CORRELATION OF PRESSURES AROUND A JET ENGINE

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It is now well known that acoustic fatigue failures can occur when a structure is vibrating in a complex manner due to the action of acoustic pressures which are varying randomly in time and space. In considering the vibration of a structure subjected to this type of loading it has been shown in work largely due to Powell (Refs. 1, 2, 3 and 4) that the effective force acting on a structure in a given normal mode of frequency ω has a power spectral density given by:-

$$W_L(\omega) = \int_A \int_A \overline{p(t,a,\omega)\, p(ta'\omega)}\; y(a)\, y(a')\, da\, da'$$

where A = total area of structure
a and a' are area elements
$y(a), y(a')$ are the mode deflections at a and a'

and $\overline{p(t,a,\omega)\, p(ta'\omega)}$ is the correlation between pressures in a narrow frequency band of central frequency ω acting on the area elements a and a'.

To evaluate this integral for a given mode it is therefore necessary to know the spatial correlation of pressures acting on the structure. This information can be obtained experimentally by passing the signal first through pairs of matched filters and correlating the filter output. Space correlograms for a range of frequency bands are shown in Figure 1. These results were obtained by Callaghan, Howes and Coles (Ref. 5) from measurements made along a line parallel to the centre line of a full scale jet from a reference position 17·3 nozzle diameters downstream and 12·1 diameters out from the jet centre line. In this case the filters were approximately a half octave wide. These curves show that as the frequency is increased the distance to first zero crossing is decreased.

Ideally the space correlograms should be obtained in narrow frequency bands which should have a width similar to the width of the resonance peaks in the structural frequency response curves. As a wide range of frequencies may be involved because of the many normal modes of the structure the difficulty of obtaining the necessary data and their presentation becomes a real problem. It is doubtful if 1/2 or 1/3 octave filters are sufficiently narrow but the use of an appreciably narrower filter would make the matching of pairs very difficult and would also require many sets of measurements to be taken to cover the required range of frequencies.

An alternative approach is to measure the cross correlation (space-time) of the overall pressures and transform these by Fourier cosine transformation to give the real part of the cross power spectrum. The cross power spectral density $C(f,x)$ is given by:

$$C(f,x) = 4 \int_0^{\infty} R(x,\tau) \cos 2\pi f\tau \, d\tau$$

where τ = time delay
and $R(x,\tau)$ is the cross correlation of the overall pressures at points separated by a distance x.

It is then possible to read off the space correlogram for any required frequency. Figure 2 shows the cross correlation curves measured near a 2 in. diameter model jet. The reference position is 2 diameters downstream and 5 diameters out from the jet boundary and the traverse line is normal to the jet boundary. The cross power spectra obtained by Fourier transformation on a digital computer are shown in Figure 3 for several spatial separations. The ordinates of the cross power spectra represent the space correlation in a narrow frequency band and thus space correlograms can be drawn for any frequency of interest. The bandwidth in this process is very narrow, being dependent only on the amount of information which is lost when the cross correlation curve is truncated at the point of maximum time delay. Figure 4 shows the space correlogram obtained from these curves and from 1/3 octave filters for a frequency band centred at 2,000 c.p.s. It can be seen that the positions of the zero crossings are approximately the same but that the curve for the narrow frequency band shows less damping.

The next point to be considered is the way in which the space correlograms change with position in the field around a jet engine. In general this region can be divided into three parts:

1. Far field
2. Jet boundary
3. Near field

Far Field

In the case of structure which is situated at a considerable distance away from the jet the noise sources in the efflux can be considered to form a point source. This point source is producing a continuous random pressure fluctuation which is being radiated acoustically into the surrounding medium. At some distant point the structure of interest is being acted upon by a plane wave which is being propagated past it. The trace wave will travel along any particular surface with a convection velocity $u_c = \frac{c}{\sin\theta}$ where θ

is the angle of incidence of the wave on the structural surface. In this travelling plane wave situation the auto-correlogram can be converted into a space correlogram by replacing time t by distance $x = u_c t$. Now as the requirement is for a space correlogram in a narrow frequency band the auto-correlogram for a narrow frequency band must be obtained. A typical auto-correlogram for a $^1/3$ octave band centred at 400 c.p.s. for a measurement in the far field of a full scale jet is shown in Figure 5. The appropriate conversion into space co-ordinates for a traverse line along the direction of propagation of the sound wave is also indicated. It can be seen that this has the form of a lightly damped cosine wave where the damping is a function of the width of the filter. For a very narrow frequency band this damping could be neglected and therefore the resultant auto-correlogram is simply:

$$R(\tau) = \cos \omega \tau$$

and hence $$R(x) = \cos \omega \frac{x}{u_c}$$

Thus in this far field case the space correlogram can be calculated directly.

Jet Boundary

Close to the boundary of the jet the pressures are influenced considerably by moving hydrodynamic pressures associated with the jet efflux. In these conditions it is reasonable to assume that over short distances the pressure field is essentially a convected field and therefore the space and auto-correlograms can be related through the convection velocity.

Full scale and model space correlograms (Refs. 5, 6 and 7) of the overall pressure along the jet boundary are very similar for considerable variations in jet velocity. The reason for this is that although the convection velocity changes the peak frequency at a given point also changes. In a typical auto-correlogram the time to the first zero crossing t_0 , is approximately inversely proportional to the peak frequency in the spectrum. For the condition of constant Strouhal number this peak frequency is proportional to jet velocity V. Thus t_0 is proportional to $\frac{1}{V}$. Now converting this to a space correlogram the distance to the first zero crossing, x_0, is proportional to $t_0 V$. Therefore to this order of approximation the distance, x_0, to the first zero crossing is independent of jet velocity.

