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MATERIALS-PROPERTY-DESIGN CRITERIA FOR METALS

Part 4: Elastic Moduli: Their Determination and Limits of Application

S. A. GORDON
R. SIMON
W. P. ACHBACH

BATTELLE MEMORIAL INSTITUTE

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FOREWORD

This report was prepared by Battelle Memorial Institute, Columbus, Ohio, under Contract No. AF 33(616)-2303. The investigation was initiated under Project No. 7360, "Materials Analysis and Evaluation Techniques", Task No. 73605, "Design Data for Metals", and was administered under the direction of the Materials Laboratory, Directorate of Research, Wright Air Development Center with Mr. D. A. Shinn acting as project engineer.

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ABSTRACT

A study was made of the modulus of elasticity at elevated temperatures for several materials as it is derived from the conventional stress-strain curve and as it is derived from the determination of the velocity of propagation of elastic waves. The two methods of determination give modulus values which agree closely in regions of low stress and where time effects are unimportant. At higher stress levels, where the stress-strain relationships are not linear and where time effects are important, moduli determined by the two methods do not agree, the dynamic modulus being higher than the statically determined modulus.

These differences determined for the magnesium alloy AZ-31 and for the aluminum alloy 2024-T4 for various temperatures are discussed with respect to their applicability to airframe design.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

M. R. Whitmore
Technical Director
Materials Laboratory
Directorate of Research

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MATERIALS-PROPERTY-DESIGN CRITERIA FOR METALS
PART 4. ELASTIC MODULI: THEIR DETERMINATION AND LIMITS OF APPLICATION

INTRODUCTION

In the design of airframe structures, it is necessary to know not only strength characteristics of the structure but also amounts of deflection and distortion of the different portions of the structure under various conditions of loading encountered in operation. The deflections and distortions are functions of the geometry of the structure and of the elastic properties of the material. It is common engineering practice to account for the elastic properties of the material in design formulas in terms of certain constants known as elastic moduli — namely, Young's modulus (E) and the shear modulus (G).

The use of elastic moduli as constant multiplicative coefficients in design formulas is an oversimplification in many situations of practical interest, especially in cases of operation at high stresses. Nevertheless, because of the great convenience of the simple formulas, attempts are usually made to extend their ranges of applicability by specifying appropriate types of elastic moduli such as secant or tangent modulus, static or dynamic modulus, etc., to fit varying situations.

The purposes of this paper are: (1) to discuss the significance of various types of elastic moduli and the limits of their application, (2) to review the methods for evaluating elastic moduli and the limitations of each, (3) to describe the effects of time, temperature, and metallurgy on the "effective" values of elastic constants, (4) to evaluate the applicabilities of elastic constants, as obtained by the various test procedures, to airframe design, and (5) to obtain experimentally, for two materials, values of Young's modulus both by a static and a dynamic method.

SIGNIFICANCE OF ELASTIC MODULI

Ordinary stress-strain curves represent experimentally obtained relationships between stress and strain, either from uniaxial tension or compression tests or torsion tests. Elastic moduli are numbers that are derived from the stress-strain curves by performing certain specified operations on the curves. These moduli are convenient constants for use in engineering formulas, but the limits of their application must be taken into consideration.

Secant and Tangent Moduli

The value of the stress required to obtain a given strain may be readily located on the stress-strain curve. The ratio of stress to strain at this point is the secant modulus. If the stress is increased by a small increment and the strain is read from the neighboring point on the curve, the ratio of the stress increment to strain increment is the tangent modulus, if the two points are close enough together compared with the local radius of curvature of the stress-strain curve. The relationship between the secant and tangent moduli depends upon the shape of the stress-strain curve. Below the proportional limit, stress is proportional to strain and the two moduli are the same to within a certain degree of accuracy. The proportional limit is not a fundamental property of the material up to which Hooke's law holds strictly, but rather an arbitrary limit up to which it holds within a specified error, depending upon the accuracy desired. Some materials have a fairly well-defined proportional limit, while for others Hooke's law is a good approximation only at very low stress levels, and nonlinearity is apparent even well below the yield strength.

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Vibrations of airframe structures involve alternating stresses, usually superimposed on steady stresses. The error involved in using the tangent modulus to relate the alternating stress to the alternating strain depends upon the curvature of the portion of the stress-strain curve covered by the alternations. Complications arise if yielding occurs over a portion of the cycle. The tangent modulus at the operating point is then no longer applicable, because work hardening would shift the static operating point part way down the curve of unloading and the alternating stress-strain variation would then be along the unloading curve. Large amplitudes of alternation, especially if complete reversal occurs together with some yielding at both ends of the cycle, cause still more complications.

In bending, the strain is not uniform but is a function of the distance from the neutral axis. The elementary bending-beam formulas assume a constant value of Young's modulus throughout the beam, independent of stress amplitude. These formulas need re-examination for a material whose stress-strain relationship departs sufficiently from direct proportionality in the operating region of interest. It is meaningless to specify that either the tangent or the secant modulus is applicable for use in a formula whose derivation depends upon the assumption of a constant modulus. If the formulas are to be used at all, some procedure is needed for obtaining a proper "effective" or "reduced" modulus from the stress-strain curve.

Static and Dynamic Moduli

Elastic moduli also are sometimes classified as static and dynamic. This classification is an entirely different one from the former one of secant and tangent moduli. This classification system is based primarily on methods of measurement.

The static modulus is obtained by loading a specimen on a test machine and recording strain as a function of stress. The rate of loading may be slow or rapid, as specified for a given application; the term "static" is used here to denote a test procedure and not speed. The stress-strain curves of some materials are functions of the rate of loading because of the finite time required for the initiation of plastic deformation, and above a certain temperature the strain is a function of time as well as of stress because of creep.

To obtain the dynamic modulus, a specimen is set into vibration, and the resonant frequencies determined. For each resonance of the specimen, there is a theoretical relationship between the wavelength and the dimensions of the specimen. This relationship usually has to be developed in view of the constraints offered by the supports and the means of driving. From the wavelength obtained from this relationship and the measured resonant frequency, the velocity of propagation of the appropriate vibrational disturbance in the medium is computed. This velocity is a function of some combination of Young's modulus (or the shear modulus), Poisson's ratio, and the density of the medium; the particular function depends upon the type of vibration involved, longitudinal or shear waves in bars or in plates, etc. For flexural vibrations of bars, the velocity is also a function of frequency and of certain linear dimensions. The derivation of the velocity formulas usually assumes a constant value of the modulus, low amplitude of vibration, the cross-section dimensions of bars or thicknesses of plates much less than a wavelength, and negligible dissipation or relaxation effects. Various corrections have been devised for departures from these assumptions to obtain improved accuracy.

