

AFCRL 203
Cleared: November 3rd, 1971
Clearing Authority: Air Force Cambridge Research Laboratories

GRD Research Notes
No. 50

AIDS FOR COMPUTING STRATOSPHERIC MOISTURE

Murray Gutnick

January 1961

Project 8624
Task 86241

Meteorological Development Laboratory
GEOPHYSICS RESEARCH DIRECTORATE
AF CAMBRIDGE RESEARCH LABORATORIES
AIR FORCE RESEARCH DIVISION (ARDC)
UNITED STATES AIR FORCE
Bedford, Mass.

Contrails

ABSTRACT

Results of experiments in measuring stratospheric humidity have been presented by various investigators in many different units. This note contains charts and tables to facilitate the time-consuming conversion of these various moisture parameters to any desired common base.

Contrails

CONTENTS

Section	Page
1. Purpose	1
2. Explanations	1
3. Examples	2
4. Symbols and Subscripts	4
5. Definitions	4
Acknowledgments	15
References	17

Contrails

AIDS FOR COMPUTING STRATOSPHERIC MOISTURE

1. PURPOSE

Conversion of various reported measurements of stratospheric humidity to a common base is necessary before such measurements can be compared. Scientists from different fields use widely differing instruments and procedures. Some report humidity in terms of frost point (F or C); others use mixing ratio or absolute humidity. Some refer to English or metric heights; others, to pressures. For rapid conversion of such readings to a common base and to have, under one cover, reference material for stratospheric moisture computations, the formulae, tables, and charts contained in this publication have been prepared.

There is yet another purpose. Not only the amount of water vapor in the stratosphere, but the variation with altitude of the various moisture parameters (mixing ratio, frost point, etc.), has proven to be a highly controversial subject.^{1, 2, 3} Plotting an ascent on the charts presented herein shows at a glance the variation with altitude of the frost point, mixing ratio, vapor pressure, and absolute humidity.

2. EXPLANATION

The two charts (Figs. 1 and 2) differ only in the scale of the ordinate. That of Fig. 1 has pressure on an approximate logarithmic scale and the corresponding ICAO Standard Atmosphere heights⁴ on a linear scale (in kilometers at the left and in thousands of feet at the right). The ordinate of Fig. 2 has pressure on a linear scale and the corresponding ICAO Standard Atmosphere heights on an approximate exponential scale.

Although both charts can be used interchangeably, interpolation is easier on Fig. 1 when only the altitude is known and Standard Atmospheric pressure for the altitude is assumed. Figure 2 should be used when the pressure is known. Errors can occur when Standard Atmospheric pressure is assumed for a given height, since actual pressures can differ markedly from those of the Standard Atmosphere.

(Author's manuscript approved 19 December 1960.)

Contrails

For example, the Standard Atmosphere pressure at 50,000 feet is 116.6 mb; yet at this same altitude at 40°N in January over North America the pressure exceeds 118 mb some 16 percent of the time, and it is less than 110 mb about 16 percent of the time (it lies outside the 110 to 118-mb range 32 percent of the time).

Both charts have for their abscissa the mixing ratio on a logarithmic scale. Slanting lines on the charts are labelled in units of vapor concentration or absolute humidity (milligrams per cubic meter) at the top, in frost point (both Celsius and Fahrenheit) in the middle, and in units of vapor pressure (microbars or thousandths of a millibar) at the bottom. The simplified formula for mixing ratio, w , in terms of saturated vapor pressure, over ice, e_{si} , and air pressure, p , was used to compute the lines: $w_1 = 622 e_{si}/p$.

These slanting lines were determined by computing the mixing ratio corresponding to pairs of values of pressure and vapor pressure; the latter were chosen to give fixed values of frost point. Lines are drawn for frost points at 10C-degree intervals with intervening values indicated by tick marks between them. For convenience, vapor pressures and absolute humidities for each frost point are given in Table 1.

Table 2 presents the relative humidity (U) as a function of the ambient temperature and frost-point depression. These tables were derived from the equation $U = 100 e/e_{si}$ for various combinations of e and e_{si} corresponding to known temperatures and frost points.