When narrow band space correlograms are studied it can be seen that the idea of a fixed convection velocity breaks down. The reason for this appears to be that for a given position the lower frequencies are associated with slower moving eddies towards the outside of the mixing region and the higher frequencies are associated with the faster moving eddies towards the centre of the jet. Figures 6 and 7 show 1/3 octave band space correlograms along the boundary for different jet velocities at a reference position 10·8 diameters downstream of a 2" model jet for frequency bands having central frequencies of 500 c.p.s. and 1,000 c.p.s. For the lower speed of 750 ft per sec. the apparent convection velocity of the 500 c.p.s. component is 0·54 V and that of the 1,000 c.p.s. component is 0·87 V where V is the velocity at the centre line of the jet. It is apparent therefore that the appropriate convection velocity can vary considerably and will depend on the position downstream and the frequency component being considered.

Near Field

The region which is usually of most interest from the structural response point of view is the region lying in between the two limits discussed above. In this region the jet forms a complex system of sources distributed over distances comparable to the distance of the point from the jet. For points on a horizontal plane through the axis of a full scale jet engine contours of equal correlation can be drawn as shown in Figure 8. These iso-correlation lines are curved and take on an elliptical form of appreciable eccentricity because correlation lengths parallel to the jet boundary are considerably greater than those normal to the boundary.

The effect of jet velocity on the space correlograms is not as great in this case as in the case of the jet boundary conditions but is not insignificant, as would be the case in the far field. Figure 9 shows a space correlogram for a $^1/3$ octave frequency band centred at 1,000 c.p.s. for a position 2 diameters downstream and 5 diameters out from the boundary of a 2 in. model jet. It can be seen that there is a small change in the spatial scale as the jet velocity is increased.

The iso-correlograms of the type shown in Figure 8 give some indication of the direction of propagation of the majority of the noise from the jet. If such curves could be obtained in filter bands it should be possible to trace back to some source area of a given band of frequencies in the mixing region. This however is a prohibitive procedure in terms of length of time involved in data processing. Alternatively the frequency band cross correlations of the pressures picked up by two microphones can be used to determine the convection velocity of the apparent wave front of a particular band of frequencies along the line joining the two microphones. Figure 10 shows the 2,000 c.p.s. filter band cross correlation curves for a position 2 diameters downstream and 5 diameters out from the boundary of a 2 in. model jet for different spatial separations of the microphone along a line normal to the jet boundary. The time delay required to recover maximum correlation between two points gives the convection velocity of the pressure wave between the points and hence the angle of incidence of the wave on this line can be determined. The drop in the maximum correlation recoverable with separation is a function of the distribution of the sources in the mixing region.

As a result of these tests it is hoped to be able to put a mathematical model to the source areas which will enable space correlograms in the near field to be estimated.

References

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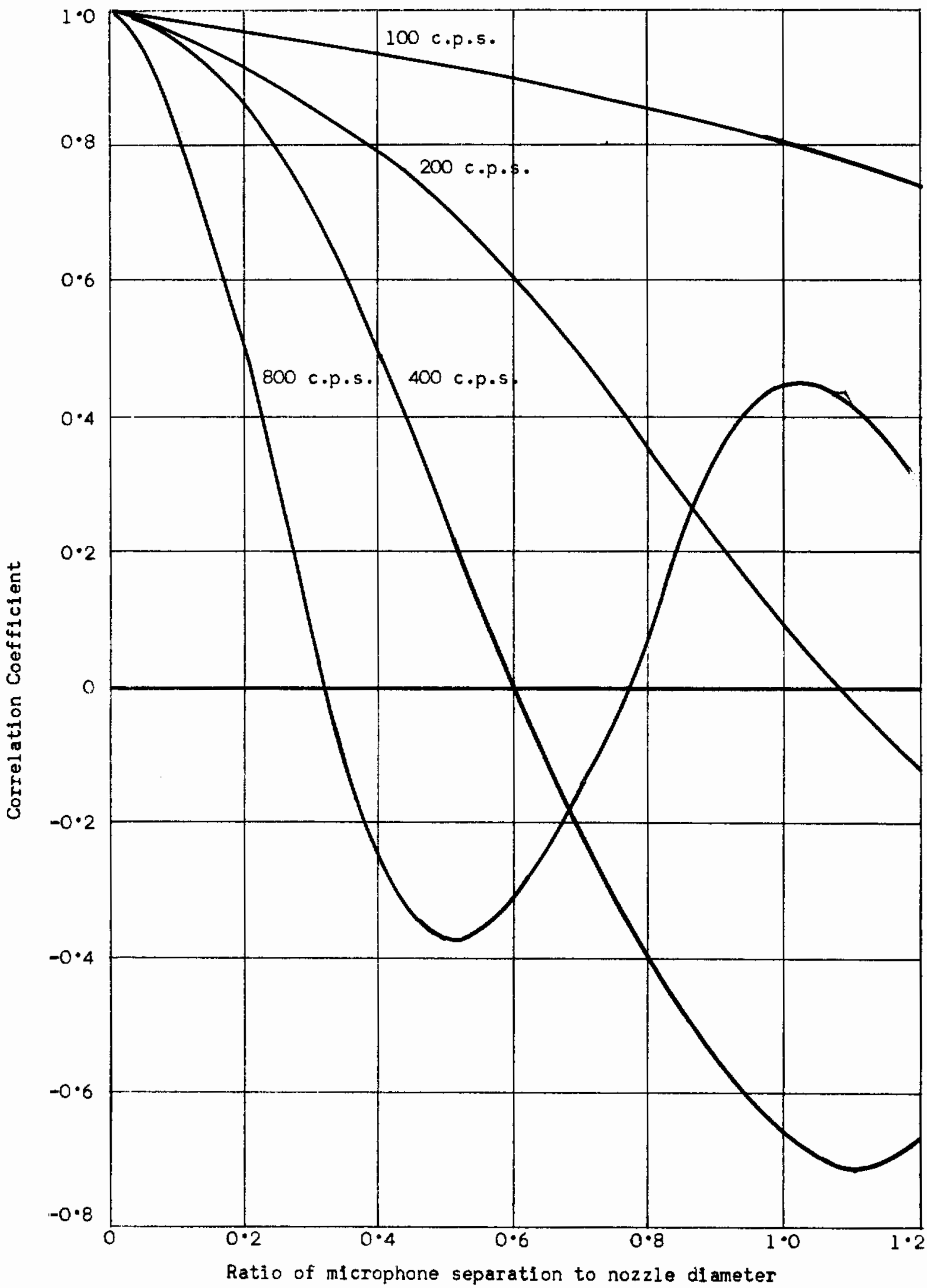


Figure 1 - Longitudinal sound pressure correlation in various frequency bands (Reference 5).