Thus, the "dynamic modulus" is a derived quantity based upon the determination of the velocity of propagation of elastic waves in contrast to the "static modulus" determined by more or less direct measurement of stress and of strain. In regions of low stress where Hooke's law holds and where time effects are unimportant, it would be expected (and is found) that moduli determined by the two methods are in close agreement. At higher stress levels, where the stress-strain curve is not linear and where time effects exist in the stress-strain relation, agreement is not to be expected. In such situations, the modulus value obtained by dynamic measurements is usually higher. The question of which value, when the two differ significantly, is more applicable in airframe design which is discussed in a later section.

DETERMINATION OF ELASTIC MODULI

Static Techniques

Tension and Compression

The tension test and the compression test are probably the most widely employed static tests for the determination of the elastic and plastic properties of a material. The experimental procedures for measuring stress strain in tension and compression are similar and employ the use of dial gages, Tuckerman strain gages, SR-4 wire strain gages, clip-type extensometer, etc. In conducting this type of test, care must be exercised in securing accurate alignment of the specimen, and extreme care must be used in the manner in which buckling is restrained in sheet specimens under compression. Normally, strain accuracy for the conventional tension or compression test is of the order of ± 20 to ± 50 microinches, giving a resulting accuracy in E of about ± 1 or ± 3 per cent. With increasing temperatures, the types of measuring instruments available become somewhat limited.

Particularly in tension tests, it is feasible to obtain stress-strain data, at the test rate of loading, well beyond the yield point of the specimen materials.

Uniform (Pure) Bending of a Simple Beam

A beam is loaded at the ends and restrained at two support points, producing bending in the center section. Strains may be measured by the same means as in the tension test. Modulus of elasticity is determined by the elastic-beam deflection formula. Accuracy of this method is usually dependent upon the ability of the specimen to adhere to the limitations and assumptions of the beam theory. Reasonably good results, however, can be obtained in determining E when the span-depth ratio of the center section is large (greater than 10) and when the material is fairly homogeneous. Instrumentation for measuring strain is similar to that used in the tension-compression test and subject to the same limitations.

The ordinary beam theory assumes a constant value of E, independent of stress level. Since the strain depends on the distance from the neutral axis, the bending test yields some kind of average value of E if the outer fibers are strained beyond the proportional limit. This value will be a function of stress level.

Combined Compression and Flexure

The technique of determining the critical load of an Euler column may be used to calculate the elastic modulus in compression. The critical load of an Euler column is independent of the mechanical strength of the material but dependent on the modulus of elasticity. In the general case of a pin-ended Euler column with a small degree of eccentricity, the lateral deflection is given by

$$y = \frac{y_o}{1 - (P/P_{cr})} \quad , \qquad (1)$$

where

y_o = initial eccentricity

P = load to produce a deflection y

$P_{cr} = \frac{\pi^2 EI}{L^2}$, the critical Euler load.

Southwell[1] suggests that the critical load can be predicted accurately by the following relationship, which eliminates the need of determining the initial eccentricity:

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \left[\frac{y_2 - y_1}{(y_2/P_2) - (y_1/P_1)}\right] \quad \text{or} \quad E = \frac{L^2}{\pi^2 I}\left[\frac{y_2 - y_1}{(y_2/P_2) - (y_1/P_1)}\right] , \qquad (2)$$

where

y_1 = deflection at load P_1

y_2 = deflection at load P_2.

Determining E by this method has not been exploited too well, but its dependence upon accurate knowledge of the end restraints in the test makes it of somewhat questionable value.

Dynamic Techniques

The dynamic type of test essentially consists of measuring the velocity or some other time function of the elastic strain wave propagated through the specimen in the longitudinal or transverse direction. Moduli determined by these methods are usually associated with very low stress levels.

Dynamic techniques for determining moduli of elasticity are inherently high precision-measurement techniques. Measurements of resonant frequencies can easily be done to 1 part in 10^4 to 10^5, depending upon the width of the resonance curve. Measurements of lengths and densities are required also to obtain moduli; with care, each of these can be accomplished to about 1 part in 10^3. The error in the modulus turns out to be roughly 0.3 per cent, according to the above figures. This is of the order of magnitude that various authors claim for the accuracy of their measurements of dynamic moduli.

The agreement between dynamic moduli and static moduli at low stress levels is only within about 2 per cent, however. At higher stress levels, the discrepancies would be expected to increase very rapidly with stress level. The 2 per cent agreement should be considered to be good, considering the number of simplifying assumptions made in the expressions for elastic moduli as functions of velocity, density, frequency, and the error in static measurement.

Several dynamic techniques of interest will be described briefly below. More complete descriptions of the experimental techniques, together with some critical evaluation, may be found in Reference 2.

Transverse Vibration Methods

Flexural vibrations may be employed to determine the elastic modulus on rectangular or circular specimens of uniform cross section along the length. The resultant dynamic modulus is, however, some average of the tension and compression modulus of elasticity.

In the free-free method, the specimen is suspended horizontally near the two nodal points for the fundamental mode of vibration. One support fiber conveys the vibration from the oscillator to the specimen, and the other is employed to transmit specimen vibrations to a receiver. The resultant fundamental resonant frequency, f_r, is related to the specimen dimensions by:

$$f_r = \frac{\alpha}{\ell^2}(EIg/A\rho)^{1/2} \text{ or } E = \frac{A\rho\ell^4}{Ig}\,\frac{f_r^{\,2}}{\alpha} , \qquad (3)$$

where

α = a constant, 3.58 for free-free bar

I = area moment of inertia ($inch^4$)

E = dynamic modulus of elasticity (psi)

g = acceleration of gravity (384 inch per $second^2$)

A = cross-sectional area ($inch^2$)

ρ = density (pounds per $inch^3$)

ℓ = length (inch).

The flexural-vibration technique is perhaps the best one for obtaining the dynamic Young's modulus. The experimental techniques involved are simple, rapid, and inexpensive, enabling a large number of specimens to be tested at various temperatures. By using thin, flat strips, resonant frequencies can be obtained at frequencies of interest (tens of cycles) with specimens of reasonable length.

Fixed-Free Method

The vibration of a single cantilever beam offers many difficulties to dynamic modulus measurements. The maximum strain is obtained by the fixed end and the accompanying shear results in a lowering of the resonant frequency and, therefore, in the apparent value of the dynamic modulus. Its use is not recommended.