All computations in the tables and on the charts are based on saturation with respect to ice, not water, and hence do not extend above -40°C, the approximate lower limit for supercooled water droplets. All values are obtained from the Smithsonian Meteorological Tables,⁵ except that vapor pressures for frost points below -100°C were computed from the Goff-Gratch formula.

3. EXAMPLES

The proper use of the charts and tables is best illustrated by the following examples:

Contrails

- Q. Find the mixing ratio at 27 km given a frost point of -40°C and assuming standard atmospheric pressure.
- A. A frost point of -40°C corresponds to a vapor pressure 128×10^{-3} mb and an absolute humidity of 119×10^{-3} gm/m³. On the left-hand scale of Fig. 1, 27 km corresponds to 88,704 feet and a standard atmospheric pressure of 18.5 mb. This horizontal line intercepts the sloping line labelled -40°C at a mixing ratio (on the abscissa) of 4.1 gm/kg.
- Q. Find the frost point, given a mixing ratio at 140 mb of 0.0116 gm/kg.
- A. On Fig. 2, the intersection of 0.0116 gm/kg (abscissa) with 140 mb (ordinate) gives -70°C (sloping line).
- Q. At 100 mb the ambient temperature is -50°C and the mixing ratio is 0.067, what is the relative humidity?
- A. From Fig. 2 we find that the mixing ratio of 0.067 at 100 mb yields a frost point of -60°C . Subtracting the frost point from the ambient temperature (-50°C minus -60°C) yields a frost-point depression of 10C degrees. Going to Table 2 and using the intersection of the -50°C temperature with a frost-point depression of 10C^o yields a relative humidity of 27 percent.

4. SYMBOLS AND SUBSCRIPTS

Symbols and subscripts used in this Note are as follows:

SYMBOLS		SUBSCRIPTS	
Absolute Humidity	ρ_w	Dew or Frost Point	d
Atmospheric Pressure	p	Ice	i
Correction Factor	f	Saturation	s
Gravitational Acceleration	g	Water Vapor	w
Length	L		
Mass	M		
Precipitable Water	W		
Specific Humidity	q		
Ratio	K		
Relative Humidity	U		
Temperature	θ		
Vapor Pressure	e		
Volume	V		

5. DEFINITIONS

Commonly used formulae and/or definitions for stratospheric humidity computations were extracted from the Smithsonian Meteorological Tables⁵ unless otherwise noted.

Absolute Humidity or Vapor Concentration (ρ_w): The density of water vapor and dry air is defined as the ratio of the mass of water vapor (M_w) to the volume (V) occupied by the mixture: $\rho_w = M_w/V$.

Dew or Frost-Point Temperature (θ_d): The temperature to which a given parcel of air must be cooled at constant pressure and constant water-vapor content for saturation to occur. A dew point below 0°C is called the frost point.⁶ It is related to the mixing ratio and pressure by $e_i(\theta_{di}) = p w_i / (0.62197 + w_i)$.

Mixing Ratio (w): In general, for w_i in g/kg and e_{si} and p in mb,

$$w_i = \frac{0.62197 f_i e_{si} \times 10^3}{p - f_i e_{si}} \quad .$$

Contrails

Here f is a correction factor for the departure of the mixture from the ideal gas laws. Since f_1 ranges from 1.0001 at -100°C and 5 mb to 1.0061 at -40°C and 1100 mb, and since in the stratosphere $e_{\text{sl}} \ll p/1000$, a valid approximation is $f_1 e_{\text{sl}} = p$. Thus the relation can, in practice, be reduced to $w_1 = 622 e_{\text{sl}}/p$.

Specific Humidity (q): In a system of moist air the ratio of the mass of water vapor to the total mass of the system. For many purposes it may be approximated⁶ by the mixing ratio w :

$$q = w/(1 + w).$$

Precipitable Water (W): The precipitable water W in the layer between pressures p_1 and p_2 is given by any of the three formulas^{2, 6, 7} involving the mixing ratio $w(p)$ at pressure level p , the relative humidity U , or the absolute humidity ρ_w :

$$W = \frac{1}{g} \int_{p_1}^{p_2} w(p) dp ;$$

$$W = 10^3 K e_s U \Delta p/gp ;$$

$$W = \rho_w \times 10^{-4} L .$$

Here K is the ratio of the molecular weight of water vapor to that of dry air, p and e_s are in mb, W is in cm, U is in percent, g is in cm/sec^2 , ρ_w is in gm/m^3 , and L is the path length in meters.