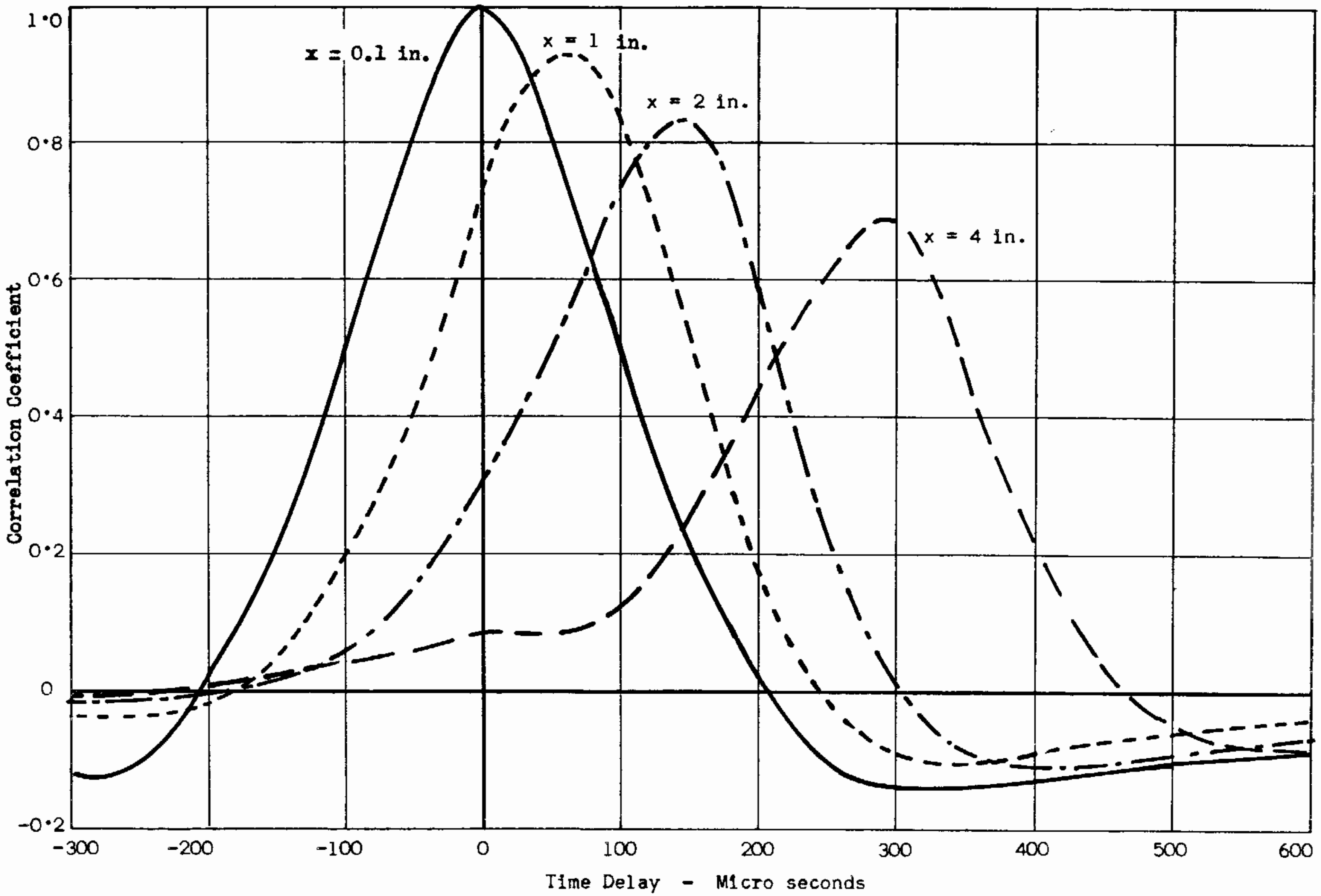


Figure 2 - Cross Correlation of overall sound pressure at position 2 diameters downstream and 5 diameters out from boundary of a 2 in. model jet, traversing normal to boundary. Velocity 860 ft/sec.