Torsional Vibration

The torsion pendulum consists of a specimen gripped at one end to a rigid support and hanging free. An inertia bar or disk is attached to the other end. The inertia bar or disk is twisted to an angle and released. The resulting period of vibration or frequency is related to the shear modulus of the material. The relation between the frequency of vibration and the shear modulus for a specimen of circular cross section is

$$f = \left(\frac{r^4 G_g}{8 \pi I \ell} \right)^{1/2} , \qquad (4)$$

where

r = radius of specimen (inch)

G = dynamic shear modulus (psi)

ℓ = length of specimen (inch)

I = mass moment of inertia of pendulum bob (pounds x $inch^2$)

g = acceleration of gravity (384 inch per $second^2$).

This method gives good results for the shear modulus. If Young's modulus is desired, it is given, for an isotropic material, by

$$E = 2(1 + \mu) G,$$

which presupposes knowledge of the value of Poisson's ratio. However, μ need not be known too accurately, since a 10 per cent error in μ produces only a 3 per cent error in $1 + \mu$. (Conversely, the above formula is a poor method for obtaining accurate value of μ from measurements of E and G.)

Longitudinal Vibrations

A number of techniques exist for determining dynamic moduli by exciting and measuring longitudinal vibrations of bars. These will be mentioned briefly below for the sake of completeness, but they are of little consequence for most aircraft structures, since the frequencies involved for specimens of reasonable size are in the thousands of cycles. Pulse techniques have enabled measurements to be made also in regions of megacycles to tens of megacycles. Among the techniques are the following:

(1) Resonance of bars with piezoelectric crystals for driving and detection

(2) Pulse methods employing piezoelectric crystals

(3) Special techniques employing electrostatic, electromagnetic, or magnetostrictive drivers or detectors.

METALLURGICAL EFFECTS AND TIME-TEMPERATURE DEPENDENCE

Garafalo, Malenock, and Smith[3] summarize results of extensive study of static and dynamic moduli of several materials as function of temperature. Experimental results presented later in this report are compatible with their observations. The following trends appear:

(1) Elastic moduli (static and dynamic values being nearly identical for the low-stress elastic region) decrease approximately linearly with increasing temperature up to some limiting temperature.

(2) Beyond this limiting temperature, static moduli decrease much more rapidly. Dynamic values sometimes continue the linear decrease for some range of temperature and then decrease more rapidly.

(3) Temperatures at which the moduli begin to diverge vary with materials. Approximate values are 300 F for magnesium alloys, 350 F for aluminum alloys, 700 F for carbon steels, 900 F for ferritic stainless steels, and 1300 F for austenitic stainless steels.

These trends are generally explainable in view of time-temperature considerations.

The dynamic moduli were determined at much higher strain rates (and lower stress levels) than the static moduli. Whenever the stress-strain relation becomes time-dependent, the moduli determined at slow strain rates would be expected to appear smaller than values determined at higher strain rates.

As temperature is increased, changes in metallurgical structure will occur; these changes will usually be accompanied by changes in the strain at a fixed stress level. Usually, the changes are, for a fixed stress, time-dependent. The structural changes may involve strain hardening or aging and/or recrystallization for aluminum and magnesium alloys; spheroidization and (at higher temperatures) austenitization in steels; graphitization in irons and steels; formation of sigma-phase in high chromium steels; and other metallurgical changes.

In fact, observation of the relative changes in static (high stress, low strain rate) moduli an and dynamic (low stress, high strain rate) moduli with temperature affords useful clues as to conditions at which metallurgical structural changes occur. However, it would be expected that static values determined at the same stress level* and strain rate as dynamic values would agree closely with each other.

In general, at temperatures beyond which the static and dynamic moduli of a material differ by a large amount, the material is unstable and its strength properties must be considered time-dependent. In such situations, design should not be based upon conventional approximations; but stress-strain-time considerations must be used.

STATIC AND DYNAMIC MODULI IN AIRFRAME DESIGN

It has been indicated that in conditions of sufficiently low stress and low temperature, where a material behaves elastically, static moduli and dynamic moduli are the same. For an airframe structure under such loading conditions, a modulus value determined by either method is applicable to design formulas.

For stressing beyond the proportional limit or at elevated temperatures, the ratio of stress to strain may be variable and time-dependent. Under such conditions, significantly different values of modulus may be obtained by static and dynamic methods of measurement. Which (if either) of two values from the two types of measurement should be used in a specific design problem requires careful consideration of the problem.

As noted earlier, there are several reasons moduli determined by the two methods may differ in value. However, two are outstanding: (1) difference in stress level for the two methods and (2) difference in strain rate. Dynamic modulus determinations are usually made at very low stress levels (although this may not always be a necessary limitation). If, at a specific temperature and strain rate, the stress-strain curve for a material is nonlinear, it usually has highest slope at low stress levels. Thus, a dynamic-modulus value determined at low stress is higher than a static-modulus value determined at high stress. Under conditions of time dependence of the stress-strain relation, the stress-strain curve determined at a very low strain rate usually has less slope than one determined at high strain rate. The dynamic modulus, measured at high frequency (and corresponding high strain rate), may be higher than the static modulus measured at a low strain rate. Both differences in measurement conditions tend to result in a dynamic-modulus value higher than a static-modulus value.

It seems logical to use, in airframe-design problems, a modulus determined under loading conditions closest to those anticipated in service loading of the part. For steady or slowly varying high stresses, static modulus values should be appropriate. For rapidly varying low stresses, dynamic moduli might be more suitable.

Dynamic moduli reported in the literature are often obtained at frequencies of 1500 cycles per second or higher. It may be questioned whether significant stresses at such high frequencies often occur in airframe primary structures. Such structures commonly have lower resonant

*The nature of the dynamic method involves loops and nodes of stress in the resonant specimen and, consequently, large variation of stress amplitude along the specimen. For this reason, correspondence of stress level of dynamic and static tests is difficult to define.

frequencies and do not vibrate at such high rates even in response to sharp gusts.* It, therefore, seems that most reported dynamic moduli may be for frequencies and strain rates that are inappropriate to many airframe-design problems.

Dynamic moduli are commonly measured at low stress levels (for example, ±100 psi), and airframe structures usually operate at higher levels (sometimes a high mean stress with a low superimposed alternating stress). This is another reason to expect many reported dynamic moduli to be inappropriate for many airframe-design problems.

Thus, when different values of modulus are reported from static and dynamic determinations, it would seem that the static value may often be more appropriate than the dynamic value for airframe design.