Relative Humidity (U): Strictly speaking, relative humidity is defined as the ratio of the mol fraction of water vapor in a mass of air to the mol fraction of water vapor in a mass of air saturated with water vapor at the same barometric pressure and temperature. However, for practical purposes, U is equivalent⁶ to $100 w/w_s$ or $100 e/e_s$.

Contrails

Vapor Pressure (e): The saturation vapor pressure with respect to ice (e_{si}) at pressure p and temperature θ is

$$e_{si} = p w_i / (0.62197 + w_i) \ .$$

Table 1. Vapor Pressure and Absolute Humidity of Water Vapor Saturated with Respect to Ice. (All values extracted from Smithsonian Meteorological Tables)

°C	microbars	g / m ³	°C	microbars	g / m ³
-41	111.500	0.106 90	-71	2.252 00	0.002 414
-42	102.100	0.095 70	-72	1.936 00	0.002 085
-43	90.980	0.085 65	-73	1.662 00	0.001 799
-44	80.970	0.076 56	-74	1.425 00	0.001 550
-45	71.980	0.068 36	-75	1.220 00	0.001 334
-46	63.930	0.060 98	-76	1.042 00	0.001 145
-47	56.710	0.054 33	-77	.869 40	0.000 982
-48	50.260	0.048 37	-78	.757 70	0.000 841
-49	44.490	0.043 01	-79	.644 40	0.000 719
-50	39.350	0.038 21	-80	.547 20	0.000 614
-51	34.760	0.033 90	-81	.463 80	0.000 523
-52	30.670	0.030 05	-82	.392 50	0.000 445
-53	27.030	0.026 50	-83	.331 60	0.000 378
-54	23.800	0.023 53	-84	.279 60	0.000 320
-55	20.920	0.020 78	-85	.235 30	0.000 271
-56	18.380	0.018 34	-86	.197 70	0.000 229
-57	16.120	0.016 16	-87	.165 80	0.000 193
-58	14.130	0.014 23	-88	.138 80	0.000 162
-59	12.360	0.012 51	-89	.116 00	0.000 136
-60	10.800	0.010 98	-90	.096 72	0.000 114
-61	9.432	0.009 63	-91	.080 49	0.000 096
-62	8.223	0.008 44	-92	.066 85	0.000 080
-63	7.159	0.007 36	-93	.055 42	0.000 067
-64	6.225	0.006 45	-94	.045 84	0.000 055
-65	5.406	0.005 63	-95	.037 84	0.000 046
-66	4.688	0.004 90	-96	.031 17	0.000 038
-67	4.060	0.004 27	-97	.025 61	0.000 032
-68	3.511	0.003 71	-98	.021 01	0.000 026
-69	3.032	0.003 22	-99	.017 19	0.000 021
-70	2.615	0.002 79	-100	.014 03	0.000 018

1 microbar = one thousandth of a millibar (10⁻³ mb)