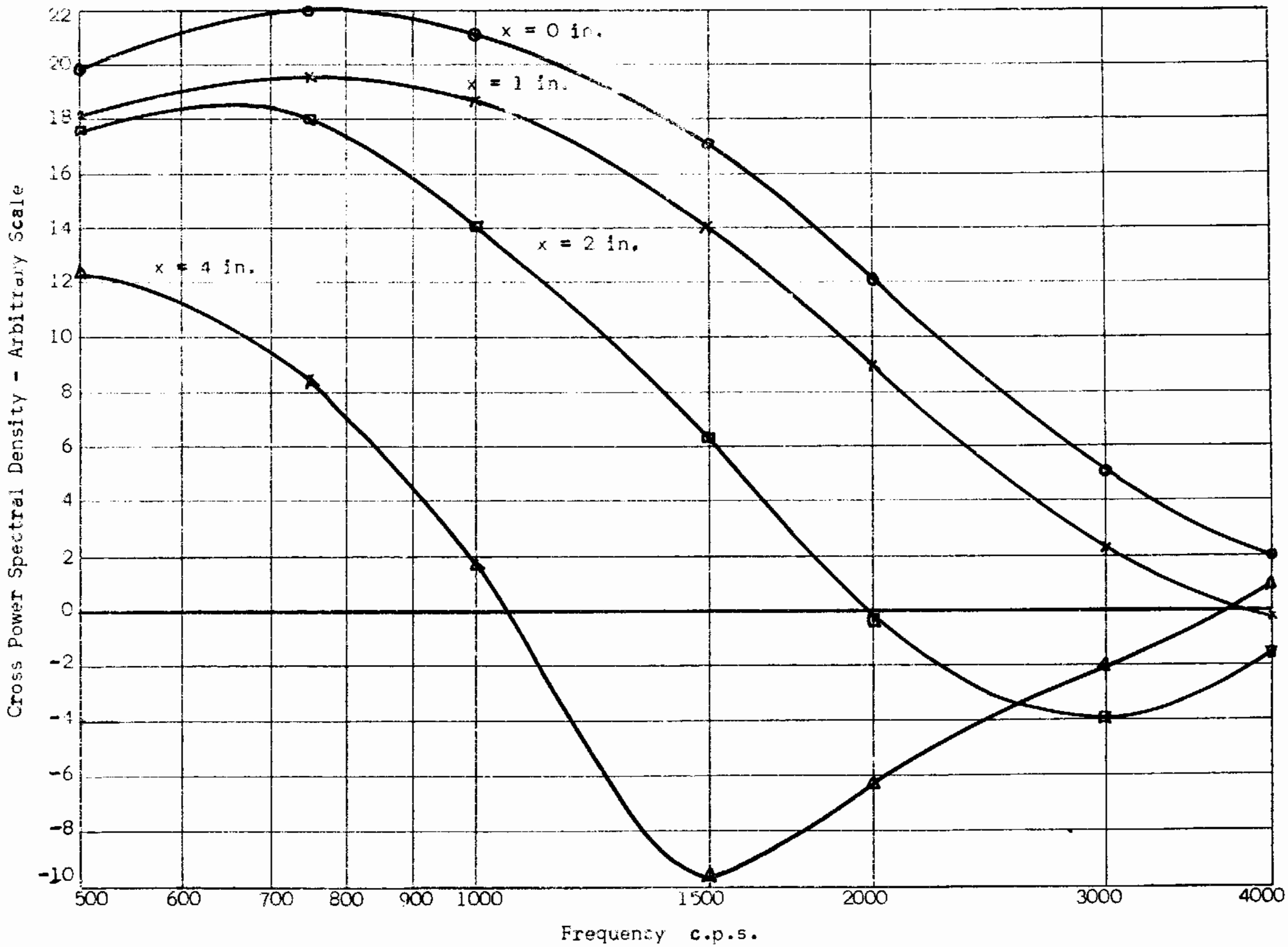


Figure 3 - Cross Power Spectrum of overall pressure at Reference Position 2 diameters downstream, 5 diameters out from boundary of 2 in. model jet, traversing normal to the boundary. 860 ft/sec.

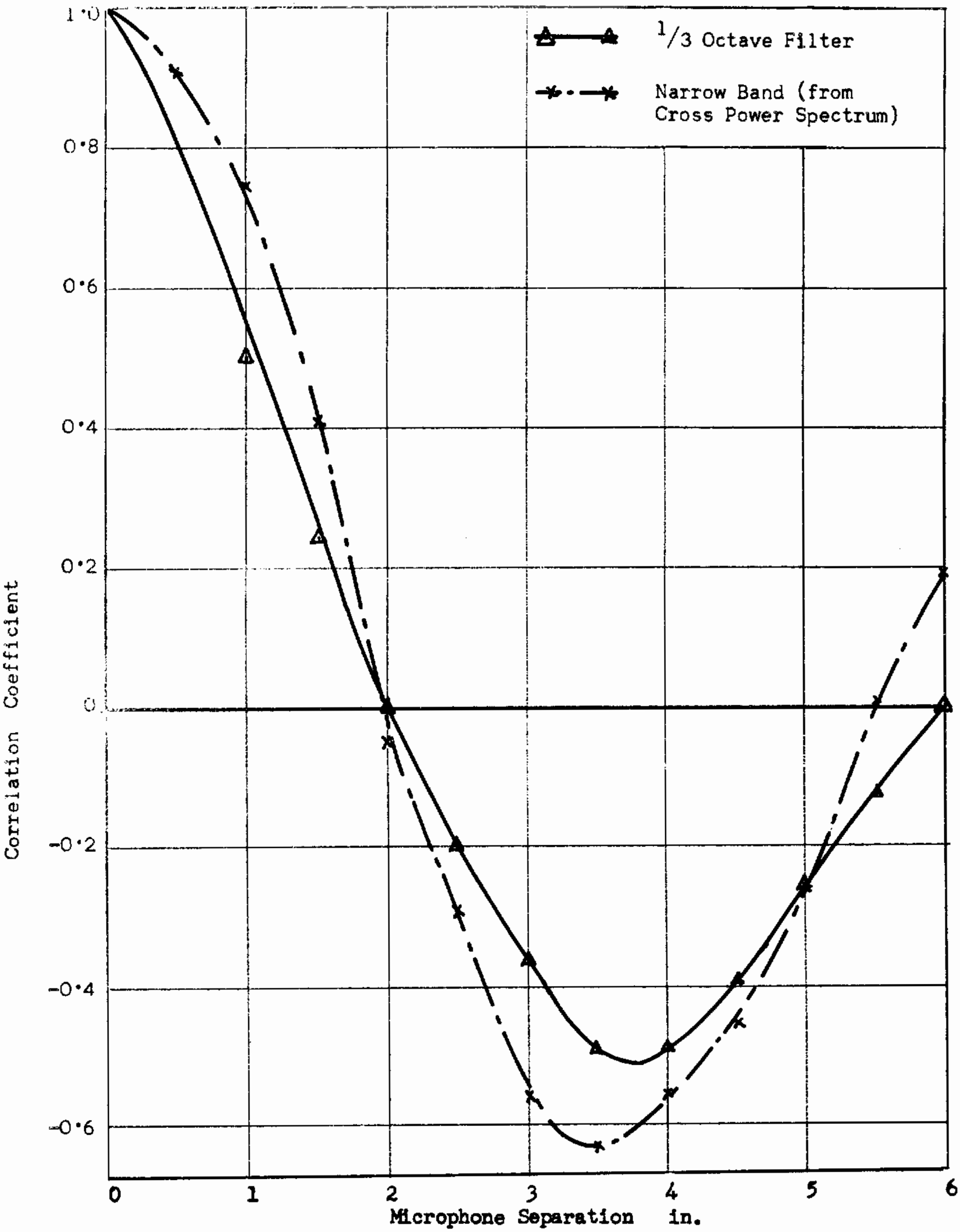


Fig. 4 - Effect of Bandwidth on Space Correlogram. Centre frequency 2,000 c.p.s. Ref. position 2 diameters downstream 5 diameters out from boundary of 2-in. model jet, traversing normal to jet boundary. 860 ft/sec.