This question may be raised: Suppose that, at some elevated temperature, a dynamic modulus (for ±100 psi at 1800 cycles per second) was 20 per cent higher than a static modulus (0 to 2000 psi at 0.02 inch-per-inch-per-minute strain rate). For some design consideration, suppose the anticipated strain rate was higher than that at which the static modulus was determined but lower than that at which the dynamic modulus was measured. Is it reasonable to use a modulus value between the two reported? Of course, more information from future experiments involving static determinations at several strain rates and dynamic determinations at several frequencies (especially lower frequencies) can answer that question. However, from what is known, it appears that reasonable interpolation will afford values as close as warranted by the present precision of measurement or of design formulas.

The following section describes an experimental determination of both static and dynamic moduli of two materials over a range of temperatures. The experimental results illustrate many of the points mentioned.

EXPERIMENTAL PROGRAM

The two materials selected were AZ-31 (QQ-M-44 Condition H) hard-rolled-plate magnesium and extruded 2024-T4 (QQ-A-354) aluminum.

The two methods of determination were: (1) obtaining a conventional static-tension stress-strain curve, and (2) measuring the characteristic frequency of transverse vibration of a bar. These methods would be expected to provide closely similar results under sufficiently low stresses (below the proportional limit in tension) and up to temperatures at which the stress-strain behavior becomes time-dependent.

The AZ-31 in the hard-rolled condition is a stable alloy at low temperatures but will undergo metallurgical changes, such as recrystallization if annealed at some elevated temperature. The 2024-T4 (solution treated and room-temperature aged) is susceptible to aging (precipitation) at elevated temperatures.

These two metallurgical changes (recrystallization and aging) influence plastic behavior at elevated temperatures. They may result in irreversible plastic deformation under stress. Numerous other factors may, in addition, produce deformation that is recoverable but not instantaneously, behavior that has been termed anelasticity. Either plastic or anelastic behavior results in a modulus that is time-dependent and sensitive to strain rate or to frequency.

To explore such effects, the alloys were tested in two conditions. The AZ-31 alloy was tested in the as-received (hard-rolled condition at room and elevated temperatures immediately upon attaining test temperature and after annealing (recrystallization). Thus, at some test temperatures, the effect of a metallurgical change occurring during the test on the mechanical properties could be evaluated.

*Rigorous calculation of transient stresses from impact loads is a complex problem and beyond the scope of simple engineering formulas.

The 2024-T4 alloy was tested in the as-received (T4) condition and after 15 hours at test temperature, in order to indicate the effect of aging on the mechanical properties.

Experimental Procedure

The magnesium alloy used in this investigation was commercial AZ-31 (QQ-M-44, Condition H) plate, 0.250 inch thick, in the hard-rolled condition. This plate was obtained from the Dow Chemical Company, and was of the following composition:

Weight, per cent							
Al	Mn	Zn	Ca	Si	Cu	Ni	Fe
2.8	0.54	1.2	0.08	0.014	0.007	< 0.004	0.004

The aluminum alloy was commercial 2024-T4 extruded bar, 0.25 inch thick by 1.5 inches wide. This extruded stock was obtained from North American Aviation, Inc., and was of the following composition:

Weight, per cent						
Cu	Mg	Mn	Fe	Si	Cr	Zn
4.5	1.5	0.71	0.27	0.12	0.01	0.02

The tensile specimens and dynamic-modulus specimens were machined so that the longitudinal axis of the specimens was parallel to the axis of rolling. The tensile specimens had a gage section 0.25 inch thick by 0.50 inch wide by 2.50 inches long. The dynamic-modulus specimen of the aluminum alloy was 0.25 inch in diameter by 5 inches in length. For the magnesium alloy, the dynamic-modulus specimen was 0.25 inch thick by 0.31 inch wide by 5 inches in length.

The tensile specimens were tested in a Baldwin-Southwark testing machine. The load on the specimens was obtained directly from the dial of the machine, and was applied at slow rate (0.001 inch per minute) since sufficient time was required to measure and record the data. The load scale could be read in 2-pound increments. The strain was measured at all temperatures by a clip-type extensometer having a sensitivity of 15 microinches in a 2-inch gage length. The accuracy of the strain measurements was approximately ±15 microinches. SR-4 strain gages were also employed at 75 F to check the calibration of the extensometer, expecially at low strains. These gages were used also to obtain specimen alignment, since it was observed that misalignment affected modulus values markedly. Single test specimens of each material were first aligned on the machine at room temperature. Then without completely releasing the load, the specimen was heated to the desired temperature, after which load increments in the elastic range were applied and the desired data obtained.

The dynamic modulus of elasticity was determined by the transverse-vibration method in which the specimen is employed as a free-free beam. The modulus value was calculated from Equation (3). Certain correction factors (such as for thermal expansion) are applicable to the modulus values, but these corrections are considerably less than the experimental error, which is approximately ±1 per cent.

Static and Dynamic Moduli

The static- and dynamic-moduli values versus temperature are presented in Table 1 and are plotted in Figure 1. Superimposed in Figure 1 are also data obtained from the WADC TR 6517, Pt 1 and ANC-5 values for comparison. It is of interest to note that the ANC-5 values compare very closely with the static-modulus curve obtained by Battelle.