Contrails

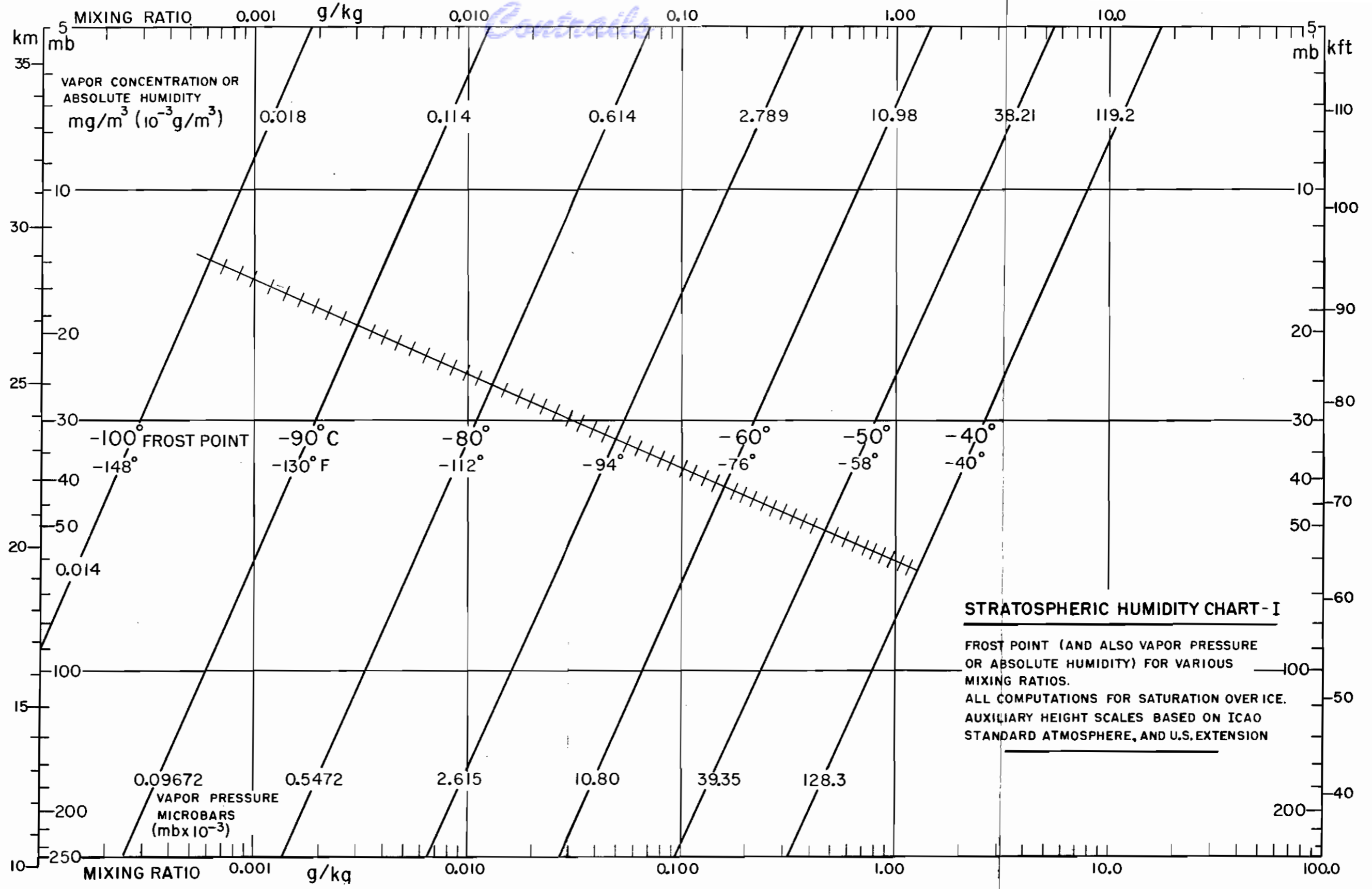


Figure 1.