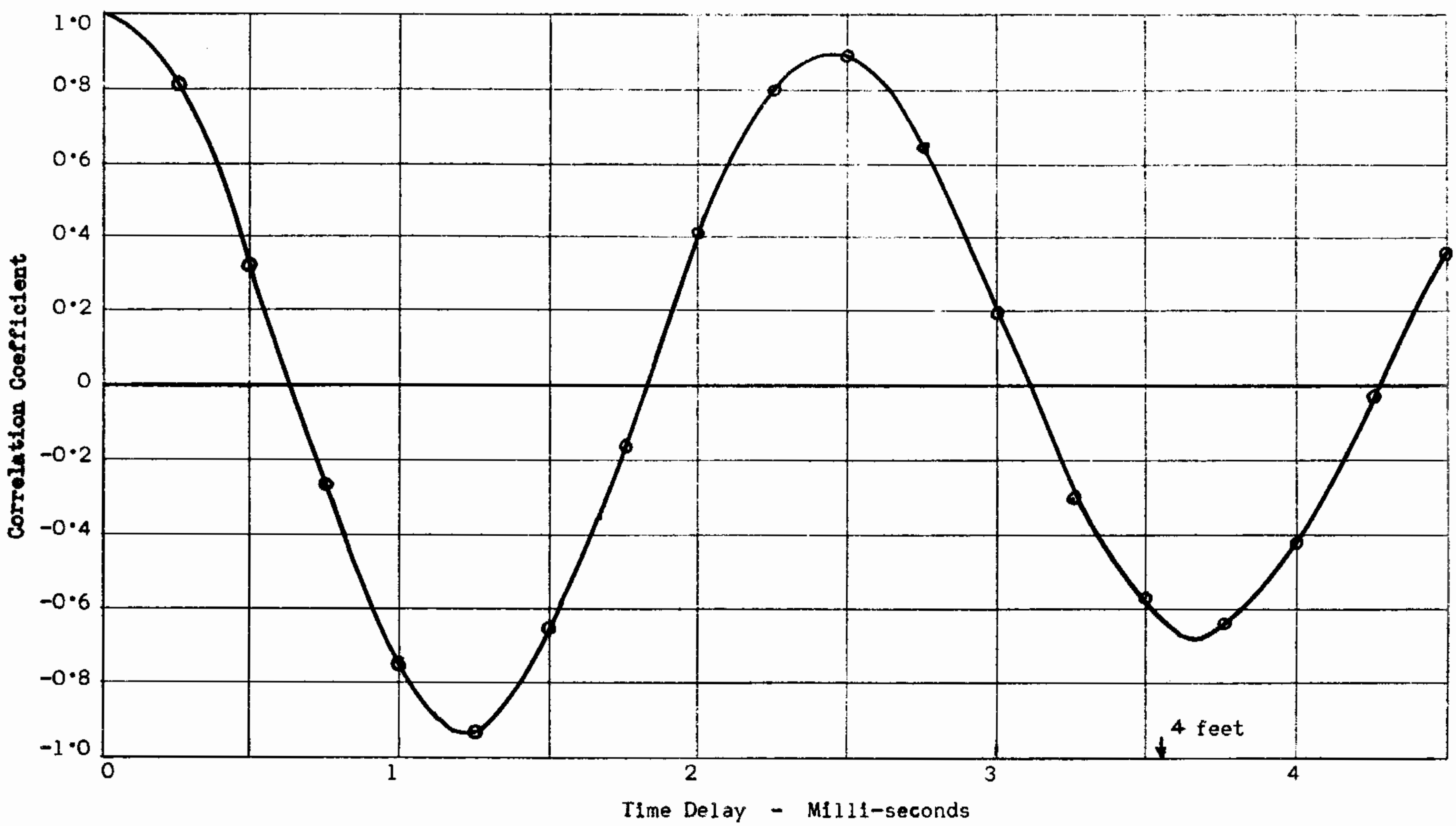
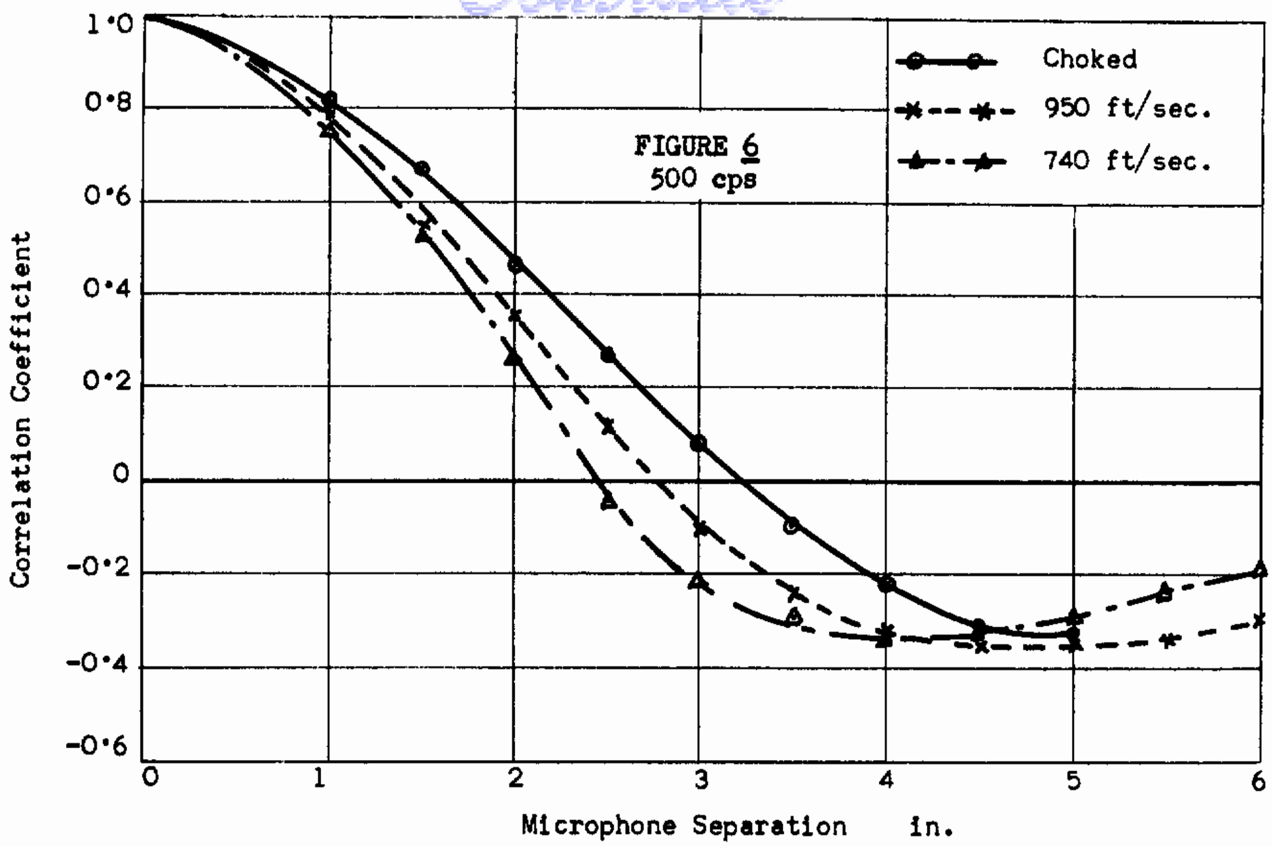