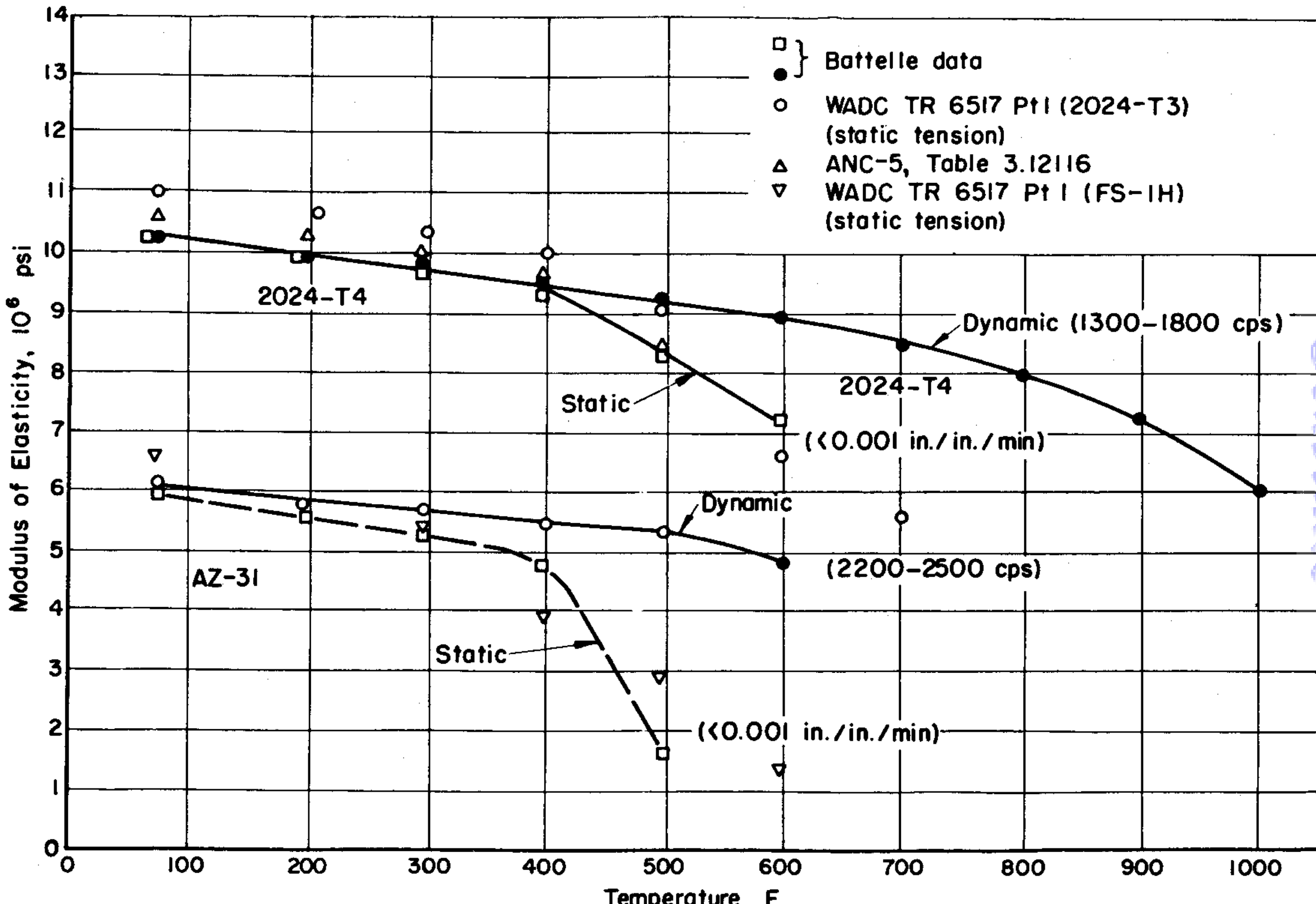

FIGURE 1. VARIATION OF THE DYNAMIC AND STATIC MODULUS OF ELASTICITY WITH TEMPERATURE FOR 2024-T4 AND AZ-31

TABLE 1. DYNAMIC AND STATIC MODULI OF ELASTICITY OF AZ-31 AND 2024-T4 AT VARIOUS TEMPERATURES

Temperature, F	AZ-31 Modulus of Elasticity, 10^6 psi Static	AZ-31 Modulus of Elasticity, 10^6 psi Dynamic	AZ-31 Dynamic Resonant Frequency, cps	2024-T4 Modulus of Elasticity, 10^6 psi Static	2024-T4 Modulus of Elasticity, 10^6 psi Dynamic	2024-T4 Dynamic Resonant Frequency, cps
75	5.9	6.1	2450	10.3	10.3	1779
200	5.5	5.8	2401	9.9	10.0	1751
300	5.3	5.7	2374	9.7	9.8	1728
400	4.8	5.5	2346	9.3	9.5	1702
500	1.7	5.3	2284	8.2	9.2	1679
600	--	4.8	2182	7.2	8.9	1651
700	--	--	--	--	8.5	1610
800	--	--	--	--	8.0	1567
900	--	--	--	--	7.3	1494
1000	--	--	--	--	6.1	1371
75(a)	--	6.1	2450	--	10.2	1770

(a) These moduli values were obtained at 75 F after the specimen had been heated to the temperature shown in this table.

Stress-Strain Characteristics

Since the determination of the static modulus required the development of stress-strain curves, these curves were completed to obtain such information as yield strength and ultimate strength and elongation for each temperature.

The AZ-31 was tested in the hard-rolled condition and in the annealed (2 hours at 400 F) condition. The data are presented in Table 2. The stress-strain curves are plotted in Figure 2, and the ultimate strength and yield strength are plotted against temperature in Figure 3. From Figure 3, it is observed that the yield strength for the hard-rolled material shows lower values above 200 F, while the yield strength of the annealed material is lower, below 200 F. The ultimate strength and the yield strength are the same for both materials at about 500 F.

The reversing of the yield-strength curves at 200 F is explained as follows. Although the annealing condition for AZ-31 is 1 hour at 400 F, the annealing process is time-temperature dependent and can proceed to equilibrium below 400 F if time is sufficiently greater than 1 hour. At both 300 F and 400 F when hard-rolled material is tested, the annealing process proceeds at an appreciable rate while the test is being made. Thus, the magnesium alloy in the hard-rolled condition is unstable during the tensile tests at 300 F and 400 F, resulting in strains greater than in the stable (annealed) condition. The effect of such instability has been observed in creep studies and results in lower creep strength in hard-rolled material than in annealed material. At 500 F, very little or no effect of instability was observed as the material anneals rapidly at 500 F and is probably completely annealed prior to testing.

The 2024-T4 is a precipitation-hardening alloy. In the T4 condition, in which the alloy was obtained, it had been solution treated, quenched, and subsequently aged at room temperature. This alloy is subject to precipitation hardening above room temperature. This can affect the mechanical properties, depending on the time-temperature relationship of the precipitation process.

