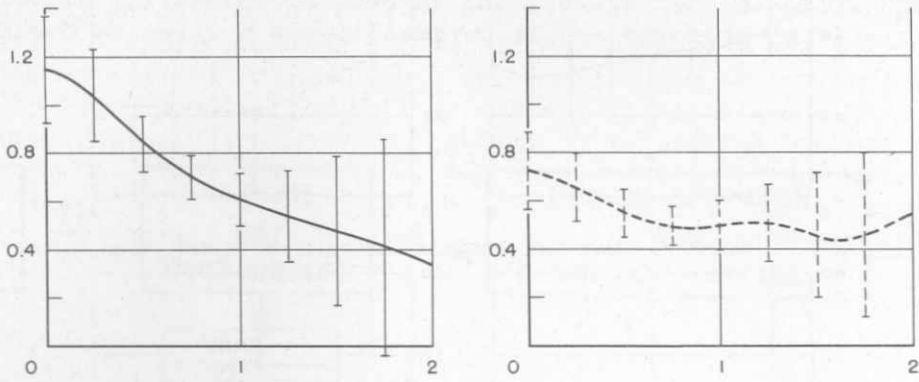


FIGURE 13 AMPLITUDE FREQUENCY RESPONSE OF  $|\theta_o/\epsilon|$

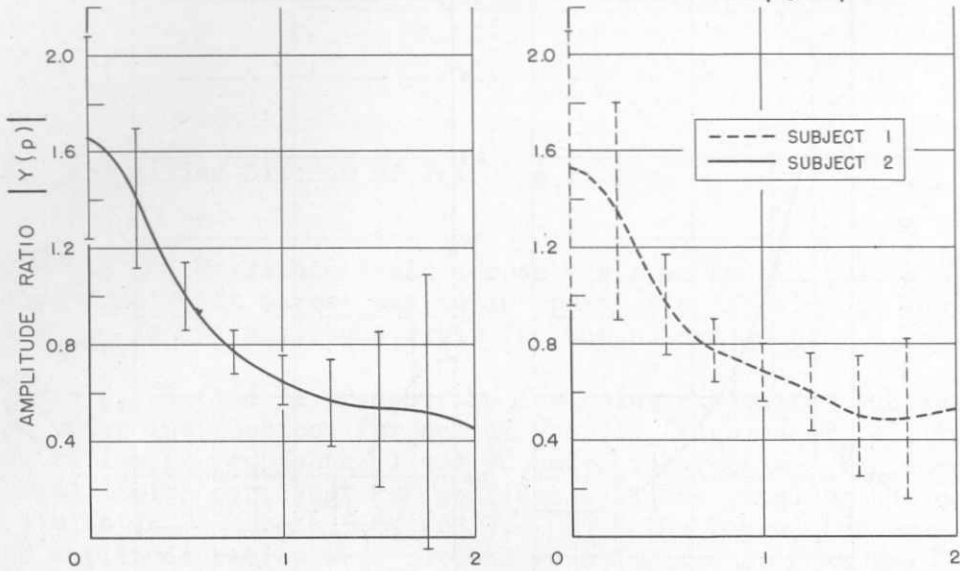
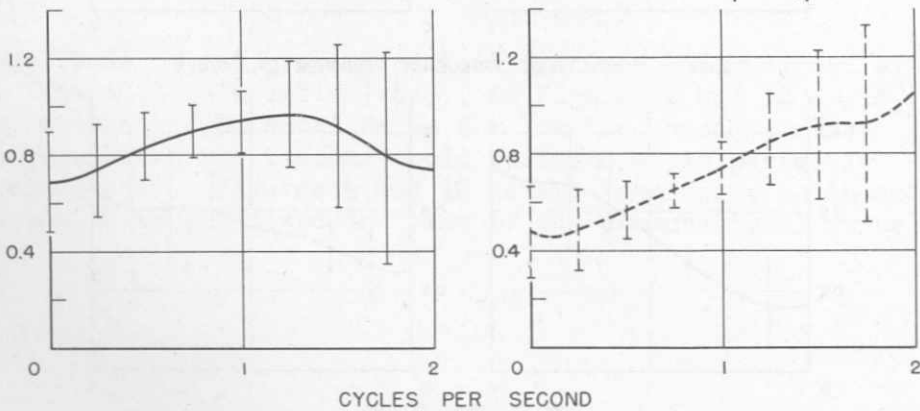


FIGURE 14 AMPLITUDE FREQUENCY RESPONSE OF  $|\epsilon/\theta_i|$



Tracking Under Instruction for Accuracy

WADC TR 52-11, Part 1

confidence bands. If, however, one considers Tukey's [3] concept of assigning degrees of freedom to spectral densities and thus arriving at confidence limits for the sampled estimate of the true spectrum, it is possible to set additional confidence bands about the amplitude ratios. The spectral densities on Figures 4, 5, 6 and 7 represent 28 degrees of freedom [1] and for this many degrees of freedom Tukey has shown that the ratio of the ratios of observed powers in two frequency bands to the actual ratio of powers in these two bands will exceed the limit of 1.88 ten percent of the time. Since the amplitude ratios are the square root of the power spectra ratios, the 90% level limit of the ratio of the ratios would be 1.37.

This fact makes the deviation of the amplitude ratio from unity at low frequencies somewhat less unreasonable. A further explanation of the results may lie in the fact that the pip on the scope was not circular but elliptical with axes of 0.2 and 0.1 centimeters, and aligning one or the other extreme position would result in a d-c error. In addition, zero set errors in the tracking apparatus and the recording devices may have existed. The foregoing explanations are also applicable to the inconsistency with expectations for  $\epsilon/\theta_1$  found in Figures 11 and 14. Here one would expect the low frequency values of the ratio to be zero, since the tracker should be able to eliminate the error in the d-c case.

Although it might be said that the amplitude ratios for  $\theta_0/\theta_1$ ,  $\theta_0/\epsilon$ , and  $\epsilon/\theta_1$  do not differ significantly over differences in subjects and instructions in view of the computational error and sampling error confidence limits, the small amount of data and the few degrees of freedom available, make any such conclusions meaningless at this time. It may, however, be seen that regardless of whether there is any validity in an attempt to obtain a transfer function from equation (1), the spectral densities of output and input signals afford an insight into the nature of the operator's tracking response. Further research with different programs, control devices, and types of input using statistically sound numbers of subjects as well as the computation of cross spectral densities is needed before this method can be properly evaluated.

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