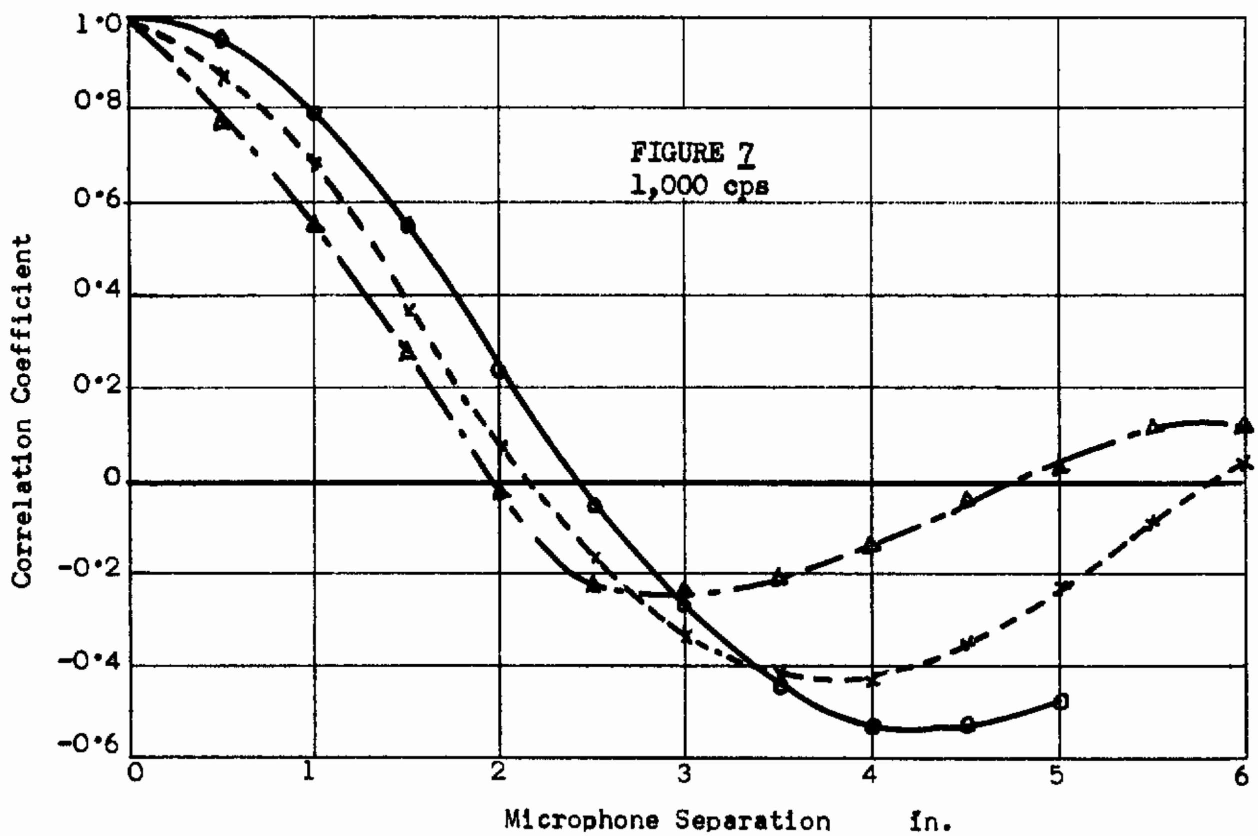