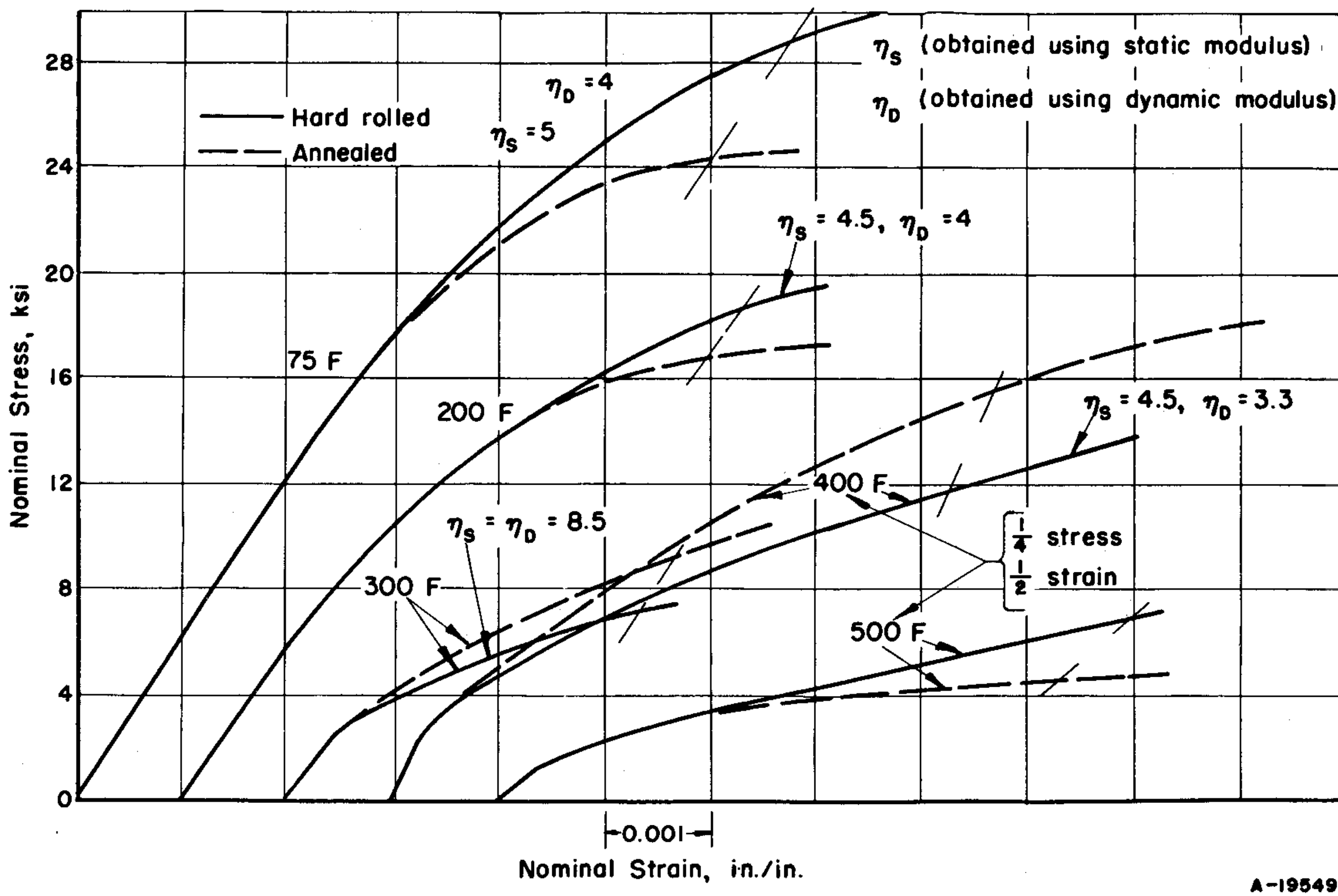

FIGURE 2. TENSILE STRESS-STRAIN CURVES AT VARIOUS TEMPERATURES FOR AZ-31 IN THE HARD-ROLLED CONDITION AND IN THE ANNEALED CONDITION

TABLE 2. TENSILE PROPERTIES UP TO 500 F OF AZ-31 IN THE HARD-ROLLED (CONDITION H) AND ANNEALED CONDITIONS

Test Condition	Temperature, F	Hardness, Rockwell E	0.2% Offset Yield Strength, ksi		Ultimate Tensile Strength, psi	Elongation in 2 Inches, per cent
			Static E	Dynamic E		
Hard rolled	75	80	28,700	28,500	37,050[a]	22[a]
Annealed	75	65	24,400	24,300	>35,000[a]	--
Hard rolled	200	--	18,600	18,400	29,750[a]	30[a]
Annealed	200	--	16,700	16,600	23,500[a]	37[a]
Hard rolled	300	--	7,100	7,000	21,850	75
Annealed	300	--	9,300	9,200	16,000[a]	28[a]
Hard rolled	400	--	2,900	2,850	11,200	--
Annealed	400	--	3,850	3,780	12,100	97
Hard rolled	500	--	1,700	1,400	7,100	48
Annealed	500	--	1,150	1,080	6,630	100

(a) These specimens broke in the knife-edge of the extensometer and these values may be low.

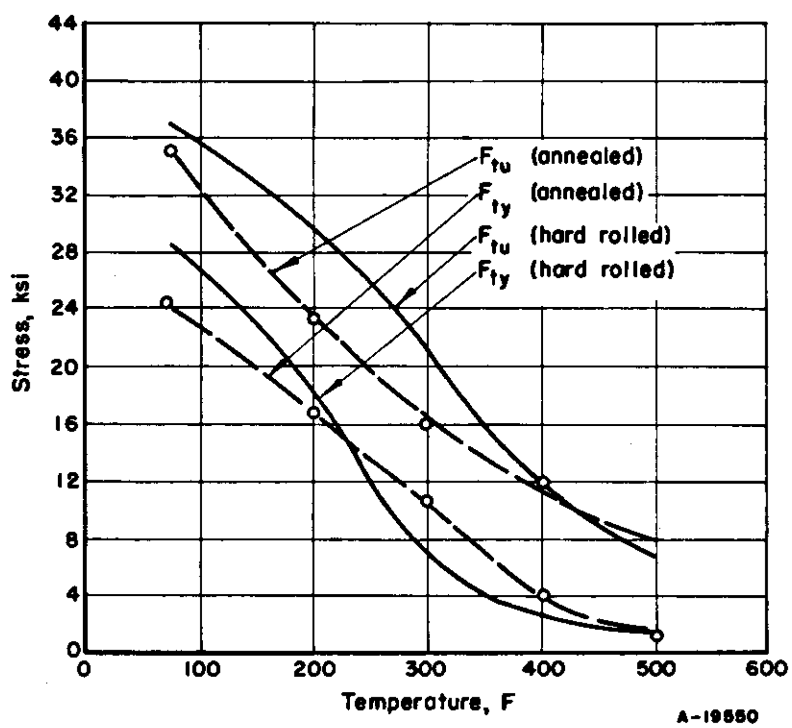

FIGURE 3. YIELD AND ULTIMATE STRENGTH OF AZ-31 VERSUS TEMPERATURE

Tension tests on 2024-T4 were made on specimens immediately upon reaching the test temperature, and also after 15 hours of exposure at test temperature. The results of these tests are presented in Table 3. The stress-strain curves are plotted in Figure 4 and the ultimate and yield strengths versus temperature are plotted in Figure 5. It is to be noted from Figure 5 that no apparent difference can be observed in the yield and ultimate strengths of this material up to 300 F resulting from time of exposure at test temperature. Above this temperature, the effect of heating for 15 hours results in lowering the yield and ultimate strength when compared to the same values for very-short-time exposure. This presumably results from overaging at these temperatures for 15 hours of exposure.