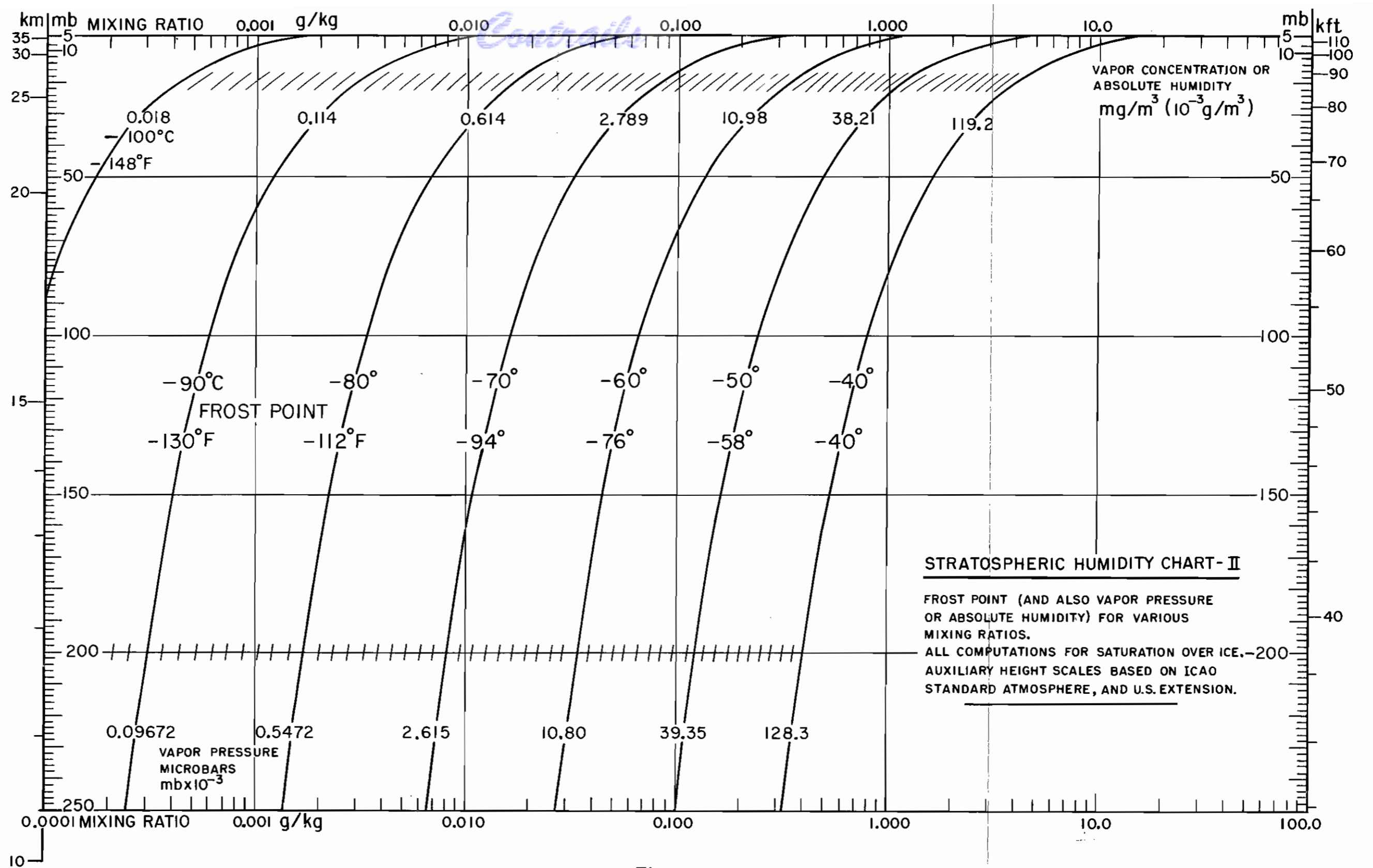


Figure 2.

ACKNOWLEDGMENTS

Members of the Applied Climatology Branch of the Meteorological Development Laboratory offered many helpful suggestions and constructive criticisms. Personnel of the Mathematics Department of Regis College, Weston, Massachusetts, provided diligent computational assistance.

Contrails

REFERENCES

1. Greenfield, S.M. and W. W. Kellog, "Calculations of Atmospheric Infrared Radiation as Seen from a Meteorological Satellite," Journal of Meteorology 17: 283-290 (June 1960).
2. Gutnick, M., "An Estimate of Precipitable Water Along High-Altitude Ray Paths," Air Force Surveys in Geophysics No. 120, Geophysics Research Directorate, Bedford, Mass., (March 1960).
3. Sissenwine, N., and M. Gutnick, "Precipitable Water Along High-Altitude Ray Paths," Proceedings of the Infrared Information Symposia, 5: 5-12 (July 1960).
4. Minzner, R. A., W. S. Ripley, and T. P. Condron, "U. S. Extension to the ICAO Standard Atmosphere," GRD and U.S. Weather Bureau, Washington, D.C., (1958).
5. Smithsonian Meteorological Tables, Sixth Revised Edition, Smithsonian Institution, Washington, D.C., (1951).
6. Glossary of Meteorology, American Meteorological Society, Boston, Mass. (1959).
7. Reitan, C.H., "Mean Monthly Values of Precipitable Water Over the United States," Monthly Weather Review, 88: 25-35 (January 1960).