Figure 5 - Auto-correlogram of filtered far field Noise, centre frequency 400 c.p.s.





Figures 6 and 7 - Effect of Jet Velocity on Filtered Space Correlograms Along Boundary of 2-in. Model Jet. Ref. Position 10.8 Diameters Downstream

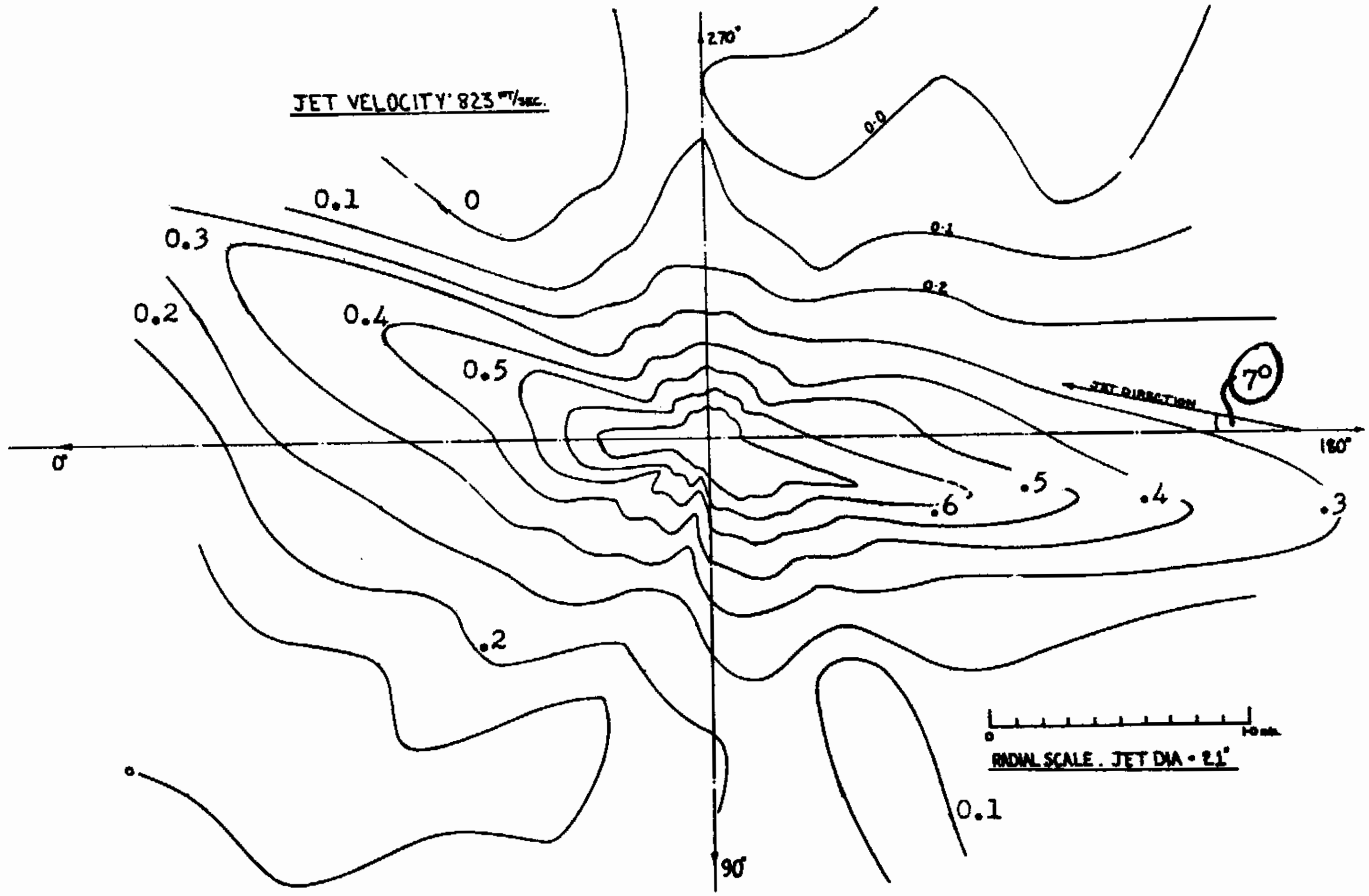


Fig. 8 - Iso-Correlation Contours for Full Scale Jet. Ref. Position 2 Diameters Downstream, 5 Diameters Out From Jet Boundary