TABLE 3. TENSILE PROPERTIES UP TO 600 F OF 2024-T4 IMMEDIATELY UPON HEATING TO TEST TEMPERATURE AND AFTER 15 HOURS AT TEST TEMPERATURE

Test Condition	Temperature, F	Hardness, Rockwell A	0.2% Offset Yield Strength, ksi		Ultimate Tensile Strength, psi	Elongation in 2 Inches, per cent
			Static E	Dynamic E		
As received	75	47	51,500	51,500	65,600[a]	12[a]
Upon reaching temperature	200	--	50,000	50,000	62,700	19
After 15 hr at temperature	200	--	49,500	--	63,600	21
Upon reaching temperature	300	--	46,500	46,500	60,000	23
15-hour exposure	300	--	47,900	--	60,700	24
Upon reaching temperature	400	--	44,500	44,500	55,300	15
15-hour exposure	400	--	41,500	41,300	45,300[a]	27[a]
Upon reaching temperature	500	--	30,400	30,000	35,700	15
15-hour exposure	500	--	22,800	22,800	26,100[a]	29[a]
Upon reaching temperature	600	--	14,300	14,250	16,200[a]	15[a]
15-hour exposure	600	--	8,500	8,500	11,100	20

(a) These specimens broke in the knife-edge of the extensometer and these values may be low.

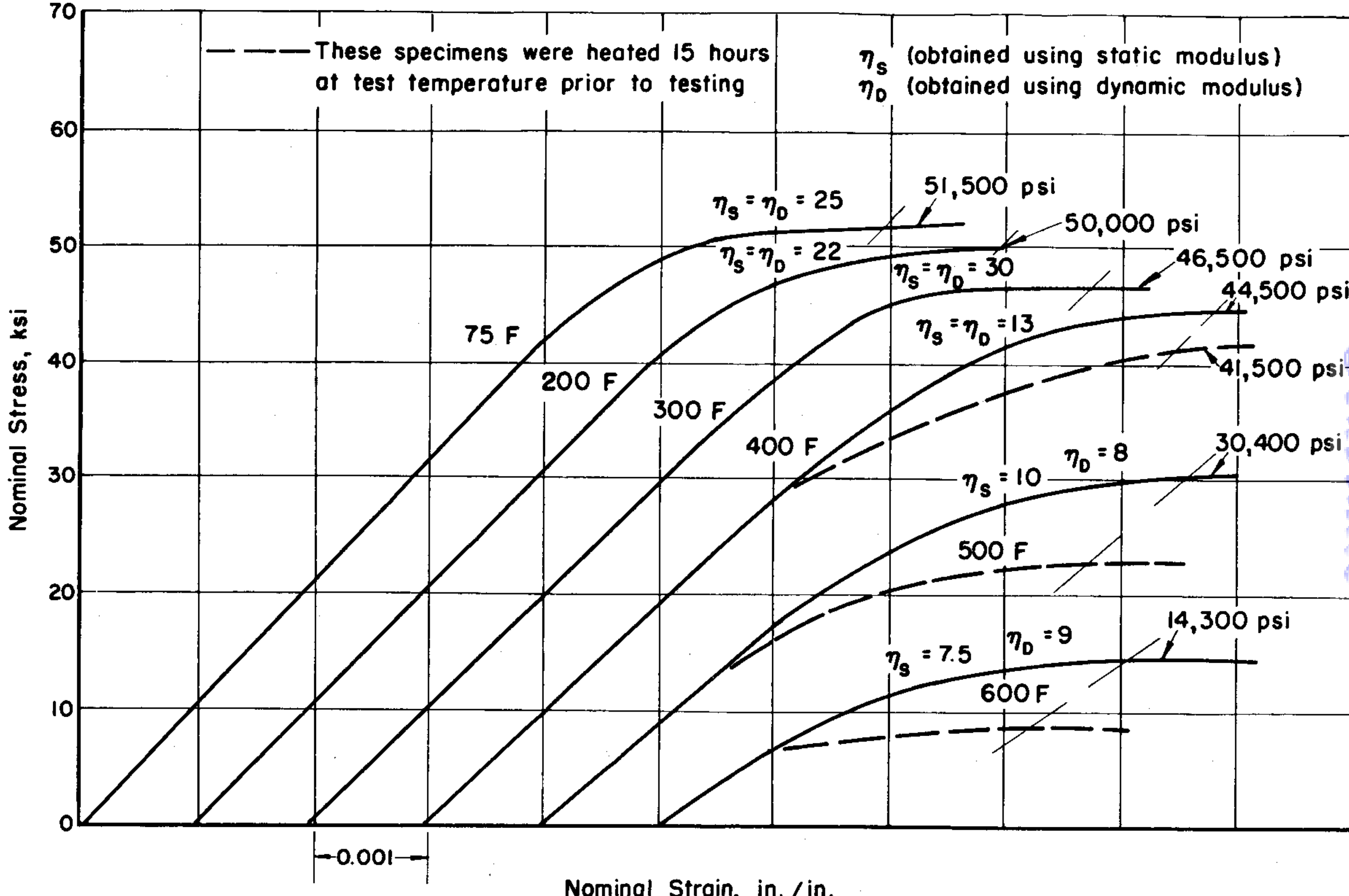

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FIGURE 4. TENSILE STRESS-STRAIN CURVES FOR 2024-T4 ALUMINUM ALLOY AT VARIOUS TEMPERATURES

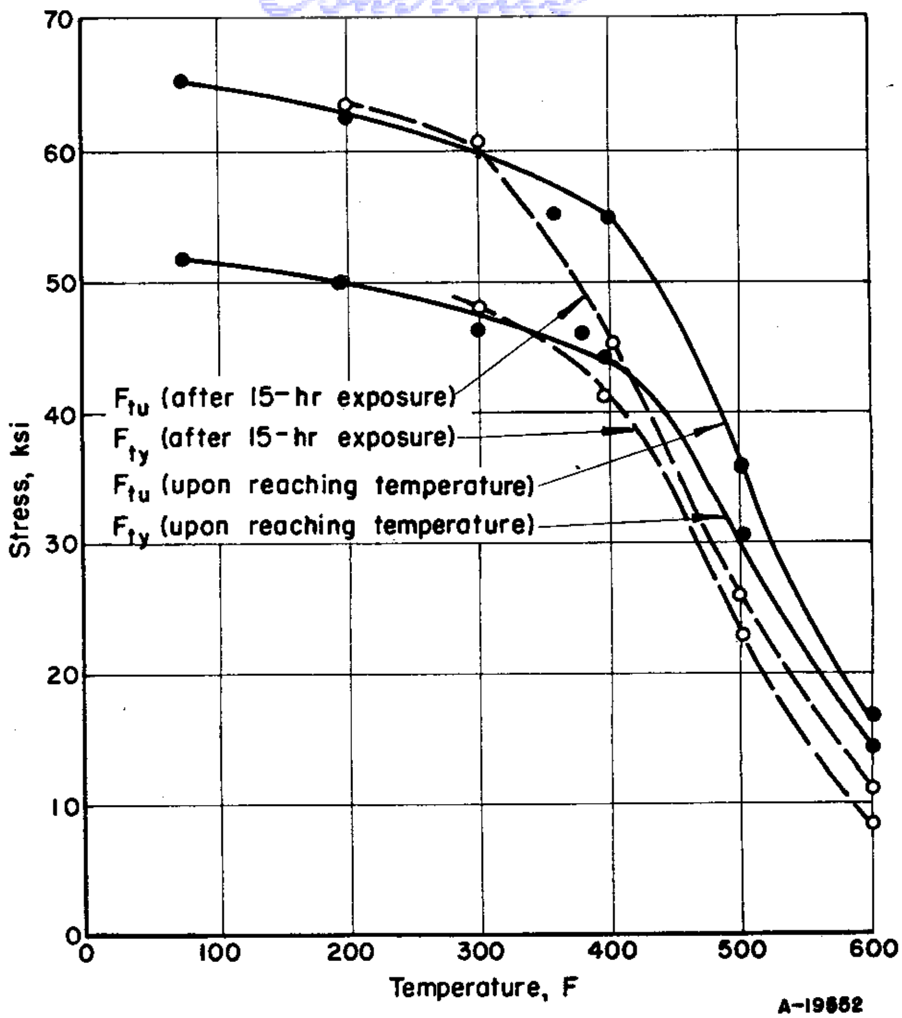

FIGURE 5. YIELD AND ULTIMATE STRENGTH OF 2024-T4 VERSUS TEMPERATURE

DISCUSSION OF RESULTS

Tables 2 and 3 report the 0.2 per cent offset yield strength for both materials tested. It is of interest to note that the yield stress so defined varies very little whether it is determined on the basis of the static or dynamic modulus of elasticity even in the AZ-31 tested at 400 F. Where the dynamic modulus is 3 times the magnitude of the static modulus, the yield stress varies by less than 100 psi. It is also noted that in the range where the static and dynamic moduli vary greatly, the material is unusable for structural application without serious consideration for creep.

If the modulus of elasticity is used in evaluation of the elastic behavior of a composite structure, the approach is valid only when the dynamic stresses are within the elastic range. Where these stresses exceed the elastic behavior of the material, a more rational consideration would involve the secant modulus which defines more accurately the ratio of stress to deformation.

In the range where the static and dynamic moduli differ in the elastic range, it is more nearly correct to use the static modulus unless the dynamic modulus is obtained at a much lower frequency (in the tens of cycles per second).

The third consideration of the modulus of elasticity is in the determination of the critical L/ρ of a column. These values are calculated for both materials and at all test temperatures, using both static and dynamic moduli, assuming that the critical L/ρ is a function of $\sqrt{E}$ as indicated in ANC-5. This is shown in Table 4.

It is to be noted that within the usable structural range of each alloy, the difference in the critical L/ρ is quite small when calculated by either the static or dynamic moduli.

The equation of the stress-strain curves using either the static or dynamic modulus of elasticity is

$$\epsilon = \frac{f}{E} + .002 \left(\frac{f}{F_{cy}}\right)^{\eta} .$$

Reference to the curves in Figures 2 and 4 will show that the variation in the Exponent η is quite small whether η is calculated on the basis of E being the static or dynamic modulus, at least in the temperature range where the material is structurally useful.

TABLE 4. EFFECT OF DIFFERENCE IN STATIC MODULUS AND DYNAMIC MODULUS ON CRITICAL L/ρ AS DEFINED BY $\sqrt{E_d/E_s}$

Temperature, F	AZ-31			2024-T4		
	E_{static}, 10^6 psi	$E_{dynamic}$, 10^6 psi	$\sqrt{\frac{E_d}{E_s}}$	E_{static}, 10^6 psi	$E_{dynamic}$, 10^6 psi	$\sqrt{\frac{E_d}{E_s}}$
Room temperature	5.9	6.1	1.016	10.3	10.3	1.
200	5.5	5.8	1.025	9.9	10.0	1.
300	5.3	5.7	1.038	9.7	9.8	1.01
400	4.8	5.5	1.07	9.3	9.5	1.02
500	1.7	5.3	1.766	8.2	9.2	1.06
600	--	--	--	7.2	8.9	1.11

CONCLUSIONS

Determination of the static modulus for aircraft materials at room temperature and at elevated temperatures offers the most usable information for airframe design. Additional benefits such as tangent modulus and yield stress and ultimate stress are obtained with little additional effort and expense.

Dynamic moduli determined at relatively high frequencies (over 1000 cycles per second) are not applicable to airframe design where they differ from the static modulus and, therefore, should not be considered as necessary design information. However, dynamic tests performed at selected frequencies should furnish simple and rapid indications of the temperatures at which various time-dependent effects occur.

Little error will occur in determining the yield stress from a stress-strain curve whether the static or dynamic modulus is used to get the 0.2 per cent offset, at least for the two materials studied. Also, little error will occur in determining the critical column L/ρ using either modulus, at least in the range of structural usefulness of the material.

Where the modulus is used for determining the deformation of a structure at a given stress, it is more rational to use the secant modulus rather than the elastic modulus. At high temperatures where the static and dynamic moduli differ widely, it will probably be necessary to consider a time-dependent function with the stress-strain curve, since creep will become a critical design criterion.

Since the dynamic modulus is usually determined at very low stresses, its usefulness has limitations in structural design. It is, therefore, recommended that only the static modulus of elasticity be considered for inclusion in the ANC-5 Bulletin. It is considered desirable, however, to indicate the approximate temperature at which the dynamic and static moduli begin to diverge significantly.

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3. Garafalo, F., Malenock, P. R., and Smith, G. V., "The Influence of Temperature on the Elastic Constants of Some Commercial Steels", ASTM STP No. 129 (June, 1952).