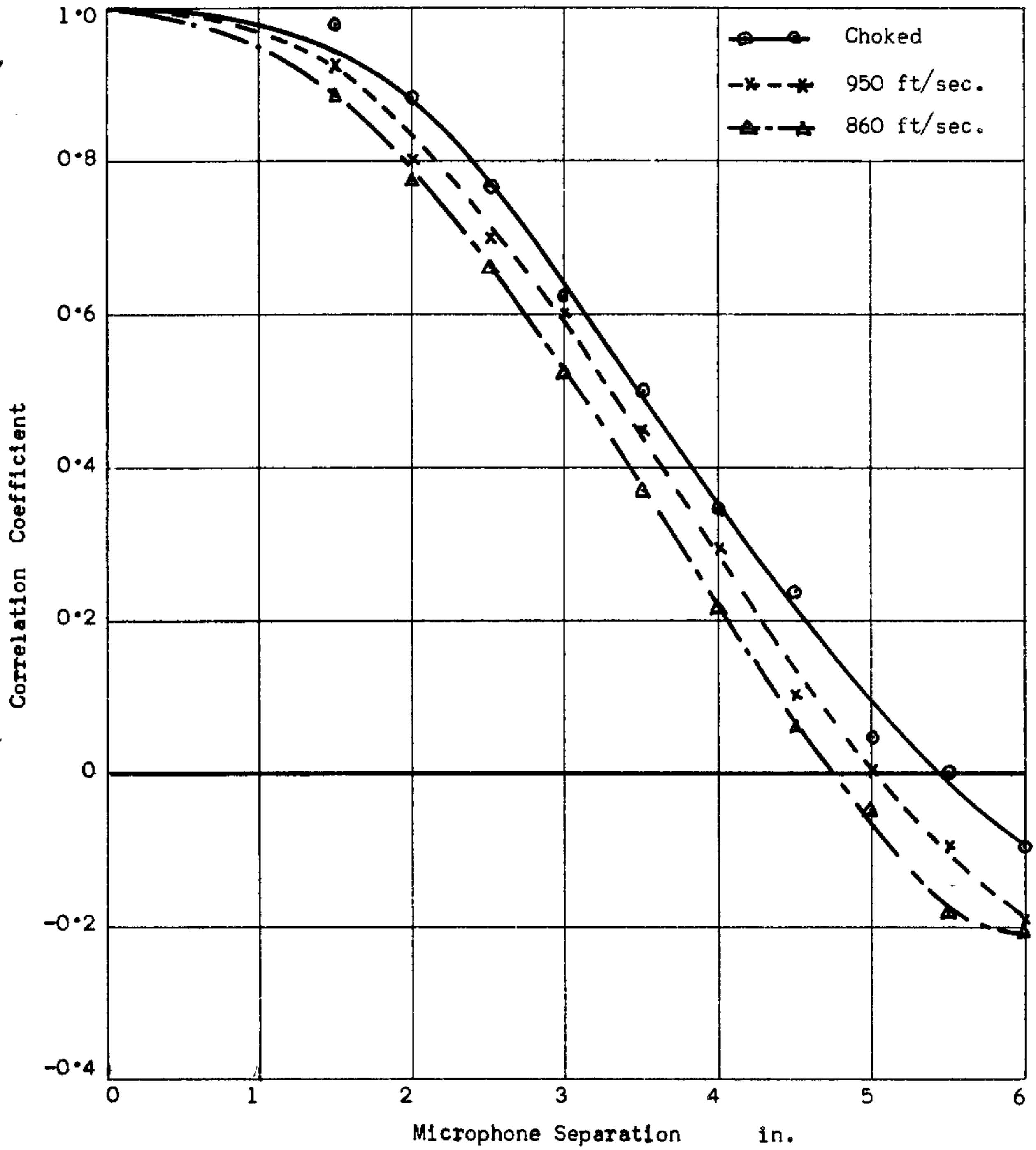
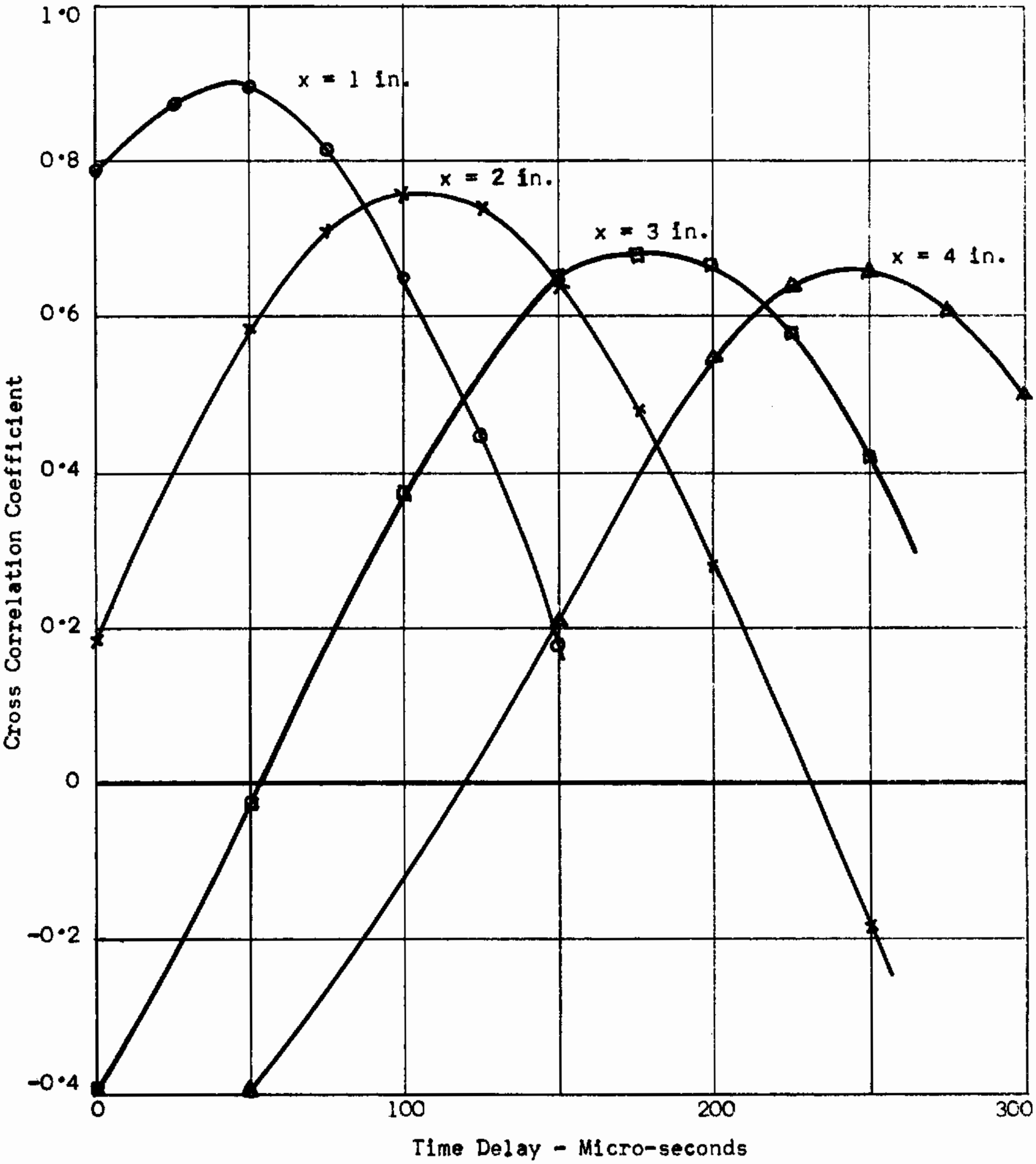


Figure 9 - Effect of Jet Velocity on Filtered Space Correlogram (1,000 c.p.s.) at position 2 diameters downstream and 5 diameters out from jet boundary.



<u>Figure 10</u> - Cross Correlation of filtered noise (2,000 c.p.s.) Reference position 2 diameters downstream, 5 diameters out from boundary of 2 in. model jet (velocity 950 ft/sec). Traversing normal to jet boundary.